# APPLICATIONS OF DIMENSIONAL ANALYSIS AND SIMILARITY THEORY IN HYDRAULICS

**Lecturer PhD. eng. Florin CIOFU**,
*Engineering Faculty, "Lucian Blaga" University of Sibiu*

**ABSTRACT**: Technological process, however complex, can be broken down into a succession of distinct component processes, in which the input materials undergo changes in shape (mechanical processes), pressure, temperature, concentration, state of aggregation (physical processes) or structure molecular (chemical and biochemical processes). The unitary operations of most process phases in process industries are based on three fundamental processes: pulse transfer, heat transfer, and mass transfer.

**KEY WORDS:** dimensional analysis, physical phenomena, hydraulic similarity

## 1. Introduction

Fluid mechanics problems can be addressed through dimensional analysis, which is essentially a mathematical procedure that studies exclusively the dimensions of physical quantities. It starts from the understanding of flow phenomena in order to determine the parameters that influence it and it is possible to group these parameters into dimensional combinations, to better knowledge and explanation of the phenomena. Dimensional analysis is of real use in experimental studies because it can indicate the magnitudes or parameters that really influence the development of physical phenomena.

According to the principle of dimensional homogeneity, all mathematical relations, which express physical phenomena, must be dimensionally homogeneous (all equation terms must have the same dimensions).

It is practically impossible to solve problems of fluid flow only theoretically. At the present state of the art, experimental research occupies an important place. Mathematical theory and experimental data have provided practical solutions for many hydraulic problems. Applications of dimensional analysis and hydraulic similarity allow the engineer to organize and simplify experiments and analyze the results obtained.

In this paper we will present the principle underlying the dimensional analysis and a case study that serve to understand the use of the dimensional analysis in the determination of the formulas for certain physical quantities specific to the fluid mechanics. Similarity relationships with specific applications will also be presented

## 2. Theoretical notions

According to the principle of dimensional homogeneity, all mathematical relations, which express physical phenomena, must be dimensionally homogeneous (all equation terms must have the same dimensions).

If the terms of a homogeneous equation in dimensional terms are shared with a quantity that is expressed in the same dimensions, an amount of terms will be added, the equation becoming a dimensionless relationship between groups of numbers and a simpler form. This is done in a dimensional analysis, grouping all the variables involved in an equation that contains groups of non-dimensional numbers, avoiding experimental research, with the non-dimensional groups being much smaller than the variables.

Dimensional analysis applications consist of:
- transformation from one system of units to another;
- establishing equations;
- reducing the number of variables required for an experimental program;
- establishing the principles of designing a model.

Buckingham's Pi Theorem is a generalization of the dimensional analysis method with a wide use today and has the main advantage of reducing the number of variables in groups of dimensional sizes.

A physical relationship involving *m* dimensions and dimensional constants can be expressed as a relation between $i = m - n$ dimensional groups, where *n* represents the number of fundamental units of the unit of measurement system used [3].

In this way, a physical equation such as:

$$f_1(x_1, x_2, \dots, x_m) = 0 \qquad (1)$$

is reduced to an equation of the type:

$$f_2(\pi_1, \pi_2, \dots, \pi_i) = 0 \qquad (2)$$

where each adimensional group $\pi$ depends on the maximum *(n + 1)* constant sizes and dimensions.

The number of adimensional groups $\pi$ is equal to *(m - n)*, the difference between the number *m* of physical quantities plus the dimensional constants involved in the phenomenon unfolding and the number *n* of fundamental sizes involved in the units of the physical and dimensional constants:
n = 3 for mechanical phenomena (length, weight, time);
n = 4 for thermal phenomena (length, mass, time, temperature);
n = 4 for electrical phenomena (length, mass, time, power intensity);
n = 5 for thermoelectric phenomena (length, mass, time, temperature, electric current intensity).

To find the desired function, Buckingam's $\pi$ theorem applies in the following way:
1 – All physical quantities and dimensional constants, which from mechanical, thermodynamic conditions, etc. go through. it is appreciated that it influences the phenomenon studied;
2 – Write the dimensional formula for each of the physical dimensions and dimensional constants considered;
3 –We choose the *n* fundamental sizes between the physical quantities and the dimensional constants that occur in the problem. The choice is made in such a way that all the quantities and constants chosen contain at least once all the fundamental dimensions of the problem;
4 –The groups $\pi_1, \pi_2, \pi_3, \pi_4, \dots, \pi_i$, are formed, consisting of each of the products of the n values chosen in item 3 plus one of the other sizes and constants. An arbitrary exponent is associated with each dimension and dimensional constants in each group $\pi$.
5 – Determine the value of these exponents, providing that each group $\pi$ is adimensional.

In many situations experimental development takes place in laboratories on plants that differ constructively from industrial ones, but allow an identical or similar unfolding of the studied phenomena. To use laboratory results at industrial plants, mathematical relationships have been established, known as similarity laws. These allow the experiment to be run with a convenient fluid for use and application of the results to a less convenient fluid for experimental use.

These laws are particularly useful because they can be used on a simpler and smaller machine (model), with the potential to substantially reduce research costs and allow the results to be transferred from model to plant or machine in natural size (prototype ). In order for the results set on the models to be used for the installation in kind, the conditions of similarity must be observed [1].

Two movements are similar when their trajectories are geometrically similar and when there are determinant ratios between the kinematic and dynamic sizes of the two phenomena at two homologous points. In order to achieve the dynamic similarity of two phenomena, it is not sufficient that the ratio of the linear dimensions be constant. Reports of kinematic and dynamic sizes must be constant.

The geometric similarity is achieved when the ratio between the linear dimensions on the prototype and the model on the model is constant [4]. The report:

$$l_0 = \frac{l_p}{l_m} \tag{3}$$

is called the ladder scale or geometric scale. The scale of the surfaces (4) and the scales of the volumes (5) can also be set:

$$S_o = \frac{s_p}{s_m} = l_0^2 \tag{4}$$

$$V_0 = \frac{v_p}{v_s} = l_0^3 \tag{5}$$

The kinematic similarity involves, in the homologous deck, the geometric similarity of the hydrodynamic field and the constant ratio of kinematic sizes of the same type (velocities, accelerations). Once the ladder scale is established, a constant ratio of the time at which the prototype phenomenon takes place and the time in which the phenomenon occurs on the model, that is the time scale:

$$t_0 = \frac{t_p}{t_m} \tag{6}$$

With these, the scales of all kinematic sizes can be determined according to $l_0$ and $t_0$. So we have the scale of speeds:

$$v_0 = \frac{v_p}{v_m} = l_0 t_0^{-1} \tag{7}$$

and the scale of accelerations:

$$a_0 = \frac{a_p}{a_m} = l_0 t_0^{-2} \tag{8}$$

The dynamic similarity requires that the ratio of all forces in nature, prototype, and model be constant. This results in the scale of forces:

$$F_0 = \frac{F_p}{F_m} \tag{9}$$

From the mechanical similarity can be defined a scale of the masses:

$$m_0 = \frac{m_p}{m_m} \tag{10}$$

Number Froude:

$$F_r = \frac{v^2}{g \cdot l} \tag{11}$$

Number Euler:

$$E_u = \frac{p}{\rho \cdot v^2} \tag{12}$$

Number Reynolds:

$$R_e = \frac{v \cdot l}{\vartheta} \tag{13}$$

Number Newton:

$$N_e = \frac{F}{\rho \cdot s \cdot v} \tag{14}$$

Number Galilei:

$$G_a = \frac{g \cdot l^3}{\vartheta^2} \tag{15}$$

Number l Strouhal:

$$S_h = \frac{v \cdot t}{l} \tag{16}$$

Number l Mach:

$$M_a = \frac{v}{v_s} \quad (17)$$

Number Weber:

$$W_e = \frac{\rho \cdot l \cdot v^2}{\sigma} \quad (18)$$

The sizes presented in relations (11-18) are also called similarity criteria. Newton's theorem states that in a group of phenomena likewise, each criterion of similarity has a unique value for all the phenomena of the group. Simultaneous observance of all these criteria leads us to a complete similarity. But in reality the simultaneous observance of these criteria is not practicable. Similarity will not be achieved by all criteria, but only after certain criteria that are determinant in the development of a phenomenon [1]. This results in incomplete similarity. Translating results from a prototype model will be affected by errors, and the influence of neglected parameters appears in the so-called scale effect.

The Euler Criterion is automatically satisfied if the Strouhal, Froude and Reynolds criteria are met simultaneously. Appears in the study of cavitation phenomenon. The Mach similarity criterion applies if the velocity of the current is high and the fluid compressibility due to the current velocity can not be neglected (in the case of a very high velocity movement of a gas in the case of a ram shot). Weber's similarity criterion is observed for movements in which superficial tension forces are determined (droplets, so when spraying liquids, small waves, in the study of fluid flow in capillary tubes or in very low depth channels). In current applications, superficial tension forces are however negligible in relation to other types of forces. The Galilean Criterion intervenes in the free movement of liquids. This number is actually a combination of criteria of similarity.

The similarity of Strouhal is used in nonpermanent periodic movements. These occur when the formed vortices alternately dislodge from one side or the other behind a body when the fluid is in a wave motion and when a body in the fluid has a periodic motion. Since most non-permanent movements of fluids in the technique are periodic movements, the Strouhal criterion is usually considered to be the similitude criterion for fluid movements. In many cases, with the Strouhal criterion, the Reynolds criterion must also be ensured.

The similarity of Froude is used when the moving element is the weight. This occurs as a predominant element in the flow of water over the spillway, the wave movement, the determination of the waveform of the surfacing surface of the surface vessels. It generally occurs when the movements have free surfaces that are not horizontal, because in these movements the effect of their own weight is determinant for the shape of the free surface. In the case of the movement of liquids over spill or wave motion, the effect of viscosity and the effect of capillary are neglected in relation to the effect of the liquid's own weight. Other times, however, in addition to the effect of the weight of the liquids, other effects must be considered.

Thus, in the movement of the fluids in the channels, in addition to the effect of their own weight, the effect of the viscosity must also be taken into consideration, and in the spills having a very thin discharge spindle and in the small waves, the capillarity effect must be taken into account.

The similarity of Reynolds must be ensured if the viscous friction has a predominant role. The higher the number of Reynolds is, the greater the viscosity influence on the fluid movement. Applies to the flow of liquids in pressurized pipes, flowing in hydraulic machines, and flowing in aerodynamic tunnels at speeds where fluid compressibility can be neglected. Generally, the length of the pipeline is the length of the pipe, the thickness of a fluid layer, the rope of an aerodynamic profile [1].

## 3. Case study

Next, using the dimensional analysis, the relationship between Reynolds number, density ρ, kinematic viscosity v, fluid velocity v of a fluid and characteristic length l shall be expressed.

Using dimensional analysis to establish the relationship between the Reynolds number and the listed sizes, we start from the fact that the Reynolds number is dependent on the ρ, u, v and l:

$$R_e = f(\rho, \vartheta, v, l) \qquad (19)$$

Dimensional analysis is based on the fact that a relationship between physical dimensions must be dimensionally homogeneous. We use the Rayleigh method which assumes that the resultant size, in our case the Re number, can be written as proportional to a product of the magnitude of the powers that determines it, that is:

$$R_e = k \cdot \rho^a \cdot \vartheta^b \cdot v^c \cdot l^d \qquad (20)$$

where k is the coefficient of proportionality. The powers a, b, c, d are found to impose the condition that this relationship be dimensionally homogeneous:

$$M^0 \cdot L^0 \cdot T^0 = k \cdot (M \cdot L^{-3})^a \cdot \cdot (L^2 \cdot T^{-1})^b \cdot (L \cdot T^{-1})^c \cdot L^d \qquad (21)$$

$$M^0 \cdot L^0 \cdot T^0 = k \cdot M^a \cdot L^{-3a+2b+c+d} \cdot \cdot T^{-b-c} \qquad (22)$$

that is, to have the following equality:

$$\begin{cases} 0 = a \\ 0 = -3a + 2b + c + d \\ 0 = -b - c \end{cases} \qquad (23)$$

By solving the system of equations we obtain:

$$\begin{cases} a = 0 \\ c = -b \\ d = -b \end{cases} \qquad (24)$$

that:

$$R_e = k \cdot \rho^0 \cdot \vartheta^b \cdot v^{-b} \cdot l^{-b} = k \cdot \left(\frac{v \cdot l}{\vartheta}\right)^{-b} \qquad (25)$$

Values of k and b are determined by experimental analysis. In the conditions of the application presented k = 1 and b = -1, case where the Reynolds number (Re) obtains the relation (13) [3].

## 4. Conclusion

Dimensional analysis is the domain that deals with establishing relationships between dimensional formulas of different physical sizes. Based on these relationships, we can sometimes determine approximate forms of valid laws in certain experimental situations. Even though formulas determined using dimensional analysis are only approximate, they can be of great help in simplifying the experiments to determine the correct form of those laws. Also, dimensional analysis can highlight dimensional reports of physical quantities, called criteria, which are used to characterize the predominance of a particular physical effect with respect to another.

## 5. Bibliography

[1] Bordeaşu, I. ş.a. – Probleme de hidrodinamică, reţele de conducte, canale şi maşini hidraulice, Timişoara, 2013

[2] Bordeaşu I., Baciu I.D., Hidraulică. Noţiuni teoretice şi probleme de hidrostatică, Editura Politehnica, Timişoara, 2002

[3] Gavrilă, L – Operaţii unitare în industria alimentară şi biotehnologii, Universitatea Bacău, 2001

[4] Gavrilă, L – Fenomene de transfer, vol. I,II, Editura Alma Mater, Bacău, 2000