A STUDY OF THE SOUND PROPAGATION IN THE ROUNDED SECTION OF A PIPE

Professor habil. dr. P. ILGAJKOJIS, Lecturer dr. E. JOTAUTIENE - Lithuanian University of Agriculture

Abstract: In this paper, the sound propagation of the rounded pipe section was studied. Solution of problems related to sound wave propagation in rounded pipes are of special practical importance, since almost any piping system includes conjugations of straight section by means of curved ones. The plane and normal sound waves propagation in the rounded pipe section has been investigated. For solving this problem it was more convenient to examine the distribution of pressure and velocity in the plane at the joint of the straight and rounded section. Applying Sturm-Liouville principle has been solved this problem. In the next part of this paper, we analysed the sound propagation when wave radiated by piston in the rounded section of a pipe. The results presented in this study can be used to determine noise reduction at pipe bends.

Keywords: coordinate, equation of the wave field, rectangular cross-section.

Introduction

The problem concerning sound propagation in curved ducts has drawn special attention of numerous researchers [1,2,3,4]. On the one hand, this may be due to the fact that this problem is in the focus since it is the generalization of the theory of waveguides. On the other hand, solution of tasks related to the propagation of waves in curved ducts is of special practical importance, because almost any system of ducts includes conjugation of straight duct sections by means of curves.

The theoretical analysis

We analyse a normal plane wave travelling along the waveguide with a rectangular cross-section. A plane normal wave travelling along the rectilinear section shall be written as [6,7]:

\[ p_1 = e^{i(kx_1 - \omega t)} \]

Here \( p_1 \) is the pressure in this wave along the waveguide, \( t \) is the time, \( \omega \) is the circular frequency and \( k = \omega/c \) is the wave number.

This wave, upon reaching the curvilinear section, shall form the reflected field [6,7]

\[ p_r = \alpha_1 e^{-i(kx_1 - \omega t)} + \sum_{n} \alpha_n \cos \frac{n\pi x_1}{a} \sqrt{\frac{k^2}{\omega} - 1} e^{-i\omega t} \]

(2)

A part of a wave energy penetrates the curvilinear section of the waveguide. The wave field that passed the curvilinear section we shall connote by \( p_t [6,7] \)

\[ p_t = \beta_1 e^{i(kx_2 + \omega t)} + \sum_{n} \beta_n \cos \frac{n\pi x_2}{a} \sqrt{\frac{k^2}{\omega} - 1} e^{-i\omega t} \]

(3)

Here the coordinate \( x_2 \) is directed along this rectilinear section, \( y_2 \) is the transversal coordinate, \( a \) is the width.

As it seen, the coefficients \( \alpha_n \) and \( j5^n \), are the coefficients of reflection and transmission of plane wave through the curvilinear section. However, for solving of the problem it is more convenient instead of the system of the coefficients \( \alpha_n \) and \( \beta_n \) to examine the distribution of pressure \( p_1(y) \) and velocity \( v_1(y) \) in the plane at the joint of the first semiwaveguide and curvilinear section, as well as pressure \( p_2(y) \) and velocity \( v_2(y) \) at the joint between the curvilinear section and the second semiwaveguide.
Numerical results are obtained for a plane wave at the joint of the first semiwaveguide and rounded section shell shown in Figure 1.

![Figure 1](image)

Figure 1. The form of the plane wave in curved duct I cross-section when $1 - \omega = 5000$; $2 - \omega = 1000$

Similarly, instead of the system of transmission coefficients $\beta_n$ we shall introduce the functions $p_2(y)$ and $v_2(y)$ (pressure and velocity distribution) in the second cross-section, i.e., at the joint of the curvilinear section and the second semi-waveguide.

We search for the field the rounded section as the solution of a wave equation in the cylindrical coordinates $(r, \theta)$ [7].

\[
\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + k^2 p = 0 \tag{5}
\]

The solution of a wave equation for the curvilinear section may be express by means of the separation of the variables in the form of the following series [6]

\[
p(r, \theta) = \sum_{n=0}^{\infty} p(y, r) \left\{ a_n \cos \gamma_n(\theta - \varphi_n) - b_n \cos \gamma_n \theta \right\} \frac{\sin(\gamma_n \varphi_n)}{\gamma_n \sin(\gamma_n \varphi_n)} \tag{6}
\]

here $y_n$ is the constant separation of the variables which form the series of eigenvalues.
The boundary conditions for the equation of the wave field in the curvilinear section are

\[
\begin{align*}
   & p(r,0) = p_1(r - R_1) \\
   & \left. \frac{1}{r} \frac{\partial p}{\partial \theta} \right|_{\theta = 0} = \rho \omega^2 v_1 (r - R_1) \\
   & p(r, \varphi_o) = p_1(r - R_1) \\
   & \left. \frac{1}{r} \frac{\partial p}{\partial \theta} \right|_{\theta = \varphi_o} = \rho \omega^2 v_1 (r - R_1)
\end{align*}
\]

(7)

here \( \varphi_o \) is the angle of dissolving of the curvilinear section, \( R_1 \) is the inferior radius of rounding, \( R_1 + \alpha \) is the superior radius.

As it seen, the boundary conditions for the field \( p(r, \theta) \) in the essence are the condition for jointing wave fields at the different sections of composite waveguide.

From the wave equation (5) we shall obtain by Fourier’s method the following problem for radial components \( p(\gamma_o r) \), i.e., for eigenforms of the given Strum-Liouville problem

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) p(\gamma_o, r) + \left( k^2 - \frac{\gamma_o^2}{r^2} \right) p(\gamma_o, r) = 0
\]

(8)

The exact solution of this equation may be described by means of Bessel’s functions \( J_{\gamma_o}(kr) \) and Neumann’s functions \( N_{\gamma_o}(kr) \) in the following form

\[
p(\gamma_o, r) = J_{\gamma_o}(kr) N_{\gamma_o}(kR_1) - N_{\gamma_o}(kr) J_{\gamma_o}(kR_1)
\]

(9)

where \( \gamma_o \) is the solution of the frequency equation [7].

Figure 3. The solution of the rounded pipe characteristic equation, when \( i?_1 = 0.2 \text{ m and } R_2 = 0.4 \)
Applying Sturm-Liouville principle has been solved this problem.

\[ \int_0^1 v_1(y_1)K_{11}(y_1,r)dy_1 + \int_0^1 v_2(y_1)K_{12}(y_1,r)dy_1 - \rho \omega^2 v_1(r-R_1) = \Phi_1(r). \]

\[ \int_0^1 v_1(y_1)K_{21}(y_1,r)dy_1 + \int_0^1 v_2(y_1)K_{22}(y_1,r)dy_1 - \rho \omega^2 v_2(r-R_1) = \Phi_2(r). \]

We had obtained the integral equations that describe the plane and normal waves in the rounded section of a pipe.

**Sound wave radiation by piston source in curved duct.**

We present the analysis of simpler problem: on one end of bent part (knee) of rectangular cross-section there is a piston which fluctuates with the velocity \( v = v_0 e^{i\omega t}, \) where \( \omega = 2\pi f \) is the cyclic frequency. The other end of the knee \( (\phi = \pi/2) \) is connected with infinite straight duct. Full sound pressure in a rounded duct are [6,7]

\[ p_1(r, \varphi, z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} R_n(k_n r) \cos k_n z (\sin \nu_n \varphi + B_n \cos \nu_n \varphi) \]  

(12)

Here \( J_n \) and \( N_n \) are Bessel and Neumann functions of order \( n, k_n = m\pi/a, \) where \( a \) is knee width (size along the axis \( z \)), \( A_{nm} \) and \( B_n \) are some constant amplitudes, \( k_m = \sqrt{k^2 - k_n^2} \) is radial wave number, \( k = \omega/c \) is medium wave number.

It follows from (12) that at low frequencies to the frequency of first resonance \( f_1 = c/(2a) \) only a plane wave may propagate in a duct

\[ p_{20}(r, z) = \frac{2iv_0 \rho \omega}{a} e^{ik z} \sum_{n=1}^{\infty} \frac{J_n^2}{v_n I_{nm}}. \]

(13)

As it is known, the piston fluctuating at an amplitude \( v_0, \) radiates a sound wave in a straight duct having pressure

\[ p_0(x) = \rho c v_0. \]

(14)

The difference of sound pressure levels

\[ \Delta L = L_{20} - L_0 = 20 \log \frac{2k}{a} \sum_{n=1}^{\infty} \frac{J_n^2}{v_n I_{nm}}. \]

(15)

where \( L_{20} = 20 \log \left| \frac{p_{20}}{p} \right|, \quad L_0 = 20 \log \left| \frac{p_0}{p} \right|, \quad p = 2 \times 10^5 Pa \) is a zero level of sound pressure.
Expression (15) shows the changes of plane wave radiation by a piston to a straight duct through a knee relatively to a sound radiation by a piston directly to a straight duct.

Conclusions

The plane and normal sound wave propagation in the curved duct has been investigated. The boundary conditions were formed between the straight and rounded section of the pipe. The investigation of the wave field propagation in rounded pipe showed that the normal wave propagation we can calculate like the plane wave.

It was determined that the sound pressure of the plane wave in the curved section depended the constructing parameter and on the sound frequency. The suggested mathematical model allows to simplify the investigation of sound propagation process in the bend. The results presented in this study can be used to determine noise reduction at pipe bends.

References: