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GENERALIZED BANACH CONTRACTION MAPPING PRINCIPLE IN GENERALIZED METRIC SPACES WITH A TERNARY RELATION

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Abstract. In the present work the use of ternary relations is introduced in fixed point theory to obtain some fixed point results in G-metric spaces. Amongst several generalizations of metric spaces suggested in recent times, G-metric spaces are the ones in which the metric is replaced by a function through which sets of three elements are assigned to non-negative real numbers. A ternary relation is assumed on the space and a generalized contractive condition is assumed for the triplets of elements related by the ternary relation. Fixed point and related results are established for such contractions as generalization of contractive mapping principle. The case without the assumption of ternary relation on the space is also discussed. There are some corollaries and illustrative examples. The illustrations establish the actuality of the generalization. The methodology of the proofs are new in the context of G-metric spaces.

1 Introduction and Definitions

Banach's contraction mapping principle is a well known result of functional analysis which is established in the general setting of complete metric spaces and has served as the basis of many fundamental results in mathematical studies [19]. It has been extended to several other spaces which are generalizations of metric spaces like cone metric spaces [18], fuzzy metric spaces [15], probabenumerateic metric spaces [25] and the like. Generalizations of these extensions have also been successfully attempted in the corresponding spaces. A particular extension of metric space is generalised metric space or G-metric space [22] where a three variable non-negative realvalued function replaces the ordinary metric which is a function of two variables. Such extensions were also attempted previously which finally culminated in the definition of G-metric space [1]. One of the salients features of G-metric spaces is its Hausdroff topology. This is a reason for a successful extension of metric fixed point theory to the space which has experienced rapid development in recent times. A comprehensive account of this development is given in the book [1]. Some more

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recent references, amongst others, are noted in [2, 3, 4, 6, 7, 8, 10, 11, 12, 13, 14, 26]. Other metric related studies have also been transferred to G-metric spaces. As instances of these works, proximity point problems have appeared in [8] and variational inequality problems have been studied in [16]. It has recently been observed that the fixed point theorems for several contractive mappings, which includes both extension of Banach's contraction as well as other types of contractions, especially discontinuous contractions, do not require in their proofs the contractive conditions to be satisfied for arbitrary pairs of points from the space. Rather the pairs of points requiring the satisfaction of the contraction between them can be restricted to a smaller set. For that purpose partial order relations are introduced in metric spaces and several contractive conditions are assumed for pairs of points conected through the partial order relation. Fixed point theory in partially ordered metric spaces has extensively devloped in recent times through works like those noted in [9, 20, 17, 23, 24].

In the same vein we show that using ternary relations it is possible to obtain fixed point results in G-metric spaces by requring a contractive condition for sets of three points from the space which must be related by the ternary relation. In fact we establish a generalization of the Banach's contraction mapping principle by the use of ternary relation. We also discuss the case of our main theorem without any such ternary relation. There are two examples which demonostrate the actuality of the generalizations. In the following we discuss some technical aspect of the G-metric space and related issues on which we base our discussion in the rest of the paper.

Definition 1 ([22])**.** *Let X be a nonempty set, and let the function $G : X \times X \times X \to [0, \infty)$ satisfy the following properties:*

(G1) $G(x, y, z) = 0$ *if* $x = y = z$ *whenever* $x, y, z \in X$*;*

(G2) $G(x, x, y) > 0$ *whenever* $x, y \in X$ *with* $x \neq y$*;*

(G3) $G(x, x, y) \leq G(x, y, z)$ *whenever* $x, y, z \in X$ *with* $z \neq y$*;*

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots,$ *(symmetry in all three variables);*

(G5) $G(x, y, z) \leq [G(x, a, a) + G(a, y, z)]$ *for any points* $x, y, z, a \in X$*.*

Then (X, G) *is called a* ***G-metric space****.*

Proposition 2 ([22])**.** *Let (X, G) be a G-metric space. Then for a sequence $\{x_n\} \subseteq X$, the following are equivalent*

(i) $\{x_n\}$ *is G-convergent to* $x \in X$.

(ii) $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$.

(iii) $\lim_{n\to\infty} G(x, x_n, x_n) = 0$.

(iv) $\lim_{n\to\infty} G(x_n, x, x) = 0$.

Proposition 3 ([22]). *In a G-metric space (X, G), the following are equivalent*

(i) The sequence $\{x_n\} \subseteq X$ is G-Cauchy.

(ii) For each $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $m, n \geq N$.

Definition 4 ([22]). *A G-metric space (X, G) is complete (or more precisely G-complete) if every G-Cauchy sequence of elements of (X, G) is G-convergent in (X, G).*

The topology of G-metric space is Hausdorff topology. Therefore the limit of a convergent sequence is unique. A comprehensive account of the struture of G-metric space is given in [1].

Proposition 5 ([22]). *If (X, G) and (X', G') are two G-metric space, then a function $T : (X, G) \to (X', G')$ is continuous at a point $x^* \in X$ if and only if whenever a sequence $\{x_n\} \subseteq X$ is G-convergent to $x^* \in X$, then the sequence $\{Tx_n\} \subseteq X'$ is G'-convergent to $Tx^* \in X'$.*

Definition 6 ([5]). *Let $(X,\ d)$ be a metric space and $T : X \to X$ be a given function. Given any $x_0 \in X$, the orbit of x_0 is denoted by $O(T(x_0))$ and defined by the sequence*

$$\{x_0, x_1 = T(x_0), x_2 = T^2(x_0), ...\}, \textit{that is} \tag{1.1}$$

the sequence $\{x_n\}$ where, for all $n \geq 1$, $x_n = T^n(x_0) = Tx_{n-1}$.

Definition 7 ([5]). *Let $(X,\ d)$ be a metric space and $T : X \to X$ be a given function. Given any point $x \in X$ is an orbital point of T if $T^n(x) = x$ for some positive integer n.*

We also recall the results by Mustafa:

Theorem 8 ([21]). *Let (X, G) be a G-complete G-metric space and let $T : X \to X$ be a mapping such that there exists $\lambda \in [0, 1)$ satisfying*

$$G(Tx, Ty, Tz) \leq \lambda\, G(x, y, z), \tag{1.2}$$

whenever $x, y, z \in X$. Then T has a unique fixed point. In fact, T is a Picard operator.

Definition 9. *Let X be a non-empty set. A ternary relation $\rho \subseteq X^3$ is said to be transitive if $(x, y, y) \in \rho$ and $(y, z, p) \in \rho$ together imply $(x, z, p) \in \rho$.*

2 Main Results

Theorem 10. *Let $(X,\ G)$ be a complete G-metric space and ρ be a ternary transitive relation defind on X. $T : X \longrightarrow X$ is mapping such that the following hold:*

(i) for each $x \in X$, there exists $n(x)$ such that

$$G(T^{n(x)}x, T^{n(x)}y, T^{n(x)}z) \leq \lambda G(x, y, z) \tag{2.1}$$

whenever $(x, y, z) \in \rho$ and $\lambda \in [0, 1)$ is a constant.

(ii) $(x, y, z) \in \rho$ implies $(x, Ty, Tz) \in \rho$ and $(Tx, y, z) \in \rho$.

(iii) there exists $x_0 \in X$ such that $(x_0, Tx_0, Tx_0) \in \rho$.

Then we have the following conclusions

a) If T is continuous then T has a unique fixed point.

b) If for any sequence $\{x_n\}$ in X, $x_k \to x$ as $k \to \infty$ and $(x_k, x_k, x_{k+1}) \in \rho$ for all $k \geq 1$, together imply that $(x_k, x_k, x) \in \rho$ for all $k \geq 1$, then there is a point x in X which is an either orbital point or a limit point of its own orbit.

Proof. Let x_0 be the same as that given in the condition (iii) of the theorem. We construct the sequence $\{x_n\}$ as follows:

$$x_{k+1} = T^{n(x_k)}x_k, \quad k \geq 1, \tag{2.2}$$

We first prove that $r(x_0) = \sup_k G(x_0, T^k x_0, T^k x_0)$ is finite. We have $(x_0, Tx_0, Tx_0) \in \rho$. Then by repeated application of the condition (ii), we have

$$(x_0, T^k x_0, T^k x_0) \in \rho \text{ for all } k \geq 1. \tag{2.3}$$

Let $p = max\{G(x_0, T^i x_0, T^i x_0) : i = 1, 2, ...n(x_0)\}$.

For every positive integer k, there exists an integer $s \geq 0$ such that

$$sn(x_0) < k \leq (s + 1)n(x_0).$$

Then

$$\begin{aligned} G(x_0, T^k x_0, T^k x_0) &\leq G(x_0, T^{n(x_0)}x_0, T^{n(x_0)}x_0) + \\ &\qquad G(T^{n(x_0)}x_0, T^{n(x_0)}T^{k-n(x_0)}x_0, T^{n(x_0)}T^{k-n(x_0)}x_0) \\ &\leq p + \lambda G(x_0, T^{k-n(x_0)}x_0, T^{k-n(x_0)}x_0) \\ &\qquad \text{(since by (2.3), } (x_0, T^{k-n(x_0)}x_0, T^{k-n(x_0)}x_0) \in \rho). \end{aligned}$$

Again by the same argument,

$$G(x_0, T^{k-n(x_0)}x_0, T^{k-n(x_0)}x_0) \leq p + \lambda G(x_0, T^{k-2n(x_0)}x_0, T^{k-2n(x_0)}x_0).$$

Combining the above two inequalities, we have

$$G(x_0, T^k x_0, T^k x_0) \leq p + p\lambda + \lambda G(x_0, T^{k-2n(x_0)}x_0, T^{k-2n(x_0)}x_0).$$

Continuing the above process, we obtain that, for all $sn(x_0) < k \leq (s+1)n(x_0)$,

$$\begin{aligned} G(x_0, T^k x_0, T^k x_0) &\leq p + p\lambda + p\lambda^2 + ... + \lambda^s G(x_0, T^{k-sn(x_0)}x_0, T^{k-sn(x_0)}x_0) \\ &\leq p + p\lambda + p\lambda^2 + ... + p\lambda^s \\ &\leq \frac{p}{1-\lambda}. \end{aligned}$$

Since k is an arbitrary positive integer and right hand side is independent of k, we conclude that $r(x_0) = \sup_k G(x_0, T^k x_0, T^k x_0) \leq \frac{p}{1-\lambda}$, which establishes the fact that $r(x_0)$ is finite.

In view of the condition $(x_0, Tx_0, Tx_0) \in \rho$, by (2.3), we obtain

$$(x_0, T^{n(x_1)}x_0, T^{n(x_1)}x_0) \in \rho.$$

Then

$$\begin{aligned} G(x_1, x_2, x_2) &= G(T^{n(x_0)}x_0, T^{n(x_1)}T^{n(x_0)}x_0, T^{n(x_1)}T^{n(x_0)}x_0) \\ &= G(T^{n(x_0)}x_0, T^{n(x_0)}T^{n(x_1)}x_0, T^{n(x_0)}T^{n(x_1)}x_0) \\ &\leq \lambda G(x_0, T^{n(x_1)}x_0, T^{n(x_1)}x_0) \text{ (by (1.1))} \\ &\leq \lambda r(x_0). \end{aligned}$$

Again, by (2.2), and by the condition (ii), we have

$$(T^{n(x_0)}x_0, T^{n(x_2)+n(x_0)}x_0, T^{n(x_2)+n(x_0)}x_0) \in \rho,$$

that is,

$$(x_1, T^{n(x_2)}x_1, T^{n(x_2)}x_1) \in \rho.$$

Then

$$\begin{aligned} G(x_2, x_3, x_3) &= G(T^{n(x_1)}x_1, T^{n(x_2)}x_2, T^{n(x_2)}x_2) \\ &= G(T^{n(x_1)}x_1, T^{n(x_1)}T^{n(x_2)}x_1, T^{n(x_1)}T^{n(x_2)}x_1) \\ &\leq \lambda G(x_1, T^{n(x_2)}x_1, T^{n(x_2)}x_1). \end{aligned} \tag{2.4}$$

Again, by (2.3), we have

$$(x_0, T^{n(x_2)}x_0, T^{n(x_2)}x_0) \in \rho.$$

Then

$$\begin{aligned} G(x_1, T^{n(x_2)}x_1, T^{n(x_2)}x_1) &= G(T^{n(x_0)}x_0, T^{n(x_0)}T^{n(x_2)}x_0, T^{n(x_0)}T^{n(x_2)}x_0) \\ &\leq \lambda G(x_0, T^{n(x_2)}x_0, T^{n(x_2)}x_0) \\ &\leq \lambda r(x_0). \end{aligned} \tag{2.5}$$

Combining (2.4) and (2.5), we obtain

$$G(x_2, x_3, x_3) \leq \lambda^2 r(x_0). \tag{2.6}$$

Proceeding in the above way, we obtain

$$G(x_n, x_{n+1}, x_{n+1}) \leq \lambda^n r(x_0) \text{ for all } n \geq 1. \tag{2.7}$$

Then, for $m > n$,

$$G(x_n,\ x_m,\ x_m) \leq \sum_{i=n}^{m-1} G(x_i,\ x_{i+1},\ x_{i+1}) \leq \sum_{i=n}^{m-1} \lambda^i r(x_0) \to 0 \text{ as } i \to \infty. \tag{2.8}$$

Hence $\{x_n\}$ is a Cauchy sequence and therefore, is convergent to $\overline{x} \in X$ (say), that is,

$$x_n \to \overline{x} \text{ as } n \to \infty. \tag{2.9}$$

If $\overline{x}$ is not a fixed point of T, then there exist disjoint open sets U and V with $\overline{x} \in U$ and $T\overline{x} \in V$ such that

$$\inf \{G(x, y, y) : x \in U \text{and } y \in V\} > 0. \tag{2.10}$$

Case-I. T is continuous.

Since T is continuous, $Tx_n \to T\overline{x}$ as $n \to \infty$. Then there exists k_0 such that for all $k \geq k_0$, $x_k \in U$ and $Tx_k \in V$.

Also by repeated application of the condition (ii), for all $k > 1$, we have

$$(x_{k-1}, Tx_{k-1}, Tx_{k-1}) \in \rho. \tag{2.11}$$

Then, by (2.2), for all $k > 1$, we have

$$\begin{aligned} G(x_k, Tx_k, Tx_k) &= G(T^{n(x_{k-1})}x_{k-1}, T^{n(x_{k-1})}Tx_{k-1}, T^{n(x_{k-1})}Tx_{k-1}) \\ &\leq \lambda G(x_{k-1}, Tx_{k-1}, Tx_{k-1}). \end{aligned} \tag{2.12}$$

Repeating the above process, we obtain

$$G(x_k, Tx_k, Tx_k) \leq \lambda^k G(x_0, Tx_0, Tx_0) \to 0 \text{ as } k \to \infty, \tag{2.13}$$

which contradicts (2.5).

Hence $\overline{x} = T\overline{x}$.

The proof that $\overline{x}$ is the only fixed point of the mapping T is by contradiction. If $y \in X$ would be another fixed point of T, then

$$G(\overline{x}, y, y) = G(T^{n(\overline{x})}\overline{x}, T^{n(\overline{x})}y, T^{n(\overline{x})}y) \leq \lambda G(\overline{x}, y, y).$$

This contradiction proves that $\overline{x}$ is a unique fixed point of T.

Hence T has a unique fixed point in X.

Case-II. Let conditions of (b) hold. We have, by our construction, and by the repeated applications of condition (ii), that for all $k \geq 0$, $(x_k, x_{k+1}, x_{k+1}) \in \rho$. Also $x_k \to \overline{x}$ as $k \to \infty$. Then $(x_k, x_k, \overline{x}) \in \rho$ for all $k > 1$.

Then, for all $k > 1$,

$$\begin{aligned} G(\overline{x}, \overline{x}, T^{n(x_k)}\overline{x}) &\leq G(\overline{x}, \overline{x}, x_{k+1}) + G(x_{k+1}, x_{k+1}, T^{n(x_k)}\overline{x}) \\ &= G(\overline{x}, \overline{x}, x_{k+1}) + G(T^{n(x_k)}x_k, T^{n(x_k)}x_k, T^{n(x_k)}\overline{x}) \\ &\leq G(\overline{x}, \overline{x}, x_k) + \lambda G(x_k, x_k, \overline{x}) \to 0 \text{ as } k \to \infty. \end{aligned} \tag{2.14}$$

Hence $T^{n(x_k)}\overline{x} \to \overline{x}$ as $k \to \infty$.

If $n(x_k) = m$ except for a finite number k's, then $T^m\overline{x} = \overline{x}$, that is, $\overline{x}$ is an orbital point of T, otherwise, $\overline{x}$ is a limit point of its own orbit.

This proves the theorem. □

Example 11. Let $X = [0, 1]$ that we write in the form

$$X = \bigcup_{n=1}^{\infty} \left[\frac{1}{2^n}, \frac{1}{2^{n-1}}\right] \cup \{0\},$$

and let's endow X with the G-metric d, defined as

$$d(x, y, z) = \max\{|x - y|, |y - z|, |z - x|\} \quad \text{for all } x, y, z \in X.$$

Let $T : X \to X$ be defined as follows:

$$Tx = \begin{cases} \frac{1}{2^{n+1}}, & \text{if } x = \frac{1}{2^n}, n = 1, 2, ... \\ 1, & \text{if } x \text{ is irrational} \\ 0, & \text{otherwise.} \end{cases}$$

Let the ternary relation ρ be defined on X as follows:

$$(x, y, z) \in \rho \text{ whenever } x, y, z \in \{\frac{1}{2^n}, n = 1, 2, ...\} \cup \{0\}.$$

It then follows that with $n(x) = 1$ for all x, inequality (2.1) is satisfied.

Then, with the choice of $x_0 = \frac{1}{2}$, all the condition of the Theorem 10 are satisfied.

By an application of Theorem 10 there exists a fixed point of T which can be seen to be $\overline{x} = 0$.

Note. The function T is not continuous here.

In the following we describe the result of Theorem 2.1 in a G-metric space without any order relation.

Theorem 12. *Let $(X,\ G)$ be a complete G-metric space and $T : X \longrightarrow X$ is a mapping, for each $x \in X$, there exists $n(x)$ such that*

$$G(T^{n(x)}x, T^{n(x)}y, T^{n(x)}z) \leq \lambda G(x,y,z) \tag{2.15}$$

for all $y, z \in X$ and $\lambda \in [0,1)$ is a constant. Then T has a fixed point.

Proof. The result follows from Theorem 10 by assuming that ρ is the universal ternary relation, that is, $(x,y,z) \in \rho$ holds for all $x,y,z \in X$. □

Note. The continuity of T is not a requirement in the theorem.

The following is an illustration of the above theorem.

Example 13. *Let $X = [0,1]$ that we write in the form*

$$X = \bigcup_{n=1}^{\infty} \left[\frac{1}{2^n}, \frac{1}{2^{n-1}}\right] \cup \{0\},$$

and let's endow X with the G-metric d, defined as

$$d(x,y,z) = \max\{|x-y|, |y-z|, |z-x|\} \quad \text{for all } x,y,z \in X.$$

Let $T : X \to X$ be defined as follows:

$$Tx = \begin{cases} \frac{1}{2^{n+1}}, & \text{if } x \in \left[\frac{1}{2^n}, \frac{3n+5}{2^{n+1}(n+2)}\right] \\ \frac{n+2}{n+3}\left(x - \frac{1}{2^{n-1}}\right) + \frac{1}{2^n}, & \text{if } x \in \left[\frac{3n+5}{2^{n+1}(n+2)}, \frac{1}{2^{n-1}}\right] \end{cases}$$

and $T(0) = 0$.

Actually T maps the interval $I_n := \left[\frac{1}{2^n}, \frac{1}{2^{n-1}}\right]$ onto the interval I_{n+1}.

The function T is a continuous function on $[0,1]$ which leaves only 0 fixed but is not a contraction. Moreover, straightforward computations, considering all the possible cases for $x \in I_n$ and $y \in I_m$ (with $m \geq n$ and $m \leq n$), lead to

$$|Tx - Ty| \leq \frac{n+3}{n+4}\,|x-y| \quad \text{for all } y \in X.$$

Therefore, if we choose $\lambda = \frac{1}{2}$ in (2.15), then for each $x \in \left[\frac{1}{2^n}, \frac{1}{2^{n-1}}\right]$, one can take $n(x)$ to be $n(x) = n+3$ and for $x = 0$, one just requires that $n(0)$ be such that $n(0) \geq 1$.

In the special case when $n(x) = 1$ for all $x \in X$, we have the following corollary which is the case for Banach contraction and an established result.

Corollary 14 ([21]). *Let $(X,\ G)$ be a complete G-metric space and $T : X \longrightarrow X$ is a mapping such that the following holds:*

For each $x, y, z \in X$,

$$G(Tx, Ty, Tz) \leq \lambda G(x, y, z)$$

where $\lambda \in [0, 1)$.

Then T has a fixed point.

The following is a corresponding result in partially ordered G-metric spaces.

Note. Here the mapping T is automatically continuous.

Corollary 15. *Let $(X,\ G)$ be a complete G-metric space with a partial order relation " $\preceq$ " defined on X. $T : X \longrightarrow X$ is such that the following hold:*

1. *for each $x, y, z \in X$ such that*

$$G(Tx, Ty, Tz) \leq \lambda G(x, y, z)$$

   whenever $x \preceq y \preceq z$ and $\lambda \in [0, 1)$.

2. *T is isotone, that is, $x \preceq y$ implies $Tx \preceq Ty$ for all $x, y \in X$.*

3. *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$.*

Let, either

a) T is continuous or,

b) for any sequence $\{x_n\}$ in X, $x_n \to x$ as $n \to \infty$ and $x_k \preceq x_{k+1}$ jointly imply that $x_k \preceq x$ for all k.

Then T has a fixed point. If T is continuous then additionally the fixed point is unique.

Proof. We define $(x, y, z) \in \rho$ if $x \preceq y$ and $y \preceq z$ implies $x \preceq y \preceq z$. From (iii) follows that $x_0 \preceq Tx_0 \preceq Tx_0$. The proof then follows by following the same steps as in Theorem 10 with $n(x) = 1$ for all $x \in X$ except for a minor modification with respect to the implications of condition ii) of the Theorem 10. We omit the details and note that this is only possible under the condition $n(x) = 1$ for all x. □

3 Conclusion

There are two spacial features of the present work which we mention here. One is the type of generalized contraction and the methodology used to deal with them in order to establish the fixed point result. The other is the use of ternary relation. We hope that both the features can be incorporated in fixed point theory elsewhere to address other circumstances. These can be taken up in future works.

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