MATHEMATICAL MODEL FOR SIMULATION OF MINE HOISTING INSTALLATION

Prof. Dr. Sc. Vladimir I. DVORNIKOV, Donbas State Academy of Civil Engineering and Architecture, Ukraine, Donetsk
Assoc. Prof. Dr. Evtim R. KYRTZELIN, University of Mining and Geology “St. Ivan Rilski”, 1100 Sofia, Bulgaria

ABSTRACT. The paper deals with the electromechanical system of the mine hoisting complex, the movement of said system being described by a system of non-linear equation in ordinary and partial derivatives. The latter are reduced, through solving the corresponding integral equations by successive approximation, to a finite system of equations in ordinary derivatives which enables the realisation of a simple numerical process giving the solution relative to the main force factors in the elements of the hoisting installation.

Up to present time the investigation of mine hoisting installation has been limited to the examination of the dynamic state of its separate element: electric drive, hoisting machine, ropes, hoisting vessels, guides etc. At the same time, one can obtain a true picture of interaction of the whole complex of the mentioned elements only by their simultaneous analysis as an electromechanical system, that is by the so called system approach. Such an approach requires, firstly, the correct mathematical description to be given to the dynamic state of all the elements of the installation within the limits of contemporary knowledge of their interrelations, and, secondly, an acceptable method to be developed for the solution of the obtained equations to enable the numerical realisation of a computational algorithm for force factors necessary to assess the stress state of the installation main elements.

The analysis of the considered electromechanical system leads to an important conclusion concerning the existence of equation system of two types: non-singular, describing the finite movement (lateral oscillations of vessels, ropes and guides), and singular, describing the infinite one-dimensional movement of hoisting installation elements with discrete and distributed masses, conditioned by the technological designation of the hoisting machine. The equations of the first type, unlike the second type, do not contain zero values in the spectre of characteristic numbers of the corresponding boundary problem, and that was the reason for the introduction of the terminology of adequate matrices of monodromy. Accordingly, method of solution for the corresponding equations of interaction are quite different.

In common case the non-singular equations can be written in a form of integro-differential equation of the type

\[ u_i(s, t) = \sum \beta P_\beta(t)G^0_{kj}(l_\beta, s) - \frac{\varepsilon B_\beta \int_0^1 G^0_{kj}(\xi, s)\partial^2u_j(\xi, t)}{\partial t^2d\xi} \] (1)

where \( G^0_{kj}(\xi, s) \) - is a Green’s function of influence, \( u_j(s,t) \) - is a function of movement to be determined; \( t \) - is time; \( P_\beta(t) \) - is a force vector of interaction between a continuous body (ropes, guides) and external bodies (vessels, pulleys) at the points \( s=l_\beta \); by two repeated Latin indices, if not otherwise specified, summation is performed from 1 to 2.

Introducing the notion of modified Green’s functions...
where $U^j_\beta(\xi), \omega^j_\beta$ - are orthonormal characteristic functions and numbers of homogeneous boundary problem, respectively, and, presenting the forces of interaction in a form of expansion in series in terms of degrees of the small parameter

$$P^\beta_k(t) = \sum_{\alpha=0}^{\infty} \varepsilon^\alpha P^{\beta\alpha}_k(t)$$  (3)

using (1) we shall get the following system of ordinary differential equations with respect to unknowns

$$\sum_{\beta} \sum_{n=1}^{m} (-B^\beta_\kappa)^n \frac{\partial^{2n}[P^{\beta(m-n)}_\kappa(l_\beta, s)]}{\partial t^{2n}} |_{s=l_\alpha} = 0$$  (4)

The singular system in a common case describes one-dimensional movement $n$ of discrete bodies (vessels, rotating elements of machine, reducer, motor), connected by intertialess couplings, and longitudinal-torsional vibration $m$ of couplings with distributed masses (of winding and balancing ropes) for which the following equation can be written

$$\frac{A^{\kappa\alpha} \partial^2 V_{i\alpha}}{\partial s^2} - \frac{B^{i\alpha} \partial^2 V_{i\alpha}}{\partial s^2} = E_{\kappa\alpha}, \quad s \in [l_\alpha, l_{\alpha+1}], \quad k = 1, 2; \quad \alpha = 1, \ldots, m$$  (5)

where $A^{\kappa\alpha}, B^{i\alpha}, E_{\kappa\alpha}$ are matrices of rigidity, inertia and a vector of distributed forces respectively. Determining the $n$-vector of discrete bodies movement as an expansion in terms of orthonormal characteristic $n$-vector $\Phi_\beta^j$, belonging to the characteristic number $\omega^j_\beta$, in the form

$$\chi^j_\alpha = \sum_{i} \Phi^j_\alpha \Phi^i_\kappa; \quad \sum_{k=1}^{n} (K^i_k - \omega^2 \delta_{ik}) \delta^i_k = 0; \quad \text{det}(K^i_k - \omega^2 \delta_{ik}) = 0,$$  (6)

we find an equation

$$\frac{d^2 \Phi^j_\kappa}{dt^2} + \gamma \omega^2 \frac{d\Phi^j_\kappa}{dt} + \omega^2 \Phi^j_\kappa = \sum_{i=1}^{n} \delta^i_k |L_i| + \sum_{\alpha=1}^{m} \int_{l_\alpha} V^{\alpha}_{\kappa\alpha} E_{\kappa\alpha} ds$$  (7)

where $V^{\alpha}_{\kappa\alpha}$ is a characteristic function of the corresponding homogeneous boundary problem (5), $L_i$ is a vector of external forces acting on the $I$-th discrete mass (the friction forces of the vessel guides, braking and driving forces).

In the relations (6) $L_i$ is a diagonal matrix containing as nonzero elements the values of discrete masses of the system. The symmetrical matrix $K^i_k$ is constructed analogous to the case of absence of inertia coupling but taking into account the fact that the $I$-th and $(I+1)$-th masses in their relative movement are subjected to the action of partial forces, respectively:
\[ P_a = C^{11}_a \chi_a - C^{12}_a \chi_{a+1} ; \quad P_{a+1} = -C^{21}_a \chi_a + C^{22}_a \chi_{a+1} ; \]  
(8)

where coefficients \( C^{ik}_a \) are calculated according to a definite rule

\[ C^{ik}_a = \frac{A_{lia} d\psi_a(s, l_{a-k+2})}{ds} \bigg|_{s=l_a+i-1}, \]  
(9)

the biharmonic functions \( \psi_a(\zeta, s) \) being such that

\[ A_{lia} \nu_{ka} = A_{lia} [\hat{\Omega}^j_{a+1} \psi_a(s, l_a) - \hat{\Omega}^j_a \psi_a(s, l_{a+1})], \]  
(10)

and the boundary conditions being

\[ \psi_a(I_a, l_a) = \psi_a(I_{a+1}, l_a) = 0, \quad \psi_a(I_{a+1}, l_a) = 1, \quad \psi_a(I_a, l_{a+1}) = -1 \]  
(11)

After defining functions \( \varphi_j(t) \) from (7), the vector of the force factors (forces and torque’s) is found in inertia couplings:

\[ Q_{ka}(s, t) = \frac{\sum \varphi_j(t) d(A_{ka} V_{ka})}{ds} \]  
(12)

In that way, the problem of the mine hoisting installation has been reduced to integration of equation system in ordinary derivatives of the type (4) and (7). The developed mathematical model makes it possible to determine the force factors in the main elements of the hoisting installation (on the main shaft of the machine, in the reducer gears, in ropes and shaft guides) during the whole winding cycle performed in accordance with the predetermined program of movement.

The calculation results can be used for designing the hoisting systems, for establishing the equipment resource, for expert inspection, for the optimal adjustment and attend of a hoisting machine, winding ropes, vessels, shaft guides and system of automatical control. The special PS-program permits to ascertain the reciprocity between the mentioned main elements of a hoisting installation to avoid unfavourable sequels. Of many years experience is convincing in necessity to make a verification with this program every time in the next cases: in process of projection, before setting in exploitation, after of any reconstruction, after of substitution some equipment, after of each received mine surveyor’ datas, before and after of each systematic adjusting works. Therefore every mine machine must has offering program as one of the means of adequate maintenance of the hoisting installation.

REFERENCES: