ANOTHER VIEW ON THE RELIABILITY OF SYSTEMS WITH COMPONENTS SUBJECTED TO FATIGUE

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ABSTRACT: The paper asserts the issue of calculating the provisional reliability for products with fatigue-exposed components, during the exploitation under normal working conditions. The difficulty of these calculi is caused by the intricate assessment of reliability indexes specific to variable charges. Following a brief introduction in this field, we present the particularities of the reliability calculi for fatigue-influenced elements. We then present some elements characteristic to fatigue, outlining the Wöhler (durability) curve used in calculating the resistance to fatigue (the durability curve) and the exponential model of the reliability function. The proposed method of calculus is original, easily applicable and it allows the assessment of reliability for several types of complex products with fatigue-exposed components.

KEY WORDS: products’ reliability, calculating reliability indexes, fatigue, variable loads

1. INTRODUCTION

The problem of reliability is posed at every stage of existence of a product, even since the designing phase.

In many practical cases, products or parts of them, are subjected to dynamic charges, so static charges’ calculi can no longer suffice. An outstanding instance of a dynamic load is fatigue resistance. *Fatigue resistance* is the property of metals and alloys to resist to repeated (cyclical) charges. Fatigue charges appear as a result of certain time-variable loads. If part of a product is charged with fatigue, predicting its reliability becomes a ticklish issue, especially since there is a chance it might fail before reaching a charge lower than the maximum prescribed.
Consequently, the product could break down even if loads are lower than prescribed for normal working conditions. It is thus well-advised that we assess the reliability of fatigue-exposed components, starting from the designing phase.

The reliability of mixed-connection systems can be calculated by using the well-known relations expressed for serial - and parallel - structure products [1]. Any given hybrid structure can be divided into boughs of serial or parallel connections or into groups of parallel-connected elements. Then, through the above-mentioned formulas, we can calculate the reliability of any suchlike-structured product.

The only hindrance that might occur while performing these calculations could reside in fatigue-exposed components, since the latter’s reliability is hard to assess. The reliability of such components greatly depends on the characteristics of the loading cycle and the amount of the average loads and their amplitudes, as will be shown next.

2. CHARACTERISTICS OF THE FATIGUE PHENOMENON CAUSED BY VARIABLE LOADS

Certain parts of products are exposed to variable and recurrent exterior forces. These components are replaced when they become damaged. They break down as a result of the exposure to recurrent and variable charges and are called fatigue fractures. There are differences between these ruptures and those caused by constant loads.

Fatigue fractures usually occur at much lower charges than would be necessary to cause a rupture under static circumstances. They are also very dangerous as they are preceded by no visible alterations in the product's physical aspect or dimensions.

Ruptures caused by fatigue can easily be recognized by the existence of two distinct areas: a shiny, relatively smooth area which notwithstanding this fact shows the way the incipient fracture formed and developed in time and a matte one, presenting asperities according to the final, instantaneous break. Fatigue resistance decreases alongside increasing the part's dimensions and the material's tensile resistance. Fatigue ruptures have a fragile character.

While studying variable stationary loads, we consider that the charges applied to the parts and thus the tensions they engender, vary periodically, at a certain frequency, as shown in figure 1.

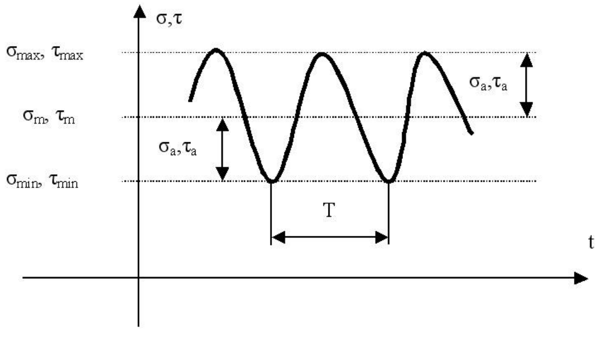

Figure 1. Variable charge

The variation of the tension (normal, σ or tangential, τ) from a random value until we reach the same value and sense of variation produces a variable-load cycle, which takes place throughout period T (fig. 1). The tension only reaches the peak value once during a cycle and it is called maximum tension (σ_{max}, τ_{max}) or upper tension limit. The lowest value – also called minimum tension (σ_{min}, τ_{min}) or lower tension limit – is only reached once during a cycle as well.

The cause of variable-load cycles is either the parts' gyration or their straight reciprocating movement. Looking at figure 1, we can easily tell that if we are familiar with the average tension and the tension's amplitude, we can calculate the extreme tensions. Thus, a variable-load cycle is either defined by its extreme values σ_{max} (τ_{max}), σ_{min} (τ_{min}) or by the mean value and amplitude σ_m (τ_m) şi σ_a (τ_a). The following ratio is also known as the cycle's index of asymmetry: $R = \sigma_{min} / \sigma_{max}$ or $R = \tau_{min} / \tau_{max}$.

According to the size of the characteristic tensions (extreme tensions, average tension and the amplitude of tension) and to the asymmetry index, we can have the following variable-load cycles: symmetrical cycles - where $\sigma_{max} = -\sigma_{min}$, $\sigma_m = 0$, $\sigma_a = \sigma_{max}$, R=-1 sau $\tau_{max} = -\tau_{min}$, $\tau_m = 0$, $\tau_a = \tau_{max}$, R=-1; asymmetrical cycles – all cycles whose $R \neq -1$; alternating cycles: tension changes its sign throughout a single period; undulated cycles – tension maintains its polarity throughout the whole period. Undulated cycles can be either positive or negative. In case one of the tension's extremes is null, the undulated cycle is called pulsating instead; the latter can be positive (R=0) or negative ($R = \pm\infty$). When the cycle's amplitude is very small, it can practically be considered null, in which case the charge is considered to be static.

Fatigue resistance is usually measured with the aid of fatigue-testing machines especially built for: pure-bending-charged test bars, pulsating-cycle testing, twisting tests, traction tests, and compound-charge tests. These machines are also equipped with cycle counters. If we perform fatigue tests on bars that are charged at tensions σ_i (τ_i) below the breaking limit σ_r (τ_r) and count the number of cycles during which the fracture occurs (N_i), we can trace a diagram similar to the one in figure 2. The curve presented in figure 2 whose asymptote quantifies fatigue resistance σ_R (τ_R), is called the durability curve or Wöhler's curve (diagram).

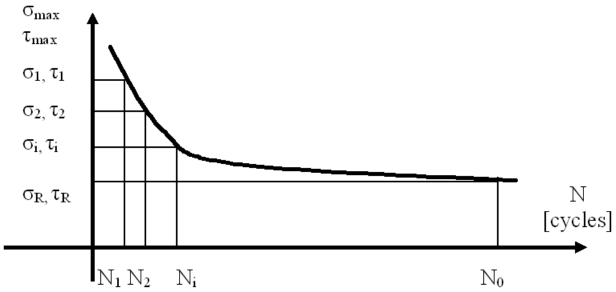

Figure 2. Wöhler’s Diagram [2]

Fatigue resistance represents the charging cycles’ maximum tension’s peak value that the test bar can undergo indefinitely without breaking. Since testing cannot be practically carried indefinitely, testing is usually limited to a certain number of cycles, N_0, also known as charging base. We usually consider $N_0 = 10^6 \ldots 10^7$ cycles for steel and $N_0 = 5x10^7 \ldots 10^8$ cycles for light alloys. We often employ the logarithmic scale for the horizontal. Any given material has infinite fatigue resistances, according to both the performed-cycles’ asymmetry index and the type of load. The most popular fatigue resistances are those of symmetrical cycles, followed by the pulsating ones. Within a given charge, symmetrical cycles enclose the lowest fatigue resistances; pulsating cycles include greater values of fatigue resistance while the static-fractured ones contain the greatest. Since the static charges’ asymmetry index is R=+1, fatigue studies usually employ the following notation, as well: $\sigma_R = \sigma_{+1}$ ($\tau_R = \tau_{+1}$), where σ_R is the static fracture resistance.

Specialty papers include empiric relations that show the connections between fatigue resistance and static rupture strengths. They also present values of fatigue resistance for various materials, in relation to the type of load and the nature of the stress cycle.

We can also use the fatigue-resistance diagram (Haig’s diagram) or the extreme cycles’ curve. The latter can be traced under σ_m, σ_a (respectively τ_R, τ_R) coordinates. We can also use Smith’s diagram, which means tracing extreme tensions (minimum and maximum) according to the average one. In practice, we usually schematize Haig’s diagram through straight lines (the Gerber, Goodman, Soderberg, Serensen methods), which allows us to set safety margins more conveniently. Fatigue-resistance diagrams help define fatigue-stress safety margins. Unfortunately, specialty papers fail to make notes on their utilization and the assessment of reliability.

3. RELATIONS BETWEEN DURABILITY CONCEPTS, FATIGUE RESISTANCE AND RELIABILITY

All products: machines, apparatuses, devices have limited durability. In this context, fatigue resistance, which according to the definition refers to unlimited durability, is an exaggeration. Thus, the modern limited-durability calculus method was developed by several authors, being very popular. It is based on Wöhler’s diagram, which is drawn – for that particular part – in the shape of a polygonal path, as shown in figure 3. If a part undergoes a variable stress, whose tension is $\sigma_{max} = \sigma_F$ ($\tau_{max} = \tau_F$) and the first occurs an indefinite number of times, respectively if $N_F > N_0$, then the extreme condition is obtained by drawing a vertical (FL_F). Point L_F can be placed anywhere on the horizontal line – BC, since the extreme condition is always defined by the fatigue resistance – σ_R (τ_R).

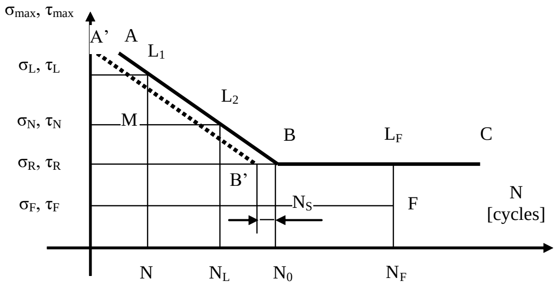

Figure 3. Limited durability [2]

However, everything changes if the stress, represented by point M is applied throughout a duration where $N < N_0$. This point represents the unitary-effort load σ_N (τ_N) that ensures durability throughout N loading cycles. Since $\sigma_N > \sigma_R$ ($\tau_N > \tau_R$), the N-cycle durability is a limited one. We can establish two extreme conditions, by drawing a vertical and a horizontal line through point M: the duration resistance, σ_L (τ_L), corresponding to N loading cycles; the lifespan, N_L, corresponding to the σ_N (τ_N) tension.

The first extreme condition is useful in strength calculations like the maximum loads a product can handle throughout a certain number of cycles. The second extreme condition is useful in assessing the product’s durability – which functions under well-known loads – and its reliability.

Segment AB is Wöhler’s curve extreme durability portion (fig. 3). Sometimes, certain parts are to function for a limited period of time, inferior to the number of cycles that would lead to the reaching of the fatigue resistance (N_0), after which they are discarded. In this case, we no longer calculate the fatigue-resistance index; instead we perform a limited-durability calculus.

The method presented is used to determine certain parts’ physical durability. In the field of design, we usually encounter new parts, which do not come with Wöhler’s curve. Durability calculus means choosing allowed superior resistances higher than those of perennial parts. In these cases, durability resistance is calculated through mathematical modeling, by using the similarity between Wöhler diagram which shows the test bar’s behavior and the studied part, using – as appropriate – one of the following relations:

$$N \cdot \sigma^m = N_0 \cdot \sigma_R{}^m = \text{const.} \qquad \textbf{(1)}$$

$$N \cdot \tau^m = N_0 \cdot \tau_R{}^m = \text{const.} \qquad \textbf{(2)}$$

where (N,σ), respectively (N, τ), are coordinates of a point situated on segment AB from fig. 3. We must note that the above relations can only apply to the limited-durability zone (segment AB, figure 3) where $\sigma>\sigma_R$ and $\tau>\tau_R$. As an average value for steel, we can consider $N_0 = 10^6 \ldots 5\cdot10^6$ and m= 9.

The limited-durability calculus allows us to use material, energy, and workmanship rationally, which is why it is essential to the designing of mass-production parts. Durability is, in fact, the time between failures, until the product is made redundant due to its fatigue-caused rupture. This is why the number of loading cycles until failure corresponds to a period that can be associated to the mean time between failures (MTBF). By knowing the duration of a loading cycle for a well-known charge, T – measured in hours/cycle, we can calculate the time between failures, TBF, by using the following relation:

$$TBF = N_L \cdot T \text{ [hrs.]} \qquad \textbf{(3)}$$

Since Wöhler’s diagram can only be traced for a limited number of parts (or test bars) made of heterogeneous materials, under close non-identical conditions and since products are used under different conditions by each user, in order to assess the mean time between failures correctly, we can correct the situation by moving segment AB into position A’B’, through an N_S shift, as shown in figure 3. We thus reduce the durability described by Wöhler’s diagram by a number of working cycles N_S, approximately equal to 5% of the traced durability. Using this approximation, the mean time between failures for a known charge (the unitary-effort stress σ_N, respectively τ_N) will be:

$$MTBF = 0{,}95 \cdot N_L \cdot T \text{ [hrs.]} \qquad \textbf{(4)}$$

If we consider that there is a relation between the mean time between failures (MTBF) and the intensity of failures (λ):

$$MTBF = 1 / \lambda \quad , \qquad \textbf{(5)}$$

we can perform reliability calculi for the considered part, based on information provided by Wöhler’s diagram. If we employ the reliability function’s exponential model, reliability and non-reliability can be calculated using the following relations:

$$R(t) = e^{-\lambda t}, \qquad \textbf{(6)}$$

$$F(t) = 1 - e^{-\lambda t}. \qquad \textbf{(7)}$$

Starting from Wöhler’s diagram, we can obtain the four reliability indexes we need:

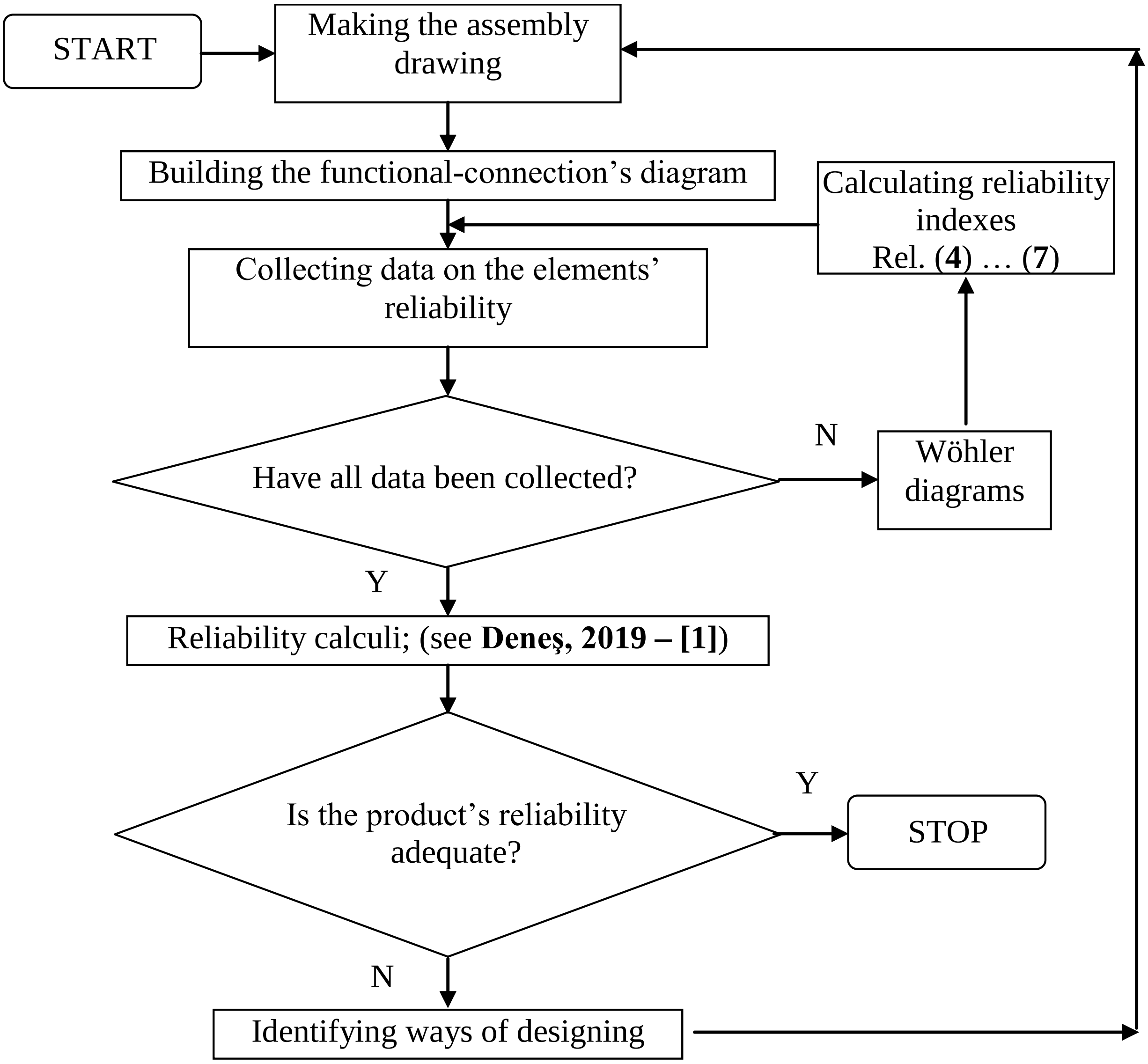

Figure 4. Working algorithm

MTBF, λ, R (t) and F (t). Therefore, we can assess a product's reliability using a diagram especially employed in performing stress calculations. If a product contains a fatigue-exposed element, it is enough that we identify Wöhler's diagram for the respective material the component is made of and already we can assess its reliability.

If Wöhler's diagram is unavailable, it can be traced by straining the element in the same way it will be stressed when part of the product, by using test bars prepared in the same way and using the same material that characterizes that element.

4. ANALYZING THE RELIABILITY OF PRODUCTS THAT CONTAIN FATIGUE-EXPOSED COMPONENTS

Products that contain fatigue-exposed components can be performed provisional reliability calculi upon only if all components' reliabilities are known. Fatigue-exposed components will finally render the product useless exclusively due to the fracturing of their material.

This is the consequence of a fatigue-exposed material, considering this happens during functioning, under lower-than-prescribed charges. In order to perform the adequate provisional reliability calculi, we propose following the working algorithm presented in figure 4. After making the assembly drawing, we can move on to the functional-connections' diagram and then we can collect data regarding the components' reliability. In the case of fatigue-exposed elements, we can use Wöhler's diagram – which can be found in specialty papers – or perform tests that will allow us to draw it.

Then, based on the relations above, we can calculate reliability indexes for both parts of the product and the product itself. Based on the results, we can come up with solutions that will improve the product's design. We can also include redundant elements that will increase the reliability of the whole assembly.

5. CONCLUSIONS

Calculating provisional reliability for fatigue-exposed products is a very ticklish problem due to the difficult appraisal of its components' reliability. Fatigue-exposed components cause products to malfunction when their materials break, at lower charges than prescribed, due to the variable charges they are subjected to.

Products' provisional reliability can be calculated if we are acquainted with both the functional-connections' diagram and with all the elements' reliability. The fatigue-exposed elements' reliability can in turn be assessed if we cover the material's Wöhler diagram.

Based on the previously exposed calculi relations and based on the working algorithm, we can calculate the reliability of any fatigue-exposed product whose components function under variable charges.

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