ANALYSIS THE SIGNIFICANCE OF RELIABLE EXPERIMENTALLY DETERMINED DISTRIBUTION LAWS

Lecturer dr. eng. Marius STAN, Petrolul-Gas University of Ploiesti

Abstract: Experimental data collected drift (random size) of the law affecting real experimental data set, so it is not the only manifestation of the set of theoretical parameters. The same law theoretically true distribution may materialize through an infinity of sets of experimental data due random factors influence. The paper presents practical ways of determining confidence intervals using Monte Carlo method. The central issue of how the analysis is to determine the parameters found empirical distribution so as accurately to model the system state.

Keywords: Gaussian, circulation system, analysis methods.

1. Structural analysis

The systems are equipped with the same gearing, and the table – with a mechanical power take off and a gear box (T) for TF, MR or P. Below we will note the structural schemes for the rotating system (SR) and for the circulation system (SC), respectively. For the centralised or group operating mode the main operating systems (SM, SR and SC) are operated by the same engine or group of engines or pumps P1 and P2, [1].

The total reliability function for this operating mode is calculated using the features of structural analysis methods [2].

The total reliability function in G1 case:

\[ R_{G1} = R_m \cdot R_t \cdot \left[1 - (1 - R_{tf-c-r})(1 - R_{pf-p})\right] \cdot R_{gf} \] (1)

where:

\[ R_{tf-c-r} = R_{tf} \cdot \left[1 - (1 - R_c)(1 - Rcv \cdot R_{mr} \cdot R_t)\right] \]

\[ R_{pf-p} = [1 - (1 - R_{pf1} \cdot R_p)(1 - R_{pf2} \cdot R_{pf2})] \]
Rm the reliability function of the group of n engines
Rch – the reliability function of swivel casing

There is also a second variant for the group operating mode, G2, where one of the pumps has its own group of engines and gearing, and for MR the power take off is connected either to TF or directly to T1, as shown in the chart below:
The total reliability function in this case is:

$$R_{G2} = [1 - (1 - R_i) \cdot (1 - R_{II})] \cdot R_{gf}$$  \hspace{1cm} (2)

where:

$$R_i = R_{m1,2} \cdot R_t \cdot \left[1 - (1 - R_{tf-c-r}) \cdot (1 - R_{pf1} \cdot R_{p1}) \right]$$

$$R_{II} = R_{m3} \cdot R_{t2} \cdot R_{pf2} \cdot R_{p2}$$

In order to carry out some purely reliability comparative analyses of finding the optimal operating mode, the designer may choose his own values for simulation so that he could decide which of the operating modes leads to a convenient value of reliability according to the number of elements taken into account and their grouping mode.

2. Distribution functions use.

Weibull distribution function is adaptive technology equipment during its life time the failures are mainly due to wear and or aging phenomena [1],[4].

$$R(\tau) = 1 - e^{-\left(\frac{\tau - \gamma}{\eta}\right)^{\beta}}$$, \hspace{1cm} \text{the values of variables are known.} \hspace{1cm} (3)

Gaussian normal distribution function or normal law is adaptable to different technical systems in which failures are due to many factors, which usually causes wear and tear phenomena of materials [1],[4].

$$R(\tau) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\tau} \exp\left[-\frac{1}{2\sigma^2}(\tau - \mu)^2\right] d\tau$$ \hspace{1cm} (4)
3. Simulations by models

The system functions (R) contained by the models described above can be continuous or discrete or signal functions (known reliability functions or chance number generating functions following various distributions simulated by means of the programming mode (MathCad)) and will replace block functions (R) within computer-assisted simulation.

The structural models thus obtained can allow us to carry out simulations useful to the design of safe and efficient systems, enhancing their performances.

This software package provides the user with a complete series of probability distributions with continuous or discrete variation and the chance number generators distributed according to the corresponding partition law [3].

The partition law WEIBULL adapts to the reliability study of technological plants during their running, when failures occur mainly because of the plant’s run out and/or ageing. The NORMAL partition law adapts to the various technical systems whose failures occur because of a great number of factors, which generally lead to materials’ wear and breakage.

Table 1. Reliability results modes

<table>
<thead>
<tr>
<th>Number simulations</th>
<th>Reliability total values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group1</td>
</tr>
<tr>
<td>100</td>
<td>0.858</td>
</tr>
<tr>
<td>1000</td>
<td>0.856</td>
</tr>
<tr>
<td>10000</td>
<td>0.848</td>
</tr>
<tr>
<td>100000</td>
<td>0.846</td>
</tr>
<tr>
<td>1000000</td>
<td>0.846</td>
</tr>
</tbody>
</table>

For the estimation of the probability of failure by frequency of into occurrence the sufficiently large number of computer simulations should be performed as Gauss and Weibull trials. Each trial consists of the generations of the random realizations all input quantities, performing the deterministic analysis of the values R as the functions of these realizations.

Analysis of accuracy must be performed using the asymptotic distribution of the obtained estimate not the quantity of gives probability. Then the average of these values from all trial is calculated with MathCad program model. The average this method its similarity and higher effectiveness it comparison which the estimation of the random probability frequency (rnorm and rweibull).
4. Conclusions

The author proposes that the method should be also applied for the other operating modes of drilling plants which were mentioned taking into account their characteristics. In are presented three structural models (G1, G2), whose graphs are displayed in descending order of reliability analysis and numerical simulation for a drilling plant.

The procedure of calculations is as follows: at which realisation of variable R is generated in accordance with its probability density function (this function MathCad is supposed to be know) and the value of the probability distribution function of quantity R corresponding is determined (this distribution is generated at MathCad system).

The average this method it is similarity and higher effectiveness it comparison which the estimation of the probability frequency. Analysis of accuracy must be performed using the asymptotic distribution of the obtained estimate not the quantity of gives probability.

The main contribution this article makes is the introduction of structural models in the calculation of complex systems’ reliability (drilling plants or production installations).

References

INVESTIGATION OF THE EFFECTS OF RUBBERIZED ASPHALT ON DECREASING THE NOISE

Prof.dr. Vasile BACRIA, Assoc.prof.dr.eng. Nicolae HERIŞANU
Politehnica University of Timișoara, Romania

Abstract: An important contribution to the noise generated by the road transportation means in the environment has the noise produced by the contact between the wheel and the rolling surface. In this paper we present an investigation of the effects of rubberized asphalt on decreasing the noise produced by the contact between the wheel and the rolling surface in the road traffic. In order to characterize the noise we have accomplished measurements in 62 points located near some of the most important crossings from Timisoara City. The results of the measurements were processed, analyzed, interpreted and compared with the admissible values defined by standards.

Keywords: acoustic pressure, statistical distribution, traffic intensity

1. Introduction

The road transportation means generate noise and vibrations which are highly detrimental for human being’s life and activity. The noise generated by the road transportation means depends on the traffic intensity and composition, as well as on the speed of vehicles and it is mainly generated by three sources: the engine, the transmission system and the contact between the wheels and the rolling surface. The noise generated by the contact between wheels and the road represents around 75% from the total noise generated by the vehicle. This depends on the nature and the state of the rolling surface.

![Fig.1. The influence of the nature of pavement on the noise.](image)

Figure 1 shows the way in which the nature of the rolling surface influences the noise level generated by different categories of vehicles. The noise generated by the contact between the tire and the road is due to the vibrations caused by the interaction between the rolling surface of the tyre and the asperities of the road clothes and in the case of the smooth surfaces it is generated by the expansion of the air contained between the profiles of the tire and the road. The noise is significant at speeds which exceed 50 km/h and the most clearly manifests itself the components with the frequencies between 30 and 50 Hz.

In order to decrease the noise generated by the contact between the wheel and the road one can use rubberized asphalt. In this paper we make some investigations on the effects of rubberized asphalt on decreasing the noise generated by the contact between the wheel and the rolling surface in the road traffic.
2. Propagation and noxious effects of the noise

The noise generated by transportation means propagates in the environment by spherical or cylindrical waves and at long distance from the sources, even by plane waves [1]. The noise is extremely injurious to human being’s nervous system generating psycho-physiological and blood circulation modifications as well as sleep disturbances. Also the visual function and endocrine gland are adversely affected and at the same time the noise generates auditory tiredness and sonorous trauma [3], [4].

In order to reduce the effects of the noise, limit values which cannot be exceeded are established. These limits are characterized by the equivalent noise level and by the noise curves (Cz). The equivalent noise level is defined by the expression

\[ L_{A_{eq},T} = 10 \log \left( \frac{1}{t_2-t_1} \int_{t_1}^{t_2} \frac{p_A^2(t)}{p_0^2} \, dt \right) \]

where \( L_{A_{eq},T} \) is the continuous equivalent level of acoustic pressure A-weighted, measured in dB, determined in a time interval which starts at \( t_1 \) and ends at \( t_2 \), \( p_0 \) is the refereed acoustic pressure (20 \( \mu \)Pa) and \( p_A(t) \) is the weighted instantaneous pressure of the acoustic signal.

The noise curves Cz define the relation between the characteristic frequency of a sound and the proper acoustic pressure level in the conditions of a subjective equivalent sensitivity.

In this respect the Romanian standard STAS 10009-88 “Urban acoustics” established the admissible limits of the noise level in urban environment, differentiated on zones and functional endorsements, technical category of streets established on the base of the technical settlements.

<table>
<thead>
<tr>
<th>Street type (according to STAS 10144-80)</th>
<th>( L_{eq} ) [dB]</th>
<th>( C_z ) [dB]</th>
<th>( L_{10} ) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I – main</td>
<td>75-85</td>
<td>70-80</td>
<td>85-95</td>
</tr>
<tr>
<td>II – linking</td>
<td>70</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>III – collecting</td>
<td>65</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>IV – local serving</td>
<td>60</td>
<td>55</td>
<td>70</td>
</tr>
</tbody>
</table>

For the noise levels generated on the streets, these values are presented in Table 1. In the same time the disposition of buildings on the streets of different technical types and also the road traffic organization must be made so that be assured the admissible limits for the street exterior noise level established in accordance with STAS 6161/1-89 to 50 dB measured at 2 m distance from the building, respectively the Cz45 curve.

Taking into consideration that the tire/road contact is an important source of noise in the road traffic from an urban area and the generated acoustic field is extremely complex, its study it is recommended to be experimentally performed.

3. Measurements accomplishment and analysis of results

Noise level measurements were performed in 62 measurement points which were located near some of the important crossings from Timișoara City [5], [6]. The measurements were performed using the Brüel & Kjaer 2237 Controller Integrating Sound Level Meter, the N.L.-20 Sound Level Meter and the Brüel & Kjaer 2250 Hand Held Analyzer. These ones allowed the recording of the most important parameters of the noise, such as: \( L_{eq} \) (equivalent noise level), \( L_{AE} \) (exposure level), \( L_{max} \) (maximum noise level), \( L_{min} \) (minimum noise level), \( L_{0,1}, L_5, L_{10}, L_{50}, L_{90}, L_{95} \) (percentage noise levels).
These parameters were determined during a continuous period of 8 hours (7.30-15.30), divided in 1 hour time intervals.

In order to perform the measurements, the microphone was placed next to the street’s border at 7.5 m distance from the axis of the first runway, at 1.30 m high from the ground.

Simultaneously with noise data recording, the traffic composition and intensity as well as the speed of the vehicles were determined.

From the obtained data it results that in 46 measurement points (from a total of 62 measurement points), which means 74.19% of the measurement points, the equivalent noise level exceeds the maximum admissible value defined by the Romanian standard STAS 10009-88 concerning “Urban acoustics”. The overtaking was included into the interval 0.1-16.1 dB and the average equivalent noise level for the 62 measured points was 70.76 dB.

**Table 2. Statistical distribution of the noise level**

<table>
<thead>
<tr>
<th>L_{eq}</th>
<th>No. of points</th>
<th>%</th>
<th>Percentage of disturbed people</th>
</tr>
</thead>
<tbody>
<tr>
<td>{54.3}</td>
<td>1</td>
<td>1.6</td>
<td>8</td>
</tr>
<tr>
<td>[55÷60]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[60.3÷64.6]</td>
<td>6</td>
<td>9.7</td>
<td>[25÷41]</td>
</tr>
<tr>
<td>[65.2÷70]</td>
<td>20</td>
<td>32.3</td>
<td>[42÷60]</td>
</tr>
<tr>
<td>[70.1÷74.9]</td>
<td>24</td>
<td>38.7</td>
<td>[60.1÷79.9]</td>
</tr>
<tr>
<td>[75.1÷79.5]</td>
<td>8</td>
<td>12.9</td>
<td>[80.1÷97.9]</td>
</tr>
<tr>
<td>{81.8}</td>
<td>1</td>
<td>1.6</td>
<td>100</td>
</tr>
<tr>
<td>[85.5÷85.9]</td>
<td>2</td>
<td>3.2</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2 presents the statistical distribution of the equivalent noise level (L_{eq}) in the measured points, as well as the percentage of disturbed people [7].

Fig.2 shows a chart of the percentage of equivalent noise levels recorded in the measurement points. In these measurement points, the admissible noise level established to 50 dB(A) measured at 2 meters distance from the buildings was generally exceeded with 1.3-32.9 dB(A).

The traffic intensity ranged during measurements between 9 and 2681 aut./h while the speed of vehicles ranged between 50 and 60 km/h. Also, the percentage of different transportation means ranged during measurements between 0.01 (tractors) and 95.27 (cars).

![Fig.2. Percentage of noise levels in the measured points.](image-url)
4. Methods used to noise reduction

In order to reduce the noise in Timișoara City, on many streets the superstructure of the runway was improved. Many crossings were modernised and semaphores were installed. One-way traffic was imposed for some streets, the speed of vehicles was limited and on many streets were installed speed limiters. It was eliminated the presence in traffic of heavy trucks in the central area of the City. In some area it was allowed the access only for certain categories of vehicles. Some acoustic screens were installed between the runways and the residential areas and protective green zones were planted. In the N-E of the City was activated the ring road, which re-direct the heavy traffic in this direction.

The effect of the implementation of these measures for noise reduction were evaluated through new measurements performed in 9 measurement points, selected near some of the most important crossings from Timișoara City. From the obtained data it results that in the 9 measured points, the equivalent noise level was reduced with 0,1-9,4 dB and in 5 points (55,5%) the noise level does not exceed the admissible value defined by STAS 10009-88.

In the following, we present a comparison between the situation existing in these 9 measurement points before and after the implementation of noise reduction measures.

<table>
<thead>
<tr>
<th>Table 3. Statistical distribution of noise level before the application of the noise reduction methods in 9 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{eq}$ [dB]</td>
</tr>
<tr>
<td>[66.2-69.7]</td>
</tr>
<tr>
<td>[70.1-74.9]</td>
</tr>
<tr>
<td>[75.6-77.1]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4. Statistical distribution of noise level after the application of the noise reduction methods in 9 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{eq}$ [dB]</td>
</tr>
<tr>
<td>[61.8-62.8]</td>
</tr>
<tr>
<td>[71.3-72.9]</td>
</tr>
</tbody>
</table>

Figures 3 and 4 present the chart of the equivalent noise levels percentage in those 9 measuring points before and after the implementation of noise abatement methods.
From these charts it can be observed the degree of reduction of the noise level and the diminution of the percentages of disturbed people as a result of the implementation of the noise abatement methods. The average equivalent noise level in these 9 measuring points was 72.07 dB(A) for an average traffic intensity of 1595.6 aut/h before the application of noise abatement methods and 67.84 dB(A) for an average traffic intensity of 1712.8 aut/h after the implementation. As regards the average equivalent noise level existent at 2 meters distance from buildings, this one was 65.6 dB(A) before the application of the noise abatement methods and 62.7 dB after that.

The percentages of transportation means in these 9 measured points ranged between 0.02 (tractors) and 95.27 (cars) before application of the noise abatement methods and these percentages ranged between 0.1 (tractors) and 93.6 (cars) after application of the noise abatement methods.

5. Effects of the rubberized asphalt on decreasing the noise

Analysing these data one can say that by implementing the decreasing methods for the noise generated by the road transportation means in Timișoara City it was obtained indeed the decreasing of noise levels but the admissible limit established by STAS 10009-88 are still exceeded.

In this case other noise abatement methods are needed to be established and implemented in order to diminish further the noise generated by transportation means.

Therefore, taking account that an important contribution to the noise generated by the road transportation means on the roads has the tire/road contact, this can be reduced by covering the road with rubberized asphalt or with rubber pavement. Using these measures it is obtained a reduction of the noise level with an average of at least 4 dB. This is used also for increasing the traffic safety through the elimination of vehicle skidding.

Rubberized asphalt consists of regular asphalt mixed with crumb rubber obtained from used tires that would otherwise be discarded or take up space in landfills.

Investigating the efficiency of the rubberized asphalt used on the streets from Timișoara City, one can conclude that from a total of 62 measurement points, in 26 of them (41.93%) the admissible limit is exceeded. The overtaking would be included into the interval 0.3-12.1 dB.

Table 5 presents the statistical distribution of the equivalent noise level ($L_{eq}$) in the 62 measurement points as well as the percentage of disturbed people after the application of the rubberized asphalt on streets.

<table>
<thead>
<tr>
<th>$L_{eq}$</th>
<th>No. of points</th>
<th>%</th>
<th>Percentage of disturbed people</th>
</tr>
</thead>
<tbody>
<tr>
<td>{50,3}</td>
<td>1</td>
<td>1.6</td>
<td>0</td>
</tr>
<tr>
<td>[56.3÷58.2]</td>
<td>5</td>
<td>8.1</td>
<td>[12÷23]</td>
</tr>
<tr>
<td>[60.6÷64.9]</td>
<td>13</td>
<td>21</td>
<td>[25.1÷43]</td>
</tr>
<tr>
<td>[65.1÷69.1]</td>
<td>28</td>
<td>45.2</td>
<td>[43.1÷59]</td>
</tr>
<tr>
<td>[70.0÷73.3]</td>
<td>11</td>
<td>17.7</td>
<td>[60÷70.1]</td>
</tr>
<tr>
<td>[75.5÷77.8]</td>
<td>2</td>
<td>3.2</td>
<td>[80.1÷95]</td>
</tr>
<tr>
<td>[81.5÷81.9]</td>
<td>2</td>
<td>3.2</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 5 shows a chart of the percentage of equivalent noise levels in this case. Comparing data from tables 2 and 5, or from the corresponding charts, it can be observed the degree of reduction of the noise level and the diminution of the percentages of disturbed people as a result of the application of the rubberized asphalt on streets in Timișoara City.

6. Conclusions
The decreasing of noise generated by the contact between the tire and the rolling surface leads to a diminution of the degree of phonic pollution and a diminution of the percentages of disturbed people. This can be obtained by covering the rolling surface with rubberized asphalt. Using this measure it is obtained a reduction of the noise level with an average of 4 dB. Once the implemented method proves its efficiency, it can be applied in every practical situation concerning traffic or industrial noise.

The alignment to European regulations imply yet the application of the Directive 2002/49/EC concerning the management of the environmental noise, this one needing beside noise mapping, a permanent monitoring of the noise in real conditions, by real measurements.

References
[2]. STAN A., et al., Considerații asupra nivelului intensității zgomotului de traffic în exteriorul clădirilor din zona centrală și semicentrală a municipiului București, Construcții, nr.10, 1973
[5]. BACRIA V., HERIȘANU N., Some aspects concerning the decrease of the noise generated by the contact between the tire and the road, Proceedings of the Symposium “Research People and actual tasks on multidisciplinary sciences”, Lozeneck, Bulgaria, 10-12 June 2008, p.137-141
[7]. *** Larmbekamfung in Wien. Enwicklung Stand Tendenzen, Magistratsabteilung 22 Umweltschutz
DEMONSTRATION AND METHOD FOR CALCULATING THE EFFICIENCY OF DIFFERENTIAL MECHANISM

Assoc. Prof. Barbu PLOSCEANU, PhD, POLITEHNICA University of Bucharest,
Assist. Prof. Ovidiu VASILE, PhD, POLITEHNICA University of Bucharest,

Abstract - To solve the problem of calculating the power circulation and efficiency, the mechanical transmission cycles are reform the notion of efficiency. Using the principle of virtual velocities presented a demonstration to calculate power flux indicator. Finally, examples of the method for calculation of differential type mechanisms.

Keywords: kinematic, forces, transfer ratio

1. Introduction

In the case of transmissions formed by mechanisms connected in series the problem of power flux transmission naturally results from the input to the output, which means from the motor element (driver) to the resistant element (driven).

For transmissions with connections in series and in parallel, which form cycles (see fig. 1.a) the power flux transmission doesn’t result in a naturally way. In that case the transmission of power flux can take place, by example, according to one of variants outlined in figure 1.b. In figure 1 is noted: MM - car engines; ML – work machine; R – reducer; D- differential mechanism.

As a consequence in such situation one asks which is the power flux through transmission branches, how much of external motor power pass through each branch, which is the torsor forces which operates on transmission elements, how one calculates the efficiency, and so on. For answering at such questions, we must make first some reformulations of notions we are talking about.

Significant in resolving the relationship for calculating the coefficient of distribution of power transmission industries. Based on data known in literature [2], [3] [4] [5] [6], give a demonstration unit for this deduction factor and exemplifies

![Diagram of transmission cycles](image)

2. Kinematic transmission ratio, forces transfer ratio, efficiency [1], [2], [3]

Let’s consider the transmission with gears with fixed axis (see figure no. 2) at witch the element 1 is the motor element, and then $\omega_M^1$ and the element 2 is driven element and then $\omega_M^2$.

As a consequence, the transmission efficiency (gearing) is:

$$\eta_{12} = -\frac{P_U}{P_m} = -\frac{M_2\omega_2}{M_1\omega_1},$$

(1)
One notes:
- the forces transfer ratio (of torsor forces), with $\nu_{i2} = -\frac{M_2}{M_1}$
- the ratio of transmission velocities (of torsor velocities) with $i_{i2} = \omega_1/\omega_2$

With this notation the relation (2) can be written: $\nu_{i2} = \eta_{i2} \cdot i_{i2}$

Then $\eta_{i2} = \nu_{i2} / i_{i2}$

If one neglects the efficiency ($\eta_{i2} = 1$), then $\nu_{i2} = i_{i2}$

Let’s consider now, the transmission composed by mechanisms connected in series from figure 3.

The kinematical transmission ratio is: $i_{i3} = i_{i2} \cdot i_{i3}$

The transfer forces ratio, if 1 is the motor element and 3 is the driven element, will be:

$\nu_{i3} = \eta_{i2} \cdot i_{i2} \cdot \eta_{i3} \cdot i_{i3} = i_{i2} \cdot i_{i2} \cdot \eta_{i2} \cdot \eta_{i3}$

But the relation (8) can more be written: $i_{i3} = i_{i2} / i_{i3}$ then

$\nu_{i3} = \frac{i_{i2}}{i_{i3}} \cdot \eta_{i2} \cdot \eta_{i3} = \frac{i_{i2} \eta_{i2}}{i_{i3} \eta_{i3}^{-1}}$

From relation (10) results that if kinematical transmission ratio is taken in the sense of power flux, that the forces transfer ratio can be obtained by multiplying the kinematical transmission ratio with the efficiency at power $+1$, and if the kinematical transmission ratio is taken in opposite sense of power flux, than the forces transfer ratio is obtained by multiplying the kinematical transmission ratio with the efficiency at power $-1$.

Then, for any transmission, if one notes with $x_i = \pm 1$, the exponent of efficiency, than the forces transfer ratio is a function like:

$\nu_{i2} = f(i_1 \eta_1^{x_1} ; i_2 \eta_2^{x_2} ; i_3 \eta_3^{x_3} ; \ldots ; i_n \eta_n^{x_n})$

Therefore the force’s transfer ratio is a function which depends on $i_1 , i_2 , \ldots , i_n$ - the kinematics transmission ratios of component mechanism; on their efficiencies $\eta_1 , \eta_2 , \eta_3 , \ldots , \eta_n$ and also depends on sense of power circulation through the exponents $x_1 , x_2 , x_3 , \ldots , x_n$.

The transmission velocities ratio is a function intermediate transmission ratio, that means:

$i_{in} = f(i_1 , i_2 , \ldots , i_n)$

Then, generalizing the relation (6) the efficiency one calculates with the expression:
In the relation (13), the exponents \( x_1, x_2, x_3, ..., x_n \) take the value +1 if the power flux coincides (has the same sense) with the kinematical one, and take the value -1, to the contrary.

As a result, for knowing which is the input and which is the output (which is the motor element and which is the driven element) for a component mechanism of a mechanic transmission as that from figure 1.b, it must be investigated (established) the exponents \( x_i \).

Also one asks how much of external power circulates through each branch, and final which is the efficiency of transmission.

3. Coefficient of power repartition. Power flux indicator

Let’s consider the transmission from figure 4, where 1 is the motor element and then \( M_1 \omega_1 \), and "n" is the driven element and then \( M_n \omega_m \), and \( R_i \), \( i = 1, ..., p \), are the component elements.

For the establishing how much of motor power circulates through each branch we can compare the motor power \( P_i = M_1 \cdot \omega_1 \) with the power applicant to the element \( k \) which is in a rotation motion, \( P_k = M_k \cdot \omega_k \), which belongs to component mechanism \( R_m \).

The ratio of two powers show how much of external motor power pass through that mechanism. This ratio is called the coefficient of power repartition [3], [4], [5] and one note:

\[
\lambda_k = \frac{P_k}{P_m}
\]  

(14)

With this, for knowing if the \( k \) element is a motor one or a resistant one, we will calculate the sign of this ratio, which means the indicator of power flux:

\[
x_k = \text{sign} \lambda_k
\]  

(15)

With this, it means that we can say which is the sense of power flux through each mechanism, respective through each branch of transmission; how much of motor power (the external) circulates through each branch; and taking into account the relation (13) which is the efficiency of transmission.

The solution of problem – the coefficient of power repartition – one obtains application the principle of virtual velocities (virtual powers) to transmission in which the connection – let be this a gearing coupling – from elements \( k \) and \( h \) is changed with forces \( F \) and \( -F \), and these (the elements) take the virtual values \( \tilde{\omega}_k \) and respective \( \tilde{\omega}_h \) (fig. 4.b).

For the considered transmission, the motor power is \( P_1 = M_1 \cdot \omega_1 \). As a result of connection changing between elements \( k \) and \( h \), the element \( h \) gets the angular velocity \( \tilde{\omega}_h \), with unknown size and sense. Let be than \( \tilde{\omega}_n \) with the same sense of resistant moment \( M_n \), the real sense following to result from calculus. Therefore \( P_n = M_n \cdot \tilde{\omega}_n \).

The power according to element \( k \) is \( P_k = -Fr_k \tilde{\omega}_k \), and the power according to element \( h \) is \( P_h = Fr_h \tilde{\omega}_h \).

Then, according to principle of virtual powers:

\[
P = M_1 \omega_1 + M_n \tilde{\omega}_n - Fr_k \tilde{\omega}_k + Fr_h \tilde{\omega}_h
\]  

(16)
And because at the application of principle of virtual powers, one neglects the frictions, according to (7) \( \nu_{12} = i_{12} = -\frac{M_2}{M_1} \). As a result relation (16) one writes:

\[
P = M_1 \omega_1 - M_1 i_{1n} \tilde{\omega}_n - F r_k \tilde{\omega}_k \left( 1 - \frac{r_h \tilde{\omega}_h}{r_k \tilde{\omega}_k} \right)
\]  

(17)

One notes:

\( \tilde{i}_{nn} = \omega_i / \tilde{\omega}_n \) → the kinetic transmission ratio according to virtual velocities impressed,

\( M_k = F r_k \) → the moment applicant to element \( k \),

\( i_{kh} = r_h / r_k \) → the kinetic transmission ratio according to real velocities,

\( \tilde{i}_{kh} = \tilde{\omega}_k / \tilde{\omega}_h \) → the kinetic transmission ratio according to virtual velocities,

With this notation the relation (17) can be written:

\[
P = M_1 \omega_1 \left( 1 - \frac{i_{1n}}{\tilde{i}_{nn}} \right) - M_k \tilde{\omega}_k \left( 1 - \frac{i_{kh}}{\tilde{i}_{kh}} \right) = 0
\]

or:

\[
P = M_1 \omega_1 \left( \frac{\tilde{i}_{nn} - i_{1n}}{\tilde{i}_{nn}} \right) - M_k \tilde{\omega}_k \left( \frac{\tilde{i}_{kh} - i_{kh}}{\tilde{i}_{kh}} \right) = 0
\]

From where:

\[
\frac{M_k \tilde{\omega}_k}{M_1 \omega_1} = \frac{\tilde{i}_{kh} \left( \tilde{i}_{nn} - i_{1n} \right)}{\tilde{i}_{nn} \left( \tilde{i}_{kh} - i_{kh} \right)}
\]  

(18)

But \( \tilde{i}_{1n} \) is a function of \( \tilde{i}_{kh} \) and \( i_{1n} \) is a fixed value of function \( \tilde{i}_{1n} \) for \( i_{kh} \) fixed (\( \tilde{i}_{kh} = i_{kh} \)).

Than the coefficient of power repartition on element \( k \) is:

\[
\lambda_k = \lim_{\tilde{\omega}_h \to \omega_h} \frac{M_k \tilde{\omega}_k}{M_1 \omega_1} = \lim_{\tilde{i}_{1n} \to i_{1n}} \frac{\tilde{i}_{kh} \left( \tilde{i}_{nn} - i_{1n} \right)}{\tilde{i}_{nn} \left( \tilde{i}_{kh} - i_{kh} \right)} = \frac{\tilde{i}_{kh}}{i_{1n}} \cdot \lim_{\tilde{i}_{1n} \to i_{1n}} \frac{\tilde{i}_{nn} - i_{1n}}{\tilde{i}_{nn} - \tilde{i}_{1n} - i_{kh} - \tilde{i}_{kh}} = \frac{i_{kh}}{i_{1n}} \cdot \frac{\partial i_{1n}}{\partial i_{kh}}
\]

Therefore:

\[
\lambda_k = \frac{i_{kh}}{i_{1n}} \cdot \frac{\partial i_{1n}}{\partial i_{kh}}
\]  

(19)

From the relation (19) results that both the power flux sense, by means (15) and the power repartition coefficient can be determined on kinematics conditions.
4. Conclusion

In conclusion for calculus of the efficiency of a mechanical transmission with cycles we suggest to pass the following steps:

a) the calculus of transmitting ratio velocities, that means function (12);

b) the setting up of function which gives the transfer ratio of torsor forces, that \( \nu_{kh} = i_{kh} \eta^x_{kh} \), and then replacing in (11) \( i_{kh} \) with \( \nu_{kh} \);

c) one calculates all coefficients of power repartition \( \lambda_k \), with relation (19);

d) one calculates the power flux indicator, that means all \( x_k \), with relation (15);

e) one changes the fluctuation set up at step "b" taking into account that \( \eta_{kh}^{-1} = 1/\eta_{kh} \) if \( \eta_{kh} \) is approximate equal to \( \eta_{kh} \); and \( \eta_{kh}^{-1} = 1/\eta_{kh} \) if \( \eta_{kh} \) differs very much of \( \eta_{kh} \);

f) one calculates the transmission efficiency with relation (13).

A very simple example of mechanical transmission witch generates the phenomena of power circulation is that of a differential mechanism and in private the planetary mechanism[8]. Thus, the differential mechanism simple aggregate relationship (19) write:

\[
\lambda_D = \frac{i_0}{i_n} \frac{\partial i_n}{\partial i_0}
\]  

(20)

where \( i_0 \) is the basic gear ratio, the ratio of kinematical transmission mechanism caused differential arm fixed planet carrier.

Whether the transmission shown in Figure 5 is a key factor driving states, led element 3, worm gear \( Z_2' = 20 \), \( Z_4 = 40 \) and \( Z_4' = 30 \), \( Z_2' = 30 \) and differential gear mechanism \( Z_1 = 20 \), \( Z_2 = 60 \) and \( Z_5 = 20 \).

The mechanical efficiency of the gear torque is equal to 0.98 and the basic mechanical efficiency (the differential arm brought fixed planet carrier) is \( \eta_{12} = \eta_0 = 0.96 \).

Is required to calculate transmission efficiency \( \eta_{13} \).

Kinematical transmission is described by the equations:

\[
\omega_4 = i_{44} \omega_1; \quad \omega_2 = i_{24} \omega_4; \quad \omega_1 = i_{12}^3 \omega_2 + (1 - i_{12}^3) \omega_3
\]

with \( i_{12}^3 = i_0 \) - base gear ratio.
From relations (20) by substitution yields:

$$i_{13} = \frac{\omega_1}{\omega_3} = \frac{1-i_0}{1-i_{41} \cdot i_{24} \cdot i_0}$$

For differential mechanism

$$\lambda_D = \frac{i_0}{i_{13}} \frac{\partial i_{13}}{\partial i_0} = \frac{i_0 (2i_{41} \cdot i_{24} - 1 - i_{41} \cdot i_{24} \cdot i_0)}{(1-i_0)(1-i_{41} \cdot i_{24} \cdot i_0)} = -0.45 \Rightarrow x_D = \text{sign} \lambda_D = -1$$

For gear wheels with $Z_1'$ and $Z_4$, $\lambda_{14} = \frac{i_{41}}{i_{13}} \frac{\partial i_{13}}{\partial i_{41}} = \frac{i_{41} \cdot i_{24} \cdot i_0}{1-i_{41} \cdot i_{24} \cdot i_0} = -0.6 \Rightarrow x_{14} = -1$

For gear wheels with $Z_4'$ and $Z_2$, $\lambda_{24} = \frac{i_{24}}{i_{13}} \frac{\partial i_{13}}{\partial i_{24}} = \frac{i_{24} \cdot i_0}{1-i_{41} \cdot i_{24} \cdot i_0} = -0.6 \Rightarrow x_{24} = -1$

Results $\nu_{13} = \frac{1-i_0 \eta_1^{-1}}{1-i_{41} \cdot \eta_1^{-1} \cdot i_{24} \cdot \eta_2^{-1} \cdot \eta_0^{-1} \cdot i_0 \cdot \eta_0^{-1}} = 1.57$ and $\eta_{13} = \frac{\nu_{13}}{i_{13}} = \frac{1.57}{1.6} = 0.98$.

References

ANALYTICAL MODEL OF CALCULUS FOR INFLUENCE THE TRANSLATION GUIDE WEAR OVER THE MACHINING ACCURACY ON THE MACHINE TOOL

Assoc.prof.dr.eng. Ivona PETRE, University Valahia from Targoviste, 
Lecturer dr. eng. Carmen POPA, University Valahia from Targoviste, 
Lecturer dr. eng. Dumitru DUMITRU, University Valahia from Targoviste, 
Eng. Ciprian MANESCU, University Valahia from Targoviste,

Abstract: The wear of machine tools guides influences favorably to vibrations. As a result of guides wear, the initial trajectory of cutting tools motion will be modified, the generating dimensional accuracy discrepancies and deviations of geometrical shape of the work pieces. As it has already been known, the wear of mobile and rigid guides is determined by many parameters (pressure, velocity, friction length, lubrication, material). The choice of one or another analytic model and/or the experimental model of the wear is depending by the working conditions, assuming that the coupling material is known. The present work’s goal is to establish an analytic model of calculus showing the influence of the translation guides wear over the machining accuracy on machine-tools.

Keywords: accuracy, machine-tools, surface.

1. Introduction
As it has already been known the accuracy of machining each piece depends on a multitude of factors connected to the technological system (the machine tool, the clamping device, the cutting tool etc) [1, 2, 3].

In the present work, the author’s goal is to establish the size and influence only of the wear of the bed the slide guide over the machining accuracy on machine-tools with use on lather. The size of the guide and (longitudinal) slide wear are important to be known because the change of the slide trajectory due to the wear that occurs in time conditions the dimensional deviations and the quality of the machined parts surface.

In order to establish the wear size of the system bed-slide, three distinct situations are being analyzed:
1 – the bed guide only is being worn
2 – the slide guide only is being worn
3 – both guides (bed-slide) are being worn

2. Analytic models establishing the error produced by the bed-slides wear
In order to establish the quantitative influence of the wear process over the machining accuracy on a lathe, the following assumptions are made [2]:
- the curve of the worn profile of the rigid guide $U(x)$ is considered known at a certain moment assessed by the number of the stress cycles;
- the wear on the current stress cycles is neglected compared to the wear produced by all the previous cycles;
- the mobile guide wear $U_1(l)$ is so produced that the profile of this guide follows continuously the rigid guide profile; based on this assumption it is admitted that the contact is always of the type according to the appropriate distribution of the normal and tangential stress;
- the mobile guide is set on the rigid one during the displacement so that normal line to the contact surface of the two profiles is unique;
- the wear is a continuous process and is characterized by the thickness of the worn layer, considered as a continuous function onto the guide length and in time.
Based on these assumptions the wear of the bed and slide guide is established for the 3 situation shown in the first part of the work.

Case 1: the bed guide is being only (fig.1)

Taking a point A \((x_0, U_{xo})\) situated at the middle of the mobile slide. The wear of the bed guide in the considered point is \(U_{xo}\) and the function of the bed guide wear \(U_x\). The deviation of the point A is considered \(\Delta_1 = AC\).

In order to establish, the coordinates of the point C the normal at the bed guide profile in the point A is considered a straight line with the slope \(m\) that can be determined based on the wear function \(U_x\).

\[
y = mx + n
\]  
\[
m = -\frac{1}{U'_x}x + n = U_x + \frac{1}{U_x}
\]  

Where: \(U_x = -\frac{1}{U'_x}x + n\) and \(n = U_x + \frac{1}{U_x}\) x

Being the conditions to limit, the co-ordinates of the point C will be \(y_c = 0\) and \(x_c = U_x U'_x + x\).

The deviation in this case considered:

\[
\Delta_1 = \sqrt{(x_c - x_A)^2 + (y_c - y_A)^2} = U_x \sqrt{1 + (U'_x)^2} \leq \Delta_{adm}
\]  

Case 2: the slide guide only is being worn according to a linear variation law (fig.2).

Considering the point \(B(0,b)\) situated at the half of the slide with the length \(l\). After a certain period of functioning, as a result of the slide wear, the point \(B\) will be displaced in the point \(B'\). In this situation the deviation of the point will be: \(\Delta_2 = BB'\)

In order to make the calculation of this deviation 2 cases can be distinguished:

a) the point A is not worn \(U_A = 0\);

b) the point A is worn by a size that can be established (measured) \(U_A \neq 0\)
• Staring from the equation of the straight line that goes through the points AO’ and following the some argument as in case 1, the deviation of the point B’ will be:

- for \( U_A = 0 \) \( \Delta_2 = 2\sqrt{a^2 + b^2} \cdot \cos \alpha / 2 \)
- for \( U_A \neq 0 \) \( \Delta'_2 = 2\sqrt{a^2 + (b + U_A)^2} \cdot \cos \alpha / 2 \)

The size of the angle \( \alpha \) depends on the size of the accuracy deviation imposed to the slide:

\[
\tan \alpha = \frac{MC}{AM} = \frac{U_c}{l}
\]  

(4)

The angle \( \alpha \) can be known if the wear of the point C, \( U_s \) and \( \alpha < \alpha_{adm} \) are measured (known).

The allowable size of the angle \( \alpha_{adm} \) is a size that can be established from the accuracy conditions imposed by the machine tool on which the slide is installed where:

\[
\tan \alpha = \frac{A_i + A_{pp}}{l} = \frac{A_i}{l} \]

(5)

where: \( A_i \) – the initial deviation that can be accepted as being the maximum allowable deviation prescribed within the accuracy parameters of the machine-tool;

\( A_{pp} \) – the accuracy deviation allowable of the parts machined on the machine tool;

\( A_{ra} \) – the allowable relative deviation of accuracy;

\( l \) – slide length

Case 3: both the mobile slide \( U_1(l) \) and the bed \( U(x) \) are worn as in figure 3.

![Fig. 3. Assessment scheme of the changes caused by wear](image)

The straight line at the bed guide in a point is a line having \( m \) slope that can be established based on the wear function \( U(x) \). Following the same argument as in cases 1 and 2 for a common point M on the bed and slide guide the changes of the co-ordinates of this point will be:

\[
\Delta y_M = AM + MB_1 = U(x_M) + U_1(l_M) \cdot \frac{1}{\sqrt{1 + (U'(x_M))^2}}
\]

(6)

\[
\Delta x_M = A_1 M \cdot \sin \gamma = U_1(l_M) \cdot \frac{y'(x_M)}{\sqrt{1 + (U'(x_M))^2}}
\]

(7)

The deviation from the abscissa implies the axial position change of different tronsons of the machined parts.
The total deviation $\Delta a$ is considered to be the size that characterizes the loss of accuracy as a result of the wear process of the mobile and fixed guide:

$$\Delta a = \sqrt{\Delta y^2 + \Delta x^2} \leq \Delta a_{\text{adm}}$$  \hspace{1cm} (8)

### 3. Explanation of the wear function

As concerns the type’s explanation of the process that has produced them there is no unanimous agreement. Actually, the different way some or other assumptions with reference to the evaluation of the complex phenomena a friction-wear-lubrication is accepted, has influenced the classification of the various types of wear noticed in the industrial practice regarding the aspect and the deterioration degree of the surfaces.

Depending on the way and size of the wear process measurement of a friction couple slide bed guide type the most accessible to any engineer is to establish the worn layer thickness namely the linear wear [4].

In order to find a more correct relation ship of the wear size a linear dependence between the wear time $t$ and the wear size $U$ is considered, the wear velocity being constant in time:

$$v_a = \frac{dU}{dt}$$  \hspace{1cm} (9)

Starting from the assumption unanimous recognized namely at the no continuous lubrication film can be formed at the normal lathers, the wear is assessed to be adhesive and/or abrasive type and the removal of a wear particle is a cumulative process of contact fatigue through elastic, plastic or elastoplastic deformations [2, 3, 4].

Under this circumstances the dimensionless linear intensity ($I_u$) considered as an indication of the wear process, depending on the material characteristics (elasticity modulus $E$, rupture stress for one cycle $\sigma_0$, the fatigue parameter Wöhler $l_0$ type), also depending on the microgeometry characteristics and on loading characteristics (contact pressure $p$, friction coefficient $\mu$) [2, 4] the form of the wear in a point $M$ situated on the couple bed slide will be:

- for the bed guide:

$$U_{\text{xM}} = \int_{l_1}^{l_2} I_{u_1} dL_{f_1} = N_o H_b \int_{l_1}^{l_2} I_{u_2} \varphi(x - l'_M) p_{r_{p}} (l'_M) p_{r_{v}} (x) dM$$  \hspace{1cm} (10)

- for the slide guide:

$$U_{\text{fM}} (l'_M) = \int_{0}^{l_2} I_{u_2} dL_{f_1} = N_o H_b I_{u_{01}} p_{r_{p}} (l'_M) p_{r_{v}} (x)$$  \hspace{1cm} (11)

where:
- $N_o$ – the number of the total strokes within the warning period;
- $l_1, l_2$ – lengths characteristic for the wear area of the bed guide;
- $\varphi(x), p_{r_{p}}(x), p_{r_{v}}(x)$ – the use coefficient, the load coefficient and velocity coefficient that are considered to have different variation laws (uniform, linear, normal);
- $H_b$ – the total length of the bed guide;
- $I_{u_{01}}, I_{u_{02}}$ – the wear intensity of the bed guide material, respective of the slide.
4. The quantitative influence of the wear process over the machining process

Based in the three cases that have been analyzed (when the bed guide only is worn, when the slide guide only is worn or both) the deviations, that characterize the accuracy loss as a result of the wear process of the friction couple elements, can be established knowing the wear size.

Thus, taking into consideration the most disadvantageous case (but the most possible one) when both the bed guide and the slide guide are worn figure 4 shows the deviation of the radius (deviations from the co-ordinate of the considered point) of axial positioning (deviations from abscises of the analyzed conditions expressed by the variation laws of the use coefficient, loading coefficient and velocity coefficient that characterize the couple wear tribologically standpoint [2].

Fig. 4. Deviations from the position of the bed-slide
5. Conclusions

As can be noticed in figure 4, the deviations of axial position (Δx) show discontinuities in the passing points from one working area to another one, function of the worn guide curve slide. This discontinuity results from the condition that the profiles of the worn mobile guide and of the fixed one are always continuously followed and they have a common contact point.

The radius deviations (Δy) are always continuous functions, so as the profiles of the machined parts in the radial direction will be “smooth” but with circular shapes of various radius.

Given the facts presented, we can conclude that by having the possibility to know the wear size beginning with the projecting status, a better choice of the material couples can be made for bed-slide, a correct evaluation of the overall dimensions of slide and of the guide can be made and an estimation closer to the ones as concerns the size of the exploitation factors (pressure, velocity, lubrication etc.).

We can’t ignore the accuracy over the machined part shape, slide that as it dread been known can be influenced in a smaller or bigger way by the shape of the bed and slide guide wear.

References
MODELING AND SIMULATION OF CYCLOID CURVES WITH APPLICATION IN ROBOTICS

Lecturer dr. eng. Lucia PASCALE¹, Assoc.prof.dr.eng. Paul Ciprian PATIC¹, Assoc.prof.dr.eng. Luminiţa DUŢĂ², Lecturer dr. eng. Adrian RUNCEANU²

1 - „VALAHIA” University of Târgovişte, Electrical Engineering Faculty, Automatics, Informatics and Electrical Engineering Department
2 - “Constantin Brâncuşi” University of Târgu Jiu, Engineering Faculty

Abstract: Mechanical power transmission by reducing the rotating speed under a constant transmission ratio represents the function of a large group of products known as speed reducers. Cycloid gearing roller (invented by L. Braren), through its qualities, has an important role in modern mechanical transmissions. The difference between numbers of teeth of the cycloid gear roller can be equal to 1 (| Z₁ - Z₂ | ≥ 1), without risk of interference, as a result, can be obtained big gear ratios in accordance with in lower overall dimensions. Thus, this paper presents the modeling and simulation of cycloid curves (epicycloids, respectively hypocycloid), which generates the cycloid gears used a lot in robotics.

Keywords: Cycloid gears, Cycloid curves, Epicycloids, hypocycloid.

1. Introduction

Reducing of the speed is a technical goal imposed by the need to adapt relatively high speeds of modern engines, at the requirements of effectors which they serve.

Speed reducers or amplifiers are systems which perform such a function. Unlike other types of gearboxes (planetary, with deformable satellites with fixed axes, etc.), cycloid reducers has the following major advantages: 1. High yields, accompanied by high transmission rates, 2. Weights and reduced-sized, 3. High reliability 4. Silence during the operating process [3], [4], [6].

Thus, in Figure 1a, is an example of a reducer which uses two internal cycloid gears with 2/3 and 2/1 rollers; in both cases, the plan of the satellite number two designates the general plan, in which: the centers of the rollers represents the generating points and, implicit the circle containing those centers means generating circle r₂. In gears 2/3 (Figure 1a), the 5 generating points (from satellite plan 2) describe a normal hypocycloid with six loops (wheel plan 3).

As a result, the middle circles from 2/3 gearbox, are in the follow ratio r₂:r₃ = a:b = 5:6. This means that when generating a full hypocycloid by a single generator point, the r₂generating circle rolls of six times over the r₃base circle and, hence, the base circle rolls of 5 times over the generator one. According to Figure 1a, the cycloid denture of the wheel 3 is obtained as the envelope of a family of circles (radius: r₄ = roller radius), whose centers are located on the hypocycloid with six loops, [7], [8], [9], [10].

Into the gear 2/1 (Figure 1a), those 5 generating points (from the 2 satellite plan) describe a normal epicycloids with four loops (in the a plane of wheel 1). As a result, the middle circles of gear 2/1 are in the ratio: r₂:r₁ = a:b = 5:4; this means that when generating a full epicycloids by a single generator point, the generating circle r₂ rolls of four times over the base circle r₁ and, thus the base circle rolls of 5 times over the generator one. According to Figure 1a, the cycloid denture of a wheel 1 is obtained as the envelope of a family of circles (radius: r₄ = roller radius), whose centers are located on epicycloids with 4 loops.

Another example of cycloid gear with bolts / rolls is illustrated in Figure 1b. According to this figure, into the 3/2 gear one observe: 3 designate the general plan (with generating circle r₃, containing the centers of reels) and 2 designate the basic plan; the curve generated is, also, normal epicycloids.
Because there are five generator points (evidenced by roller centers) and because the epicycloids has four loops, result that that the properties of 3/2 gear are similar to those of 2/1 gear from Figure 1a [3], [5], [6].

![Fig. 1. a.) Reducer comprising a planetary drive with two central wheels, b) cycloidal gear reducer with roller and clutch-type Schmid with roller](image)

The reducer illustrated in Figure 2 consists from a cycloid gear with rollers and a fixed central wheel with inner denture 3. The entry is achieved through the element H (bearing eccentric), characterized by a high speed and output is 1 element, characterized by low speeds. The reducer use 20 rollers and the number of teeth of the cycloid wheel is 21 (cycloid wheel denture being profiled by cycloid curves) [1], [2].

![Fig. 2. Bolts cycloid gear formed by coupling from bolts 1-2 and internal cycloid gear with roller 2-3](image)

2. **Modeling and simulation of the cycloid curves used in generation of the cycloid denture.**

Cycloid (the planetary curve) is the curve generated during the relative motion of two planes whose middle centers are circles: one plane contains point generator and the other plane containing the curve of generator point. The main types of cycloid are [3], [6]:

a. Epicycloids when generating circle rolls without slipping (with its outer or inside side) on the outside of the base circle (which, for simplicity, is considered fixed);

b. Hypocycloid when generating circle rolls, without slipping, with its external side, into the circle base.
Epicycloids is generated by two distinct center movements (the double generation cycloid theorem) in this way: at one of the two center movements the circles contact takes place outside the generator circle (circles with continuous line, in Figure 3) and the other, the contact has place into the generator circle (dashed circles, in Figure 3).

Unlike by the epicycloids, those two centers movements, which generate a hypocycloid, are characterized by centers circles whose contact takes place into the circle base (Figure 4). According with Figure 3, the epicycloids equations are shown in:

\[
\begin{align*}
x &= r_i (k_i + 1) \cos \phi - h_i \cos (k_i + 1) \phi \\
y &= r_i (k_i + 1) \sin \phi + h_i \sin (k_i + 1) \phi.
\end{align*}
\] (1)

According with Figure 4, the hypocycloid equations are shown below:

\[
\begin{align*}
x &= r_i (1 - k_i) \cos \phi + h_i \cos (1 - k_i) \phi \\
y &= r_i (1 - k_i) \sin \phi - h_i \sin (1 - k_i) \phi
\end{align*}
\] (2)

**Fig. 3.** Notations and geometrical structures necessary to establish the parametric equations of the epicycloids

**Fig. 4.** Notations and geometrical structures necessary to establish the parametric equations of the hypocycloids
In Figure 5 are examples of epicycloids obtained for different values of the parameters $k_1$ and $h_1$ (respectively $k_2$ and $h_2$); the circles $r_1$ and $r_1'$ are base circles that correspond to the two centers movements that generate the represented planetary center curve [9].

On the other hand, in Figure 6 are examples of hypocycloid obtained for different values of the parameters $k_1$ and $h_1$ (respectively $k_2$ and $h_2$); the circles $r_1$ and $r_1'$ are circles that correspond to the two basic movements that generate the represented planetary center curve [10].

![Epicycloids examples](image)

**Fig. 5. Epicycloids examples**

![Hypocycloid examples](image)

**Fig. 6. Hypocycloid examples**
To obtain closed cycloid curves should be that the ratio \( r_2: r_1 \), respectively \( r'_2: r'_1 \) being a rational number, otherwise the generating points do not return to its original position.

Hypocycloids and respectively epicycloids were generated in Matlab, using, for that, software special developed for this action (Figure 7).

3. Conclusion

To obtain closed cycloid curves should be that the ratio \( r_2: r_1 \), respectively \( r'_2: r'_1 \) being a rational number, otherwise the generating points do not return to its original position. If one have a two-wheel mechanism which are the follow ratio \( r_2:r_1=a:b \), where \( r_2 \) is the radius of generating circle, \( r_1 \) is the cycloid base circle radius and \( a \) and \( b \) are integers, which are common divisor only unity.

While generating of a full cycloid, the generator circle makes a number of \( b \) full rolling (over the base circle of the cycloid) and the base circle of the cycloid makes a number of a full rolling (over the generator circle.)

So, a cycloid generated by this mechanism has the following properties: is composed by a number of \( b \) identical branches arranged in equal angles; can be generated by a number of a points of the generator plan; if the ratio \( k_1=r_2/r_1=a:b \), (\( a<b \)), then the second generator planetary mechanism have the follow ratio \( k_2=1\pm k_1=(b\pm a)/b \); so the second mechanism can generate the number of \( b \) branches of the curve with a maximum \( b \pm a \) of generating points.

A class of hypocycloids and epicycloids is firstly generated, through an adequate modification of the coefficient of addendum modification \( x \); then for different values of the roller’s radius, a basis of cycloid gears is generated from which the optimal solution is identified.

The analysis of the generated gears highlights the fact that are preferred the relative high positive values of the addendum modification coefficient \( x \), together with the high values of the roller’s radius \( r \).
References


NUMERICAL SIMULATIONS FOR THE CASE OF RIGID ROTATING KINEMATIC COUPLING WITH BIG CLEARANCE

Lecturer dr. eng., Jan-Cristian GRIGORE, University of Pitești

Abstract: In this paper an algorithm based on [1] [2] are numerical simulations, achieving generalized coordinates of motion, positions, speeds of a rigid rotating kinematic coupling with big clearance in joint, case without friction

Keywords: algorithms, energy, angular velocities.

1. Numerical application

Based on algorithms developed in [1] and [2] will be solved for a revolute motion rectangular table with \( m=15 \) kg (fig. 1) and dimentions \( b=0.4 \, m; h=0.6 \, m \) with big clearance joint \( r=2 \, mm; 2L=10 \, mm \), whereas the moment \( M=-80 \, N \cdot m \), moments of inertia are

\[
J_x = \frac{mh^2}{12} = 0.45 \, kg \cdot m^2; \quad J_z = \frac{mb^2}{3} = 0.8 \, kg \cdot m^2; \quad J_y = J_x + J_z
\]

Potential energy given weight \( mg \) is

\[
V = mg \cdot Z_c \cdot \frac{b}{2} \cos \theta \cdot \sin \varphi
\]

and so the components are generalized forces,

\[
Q_{\theta} = -\frac{\partial V}{\partial \theta} = -mg \cdot \frac{b}{2} \cos \theta \cdot \sin \varphi; \quad Q_{\varphi} = -\frac{\partial V}{\partial \varphi} = -mg \cdot \frac{b}{2} \sin \theta \cdot \cos \varphi
\]
The moments are:
\[
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
M
\end{bmatrix} = M \cdot \begin{bmatrix}
sin\theta \sin\phi \\
sin\theta \cos\phi \\
0
\end{bmatrix}
\]
(3)

components of generalized forces given the moment \( M \) is
\[
\begin{align*}
Q'_\omega &= M \frac{\partial \omega}{\partial \psi} + M_y \frac{\partial \omega}{\partial \theta} + M_z \frac{\partial \omega}{\partial \phi} \\
Q'_\theta &= M_x \frac{\partial \theta}{\partial \psi} + M_y \frac{\partial \theta}{\partial \theta} + M_z \frac{\partial \theta}{\partial \phi} \\
Q'_\phi &= M_x \frac{\partial \phi}{\partial \psi} + M_y \frac{\partial \phi}{\partial \theta} + M_z \frac{\partial \phi}{\partial \phi}
\end{align*}
\]
(4)

Into account expressions
\[
\omega_x = \psi \sin \theta \sin \phi + \hat{\theta} \cos \phi; \quad \omega_y = \psi \sin \theta \cos \phi - \hat{\theta} \sin \phi; \quad \omega_z = \psi \cos \theta + \phi
\]
can writer:
\[
\begin{align*}
Q'_{\psi} &= M \\
Q'_{\theta} &= 0 \\
Q'_{\phi} &= M \cos \theta
\end{align*}
\]
(6)

and obtain expressions for generalized forces total
\[
\begin{align*}
Q_{\psi} &= Q_{\psi}' \\
Q_{\theta} &= Q_{\theta}' - mg \frac{b}{2} \cos \theta \sin \phi \\
Q_{\phi} &= Q_{\phi}' + Q_{\phi}' - mg \frac{b}{2} \sin \theta \cos \phi + M \cos \theta
\end{align*}
\]
(7)

a). If the point \( O \) is fixed

If the point \( O \) is fixed then \( \theta = \arctan \frac{r}{L} \) and obtain the equations of motion
\[
\begin{bmatrix}
J_x \sin^2 \theta \sin^2 \phi + J_y \sin^2 \theta \cos^2 \phi + J_z \cos^2 \theta \cos \theta \\
J_z \cos \theta \\
J_z \cos \theta
\end{bmatrix} \begin{bmatrix}
\ddot{\psi} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix} + \left( J_x - J_y \right) \begin{bmatrix}
\psi \phi \\
\frac{1}{2} \psi^2 \\
\frac{1}{2} \psi^2
\end{bmatrix} \sin \theta \sin 2\phi = \begin{bmatrix}
M \cos \theta - m b g \frac{M}{2} \sin \theta \cos \phi
\end{bmatrix}
\]
(8)

The initial conditions
\[
t = 0; \ \psi = \frac{\pi}{2}; \ \phi = 0; \ \dot{\psi} = 200 rad/s; \ \dot{\phi} = 0
\]
(9)

and using the calculation of annexes are obtained table 1 and diagrams of variation of parameters \( \phi, \psi \) from fig. 2 a), b) and angular velocities \( \dot{\phi}, \dot{\psi} \) from fig. 3 a), b) so.
<table>
<thead>
<tr>
<th>t</th>
<th>$\psi$</th>
<th>$\dot{\phi}$</th>
<th>$\psi$</th>
<th>$\varphi$</th>
<th>$\dot{\psi}$</th>
<th>$\dot{\phi}$</th>
<th>$z_{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.14</td>
<td>-101.74</td>
<td>1.5708</td>
<td>0</td>
<td>200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.001</td>
<td>-6.3449</td>
<td>-100.34</td>
<td>1.7708</td>
<td>-5.0752e-005</td>
<td>199.99</td>
<td>-0.10127</td>
<td>-3.7698e-006</td>
</tr>
<tr>
<td>0.002</td>
<td>-9.921</td>
<td>-96.187</td>
<td>1.9708</td>
<td>-0.00020161</td>
<td>199.99</td>
<td>-0.19977</td>
<td>-1.4975e-005</td>
</tr>
<tr>
<td>0.003</td>
<td>-15.761</td>
<td>-89.403</td>
<td>2.1708</td>
<td>-0.00044839</td>
<td>199.97</td>
<td>-0.2928</td>
<td>-3.3305e-005</td>
</tr>
<tr>
<td>0.004</td>
<td>-23.701</td>
<td>-80.179</td>
<td>2.3707</td>
<td>-0.00078442</td>
<td>199.95</td>
<td>-0.37778</td>
<td>-5.8265e-005</td>
</tr>
<tr>
<td>0.005</td>
<td>-33.516</td>
<td>-68.773</td>
<td>2.5707</td>
<td>-0.0012005</td>
<td>199.93</td>
<td>-0.4524</td>
<td>-8.9174e-005</td>
</tr>
<tr>
<td>0.006</td>
<td>-44.93</td>
<td>-55.506</td>
<td>2.7706</td>
<td>-0.0016853</td>
<td>199.89</td>
<td>-0.5147</td>
<td>-0.00172518</td>
</tr>
<tr>
<td>0.007</td>
<td>-57.628</td>
<td>-40.746</td>
<td>2.9704</td>
<td>-0.0022254</td>
<td>199.84</td>
<td>-0.56301</td>
<td>-0.0001653</td>
</tr>
<tr>
<td>0.008</td>
<td>-71.257</td>
<td>-24.903</td>
<td>3.1702</td>
<td>-0.0028061</td>
<td>199.77</td>
<td>-0.59593</td>
<td>-0.00020643</td>
</tr>
<tr>
<td>0.009</td>
<td>-85.448</td>
<td>-8.4085</td>
<td>3.370</td>
<td>-0.0034116</td>
<td>199.69</td>
<td>-0.61256</td>
<td>-0.00025341</td>
</tr>
<tr>
<td>0.01</td>
<td>-99.814</td>
<td>8.2868</td>
<td>3.5696</td>
<td>-0.0040257</td>
<td>199.6</td>
<td>-0.61256</td>
<td>-0.00029902</td>
</tr>
<tr>
<td>0.011</td>
<td>-113.97</td>
<td>24.731</td>
<td>3.7692</td>
<td>-0.0046317</td>
<td>199.49</td>
<td>-0.59604</td>
<td>-0.00034403</td>
</tr>
<tr>
<td>0.012</td>
<td>-127.53</td>
<td>40.476</td>
<td>3.9686</td>
<td>-0.0052131</td>
<td>199.37</td>
<td>-0.56344</td>
<td>-0.00038722</td>
</tr>
<tr>
<td>0.013</td>
<td>-140.11</td>
<td>55.08</td>
<td>4.1679</td>
<td>-0.0057538</td>
<td>199.24</td>
<td>-0.51558</td>
<td>-0.00042738</td>
</tr>
<tr>
<td>0.014</td>
<td>-151.38</td>
<td>68.151</td>
<td>4.3671</td>
<td>-0.0062392</td>
<td>199.09</td>
<td>-0.45378</td>
<td>-0.00046343</td>
</tr>
<tr>
<td>0.015</td>
<td>-161.04</td>
<td>79.348</td>
<td>4.5661</td>
<td>-0.0066566</td>
<td>198.94</td>
<td>-0.39777</td>
<td>-0.00049444</td>
</tr>
<tr>
<td>0.016</td>
<td>-168.85</td>
<td>88.387</td>
<td>4.765</td>
<td>-0.0069954</td>
<td>198.77</td>
<td>-0.29569</td>
<td>-0.0005196</td>
</tr>
<tr>
<td>0.017</td>
<td>-174.61</td>
<td>95.032</td>
<td>4.9636</td>
<td>-0.0072467</td>
<td>198.6</td>
<td>-0.20376</td>
<td>-0.00053826</td>
</tr>
<tr>
<td>0.018</td>
<td>-178.13</td>
<td>99.081</td>
<td>5.1621</td>
<td>-0.0074027</td>
<td>198.42</td>
<td>-0.10652</td>
<td>-0.00054985</td>
</tr>
<tr>
<td>0.019</td>
<td>-179.32</td>
<td>100.42</td>
<td>5.3605</td>
<td>-0.0074591</td>
<td>198.24</td>
<td>-0.0065789</td>
<td>-0.00055404</td>
</tr>
<tr>
<td>0.02</td>
<td>-178.16</td>
<td>99.028</td>
<td>5.5586</td>
<td>-0.0074146</td>
<td>198.06</td>
<td>0.093383</td>
<td>-0.00055074</td>
</tr>
</tbody>
</table>

Fig. 2

Fig. 3
References

SOME CONSIDERATIONS REGARDING ANALITICAL SOLVING OF THE REYNOLD’S EQUATION FROM FACE SEAL

Prof. dr.eng. Nicolae POPA, University of Piteşti,
Lecturer dr.eng. Constantin ONESCU, University of Piteşti

Abstract: Taking into consideration a mechanical face seal which runs in hydrodynamic duty, we intend to understand what’s going on under its surfaces. Until nowadays the experimental study to measure the parameters of film between two seal surfaces is very difficult. To explain these phenomena from this interface is necessary to know the theoretical pressures. The paper uses a primary seal model where the seal surfaces are considered with misalignment but the ring centre distance is constant. Considering some simplified hypothesis it determines the Reynold’s equation, it’s solving leading to the pressure equation of sealing interface.

Keywords: pressure, hydro-dynamic, thickness.

1. Introduction

The mechanical face seals are used to obtain the seals from the mechanical systems, whose good running depends, in a critical manner, of the mechanical seal, even if the cost is minimum related to the whole system.

This fact has determined the physicians, the mathematicians and the engineers to be more interest of these mechanical face seals performances. The thick films approximation, introduced by O. Reynolds in 1886 is the fundamental base of the theoretical studies in this domain.

There are two criteria of good running for the mechanical face seals: the sealing film stability and the mobile primary mechanical seal equilibrium. The equilibrium is complete when the resultant of forces given by the pressure HD or HS in the film is equal with the load of the exterior forces.

A well knowledge of the pressure field is necessary in all the experimental, theoretical or numerical studies. Until now, the experimental study is very difficult because the measurements done in the film which separates the two surfaces are very approximately. This is due to the complex phenomena which take place and have very different natures (HD, THD, elastic, etc).

We admit that this thing cannot be always accepted, that the correct running of a mechanical face seal is linked by a complete fluid film existence in the $S_1 - S_2$ interface (fig.1).

![Fig. 1 The representation of the primary seal.](image-url)
Also, we say that the mechanical seal runs in a hydro-dynamic duty. A mixed duty, different from the hydro-dynamic one and characterized by some consistent contacts existence between the surfaces, may exist if:
- the exterior forces that tend to eliminate the film are important;
- the fluid vaporizes itself;
- the surfaces geometry isn’t favorable for apparition and maintaining the fluid film.

2. Reynold’s Equation

We study the movement of a viscid fluid between two surfaces, in the following hypothesis:
- The fluid is Newtonian and the flow is laminar;
- The exterior mass forces and the inertial ones are insignificant;
- The film thickness is very small related to the other dimensions of the sealing space;
- The fluid speed on the film height is very small related to the components in the other two directions; the speeds proportion is the same with the proportion of the corresponding dimensions;
- The surfaces asperity heights are smaller comparing with the film thickness

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( \rho r v^r \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \rho v^\theta \right) + \frac{\partial \left( \rho v^z \right)}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \]  

(1)

and

\[ \frac{\partial p}{\partial r} = \frac{\partial}{\partial z} \left[ \mu \frac{\partial v^r}{\partial z} \right], \quad \frac{1}{r} \frac{\partial p}{\partial \theta} = \frac{\partial}{\partial z} \left[ \mu \frac{\partial v^\theta}{\partial z} \right], \quad \frac{\partial p}{\partial z} = 0 \]  

(2)

where: \( r, \theta, z \) are the cylindrical coordinates; \( v^r, v^\theta, v^z \) are the speed components in the cylindrical coordinates; \( p \) is the pressure, \( \rho \) is the specific consistency; \( \mu \) is the viscosity coefficient.

On \( S_1 \) and \( S_2 \) surfaces (by equation \( z = H_1 \) and \( z = H_2 \)) we put the following conditions at limit:

\[ v^r = v_1^r; \quad v^\theta = v_1^\theta; \quad v^z = v_1^z, \text{ pentru } z = H_1(r, \theta, t) \]
\[ v^r = v_2^r; \quad v^\theta = v_2^\theta; \quad v^z = v_2^z, \text{ pentru } z = H_2(r, \theta, t) \]  

(3)

We reduce the problem (1), (2), (3) to one problem for the unknown \( p \).
From (2) results \( p = p(r, \theta, t) \). We consider \( \mu = \mu(r, \theta, t) \) and then (2) becomes:

\[ \frac{\partial^2 v^r}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial r} \text{ and } \frac{\partial^2 v^\theta}{\partial z^2} = \frac{1}{\mu r} \frac{\partial p}{\partial \theta} \]  

(4)

The obtained system integrates itself twice related to \( z \), and taking into account that \( p(dr \text{ and } \frac{\partial p}{\partial r}, \frac{\partial p}{\partial \theta}) \) do not depend on \( z \), it results:

\[ v^r = \frac{1}{\mu} \frac{\partial p}{\partial r} \left( \frac{z^2}{2} + C_1 z + C_3 \right) \text{ and } v^\theta = \frac{1}{\mu r} \frac{\partial p}{\partial \theta} \left( \frac{z^2}{2} + C_2 z + C_4 \right) \]  

(5)

Replacing in (5) the constants, it results:
\[
v' = \frac{1}{2\mu} \frac{\partial p}{\partial r} [z^2 - z(H_1 + H_2) + H_1 H_2] + \frac{v'_1 - v'_2}{H_1 - H_2} (z - H_1) + v'_1 \tag{6}
\]

\[
v^\theta = \frac{1}{2\mu r} \frac{\partial p}{\partial \theta} [z^2 - z(H_1 + H_2) + H_1 H_2] + \frac{v^\theta_1 - v^\theta_2}{H_1 - H_2} (z - H_1) + v^\theta_1 \tag{7}
\]

We integrate the continuity equation (1) related to \( z \) on the range \([H_1, H_2]\) and we obtain

\[
\int_{H_1}^{H_2} \frac{\partial}{\partial r} (\rho v') dz + \int_{H_1}^{H_2} \frac{\partial}{\partial \theta} (\rho v^\theta) dz + \int_{H_1}^{H_2} r \frac{\partial \rho}{\partial t} dz = 0 \tag{8}
\]

Taking into account the derivation formula of the integral and considering \( \rho = \rho(r, \theta, t) \), then (8) becomes

\[
\frac{\partial}{\partial r} \int_{H_1}^{H_2} \rho v' dz - \frac{\partial H_2}{\partial r} \rho v'_1 + \frac{\partial H_1}{\partial r} \rho v'_2 + \frac{\partial}{\partial \theta} \int_{H_1}^{H_2} \rho v^\theta dz - \frac{\partial H_2}{\partial \theta} \rho v^\theta_1 + \frac{\partial H_1}{\partial \theta} \rho v^\theta_2 + r \rho (v^\theta_2 - v^\theta_1) +
\]

\[+ r \frac{\partial \rho}{\partial t} (H_2 - H_1) = 0 \]

In order to calculate the above integrals, we take (6) and (7) into account and it results:

\[
\frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} (H_2 - H_1)^3 \frac{\partial p}{\partial r} \right] + \frac{\partial}{\partial \theta} \left[ \frac{\partial}{\partial \theta} (H_2 - H_1)^3 \frac{\partial p}{\partial \theta} \right] = 6(H_2 - H_1) \frac{\partial}{\partial r} [v'_1 + v'_2] + \rho (v^\theta_1 + v^\theta_2) \frac{\partial (H_2 + H_1)}{\partial r} +
\]

\[+ 6(H_2 - H_1) \frac{\partial}{\partial \theta} [(v^\theta_1 + v^\theta_2)] + 6v^\theta_2 \frac{\partial (H_2 + H_1)}{\partial \theta} + 12(v^\theta_2 - v^\theta_1) \tag{9}
\]

2.1. Particularly case – face seal

We suppose that \( S_1 \) and \( S_2 \) are coaxial circular crowns, \( S_1 \) is fixed; \( S_2 \) has a rotation movement, with angular speed \( \omega \), around the Oz axis (fig. 2).

In order to find out \( H_1 = H(r, \theta, t) \), or \( M_1(r, \theta, z) \in S_1 \). We notice that \( M_1 \) and \( N_1 \) have the same \( z \).

Because \( \chi \) is small, we approximate \( \tan \chi \approx \sin \chi = \chi \), so:

\[
H_1(r, \theta, t) = \rho \chi \sin \theta \tag{10}
\]

Similarly for \( H_2 \) (but we replace \( \theta \) with \( 0 - \theta \)t because the \( S_2 \) movement), we obtain:

\[
H_2(r, \theta, t) = \rho \chi \sin (\theta - \alpha t) \tag{11}
\]

Due to the fact that the fluid is consistent, it adheres to the surfaces, so:

\[
v^\theta_1 = 0, \quad v^\theta_2 = 0, \quad v'_1 = \omega r, \quad v'_2 = 0 \tag{12}
\]

We consider \( h_0 \) a constant value, so we can suppose that \( v^\theta_2 = 0 \). Equation (9) becomes

\[
\frac{\partial}{\partial r} \left[ \rho \chi (H_2 - H_1)^3 \frac{\partial p}{\partial r} \right] + \frac{\partial}{\partial \theta} \left[ \frac{1}{\mu r} (H_2 - H_1)^3 \frac{\partial p}{\partial \theta} \right] = 6v^\theta_2 \frac{\partial (H_2 + H_1)}{\partial \theta} \tag{11}
\]

We note \( h = H_2 - H_1 \) and consider \( \mu \) as a constant value. Equation (11) becomes:

\[
\frac{\partial}{\partial r} \left[ rh^3 \frac{\partial p}{\partial r} \right] + \frac{\partial}{\partial \theta} \left( h^3 \frac{\partial p}{\partial \theta} \right) = 6\mu \rho r^2 \chi \cos (\theta - \omega t) + \chi \cos \theta \tag{12}
\]
We introduce the a-dimensional measures:

- The thickness \( h = \frac{h_0}{\theta} \), the radius \( \bar{r} = \frac{r}{R_e} \), the relative pitch \( \chi = \frac{X_1}{X_2} \).

- The pitching – film thickness parameter \( e_2 = R_e \frac{X_2}{h_0} \).

- The angular position of the rotor \( \Omega = \omega r \).

- The pressure \( \bar{p} = \frac{h_0^2}{8 \mu R_e^2 \omega} p \).

Then, for \( r \in [R_1, R_e] \) we have \( \bar{r} \in [1, 1] \), and \( \frac{\partial}{\partial r} = \frac{\partial}{\partial \bar{r}} \frac{\partial}{\partial r} \), so \( \frac{\partial}{\partial r} = \frac{1}{R_e} \frac{\partial}{\partial \bar{r}} \).

The a-dimensional film thickness becomes:
\[
\bar{h} = 1 + \bar{h}_0 \sin(\theta - \Omega) - \chi \sin(\theta)
\]
and the Reynolds equation in a-dimensional measures:
\[
\frac{\partial}{\partial \bar{r}} \left[ \bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{r}} \right] + \frac{\partial}{\partial \theta} \left[ \frac{\bar{h}^3}{\bar{r}} \frac{\partial \bar{p}}{\partial \theta} \right] = -e_2 \bar{F}^2 \cos(\theta - \Omega) + \chi \cos \theta
\] (13)

3. Integration of Reynold’s Equation In order to simplify the notations, instead of \( \bar{h}, \bar{r}, \bar{p} \) we will use \( h, r \) and \( p \). Because \( e_2 < 1 \) (from \( R_0(\chi + \chi_2) < h_0 \)), so \( e(1+\chi) < 1 \) we will develop in ranges of power after \( e_2 \) the functions \( h_1 \) and \( p \). We have:
\[
h = 1 + e_2 h_1(r, \theta), \quad \cos h_1(r, \theta) = \frac{r}{\sin(\theta - \Omega) - \chi \sin(\theta)}
\]
\[
p = p_0(r, \theta) + e_2 p_1(r, \theta) + e_2^2 p_2(r, \theta) + \ldots
\] (14)
We introduce the relations (14) in (13) and we obtain:

\[
\frac{\partial}{\partial r} \left[ r(1+e_2 h_1)^l \frac{\partial}{\partial r} \left( \sum_{k=0}^{n} e_k^k p_k \right) \right] + \frac{\partial}{\partial \theta} \left[ \frac{1}{r} (1+e_2 h_1)^l \frac{\partial}{\partial \theta} \left( \sum_{k=0}^{n} e_k^k p_k \right) \right] = -e_2 r^2 [\cos(\theta - \Omega) + \chi \cos \theta]
\]

Solving this equation and identifying the \( e_2 \) coefficients, we keep in mind the first two equations:

\[
r \frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r} \frac{\partial^2 p_1}{\partial \theta^2} + \frac{\partial p_1}{\partial r} = -r^2 [\cos(\theta - \Omega) + \chi \cos \theta]
\]

(15)

\[
r \frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \frac{\partial^2 p_2}{\partial \theta^2} + \frac{\partial p_2}{\partial r} = -3h_1 \frac{\partial}{\partial r} \frac{\partial^2 p_1}{\partial r^2} - 2r \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} \frac{\partial^2 p_1}{\partial r^2} - 3h_1 \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} \frac{\partial^2 p_1}{\partial r^2}
\]

(16)

Because \( p(r, \theta) = p(r, \theta + 2\pi) \) it is expected like \( p_1(r, \theta) = p_1(r, \theta + 2\pi) \) so we can solve in Fourier range \( p_1 \) and \( p_2 \) after \( \theta \). After identifying the trigonometrically functions coefficients we obtain equations which are particular cases of the Euler type equation. In the end it results:

\[
p_1(r, \theta) = -\frac{1}{8} \left[ r^3 - (r_i^2 + 1) r + \frac{r_i}{r} \right] \cos \Omega + \chi \cos \theta + \sin \Omega \sin \theta
\]

(17)

\[
p_2(r, \theta) = \frac{1}{32} \chi \sin \Omega \left[ 1 - 2r_i^2 + \left( r_i^4 - 1 \right) \frac{\ln r_i}{\ln r} + 2(2r_i^2 + 1)r_i^2 - 3r_i^4 \right] + \frac{1}{32} \sin 2\Omega \frac{5 + 8r_i^2 + 5r_i^4}{1 + r_i^2} r_i^2

- 2 \frac{r_i^4}{1 + r_i^2} \frac{1}{r_i^2} - 3r_i^2 - 5r_i^4 \cos 2\theta +

+ \frac{1}{32} (\cos 2\Omega - \chi^2) \left[ 5 + 8r_i^2 + 5r_i^4 \frac{1}{1 + r_i^2} r_i^2 + 2 \frac{r_i^4}{1 + r_i^2} \frac{1}{r_i^2} + 3r_i^2 + 5r_i^4 \right] \sin 2\theta
\]

(18)

From (14), (17) and (18) we assume:

\[
p(r, \theta) = e_2 p_1(r, \theta) + e_2^2 p_2(r, \theta) = -\frac{e_2}{8} \left[ r^3 - (r_i^2 + 1) r + \frac{r_i}{r} \right].
\]

\[
\cdot [\cos \Omega + \chi ] \cos \theta - \sin \Omega \sin \theta] + + \frac{e_2^2}{32} \left( \chi \sin \Omega \left[ 1 - 2r_i^2 + \left( r_i^4 - 1 \right) \frac{\ln r_i}{\ln r} + 2(2r_i^2 + 1)r_i^2 - 3r_i^4 \right] + \right.

\left. + \sin 2\Omega \cos 2\Omega - \left( \cos 2\Omega - \chi^2 \right) \sin 2\theta \right] \frac{5 + 8r_i^2 + 5r_i^4}{1 + r_i^2} r_i^2 - 2 \frac{r_i^4}{1 + r_i^2} \frac{1}{r_i^2} - 3r_i^2 - 5r_i^4 \right] \sin 2\theta
\]

(19)

We return to the notations \( \bar{p}, \bar{r}, \bar{r} \) and we write:

\[
\bar{p}(\bar{r}, \theta) = -\frac{e_2}{8} \left[ \bar{r}^3 - \left( \bar{r}_i^2 + 1 \right) \bar{r} + \frac{\bar{r}_i^2}{\bar{r}} \right] \left( \cos \Omega + \chi \right) \cos \theta - \sin \Omega \sin \theta \right] + + \frac{e_2^2}{32} \left( \chi \sin \Omega \left[ 1 - 2\bar{r}_i^2 + \left( \bar{r}_i^4 - 1 \right) \frac{\ln \bar{r}_i}{\ln \bar{r}} + 2(2\bar{r}_i^2 + 1)\bar{r}_i^2 - 3\bar{r}_i^4 \right] + \sin 2\Omega \cos 2\Omega - \left( \cos 2\Omega - \chi^2 \right) \sin 2\theta \right] \frac{5 + 8\bar{r}_i^2 + 5\bar{r}_i^4}{1 + \bar{r}_i^2} \bar{r}_i^2 - 2 \frac{\bar{r}_i^4}{1 + \bar{r}_i^2} \frac{1}{\bar{r}_i^2} - 3\bar{r}_i^2 - 5\bar{r}_i^4 \right] \sin 2\theta
\]

(19)

The relation (19) represents a solution of the Reynolds equations for the non-dimensional pressure.
4. Conclusions

Many papers have shown the presence of a fluid film between the two surfaces of the primary mechanical seal. Different hydro-dynamic bearing mechanisms have been proposed in order to explain the creation and the maintaining of this film. There is no one explanation and lots of hypotheses were released. Lots of theoretical papers which have been considered the distance between the axis constant, also have demonstrated that the film bearing varies generally in time. This paper too shows that the relation (1) is not respected. So, the two mechanical seal surfaces may approach and bear away during the rotation. We conclude that the thickness at the center of the sealing space varies in time.

References
KINEMATICS OF A MECHANISM WITH STOPPINGS, BASED ON A HYPOCYCLOID WITH 6 BRANCHES

Prof. PhD. eng. Liliana LUCA, „Constantin Brancusi „, University of Targu- Jiu
Prof. PhD. eng. Iulian POPESCU, University of Craiova

Abstract. They are studied the positions, velocities and accelerations of a mechanism based on planetary gear and a RRP dyad. The mechanism ensures the plunger stop on a certain subinterval of the cycle, although the leading element is rotating. Because of the approximation of the hypocycloid shortened arc with six branches, by a circle arc, errors occur.

Keywords: planetary mechanism, stoppings, hypocycloid

1. Introduction
In [1] is given a mechanism used to press for plastics, consisting of a planetary gear and an RRP type dyad. Points on the satellite wheel generate hypocycloids with 5 branches. Such a shortened hypocycloid is approximated with an arc, thus ensuring the temporarily stop of the final leading element, even though the port satellite arm rotates further. Below are studied the cinematic possibilities of such a mechanism, using a shortened hypocycloid, with 6 branches.

2. Positions mechanism
In fig. 1 is given the cinematic scheme of the planetary mechanism, and in fig. 2 is showed the equivalent mechanism, including also a RRP type dyad. The mechanism has a leading element with rotating movement 1, and a second leading element but with flat moving, 2 (his movement is dependent on the movement of 1) and EF dyad.

From fig. 1 it results: e=a-b, c<b.

Based on fig. 2 are written the following relations:

\[ x_B = e \cdot \cos \varphi; \quad y_B = e \cdot \sin \varphi \]
\[ x_E = x_B + c \cdot \cos \alpha \quad ; \quad y_E = y_B + c \cdot \sin \alpha \]  \hspace{1cm} (2)
\[ x_F = x_E + d \cos \beta = 0 \quad ; \quad y_F = y_E + d \sin \beta = S_4 \]  \hspace{1cm} (3)

Provided that the circle of the wheel 2 rolls without slipping on the running circle of wheel 3, it results:
\[ \alpha = 2\pi - (\psi - \phi) \]  \hspace{1cm} (4)
\[ a \phi = b \psi \]  \hspace{1cm} (5)

where \( \phi \) is the angle of rotation of the conveyor, and \( \psi \) is the arc described on base, equal to the one from the conveyor.

Were established the following dimensions: \( b = 25 \text{ mm}, \ c = 20 \text{ mm}, \ a = 150 \text{ mm}, \ e = 125 \text{ mm}, \) speed of port-satellite arm 60 rpm.

In fig. 3 is shown the generated hypocycloid. EF rod length can be mathematically determined based on the coordinates of three points on the hypocycloid branch, approximated by an arc. This solution, however, does not allow that the port-satellite arm perform a complete rotation, when ABE is in extension on x axis. Because of this, additional condition should be fulfilled:
\[ d > x_{E_{\text{max}}} \]

Based on the diagram from fig.4 it results:
\[ d = 150 \text{ mm}. \]

This new approximation causes larger errors at the end of the race.

From fig. 5, where are presented successive positions of 1 and 2 elements, it appears that item 2 is positioned at different angles relative to 1, as the 2 wheel turns more rotations at one rotation of the port-satellite arm.
In fig. 6 are showed successive positions of the EF rod, observing variable distances of the piston in relation with x axis, which shows great variations in its speed. Fig. 7 shows successive positions of the entire mechanism, for the entire cycle, and in fig. 8 the positions only for useful branch of the hypocycloid.

From fig. 9 it can be observed the generating of the useful hypocycloid branch. Law of motion of the piston 4 can be traced in fig. 10. Large oscillations are found at \( \phi > 180 \) degrees, and also small variations in the range \( \phi = 60 \ldots 120 \) degrees.
Fig. 9

Fig. 10

Fig. 11 states only the useful area of the chart; deviations appear large due to selected scale, but they do not exceed 2.9 mm.

3. Speeds

They are derived, in relation with time, the relations from positions, resulting:

\[ \dot{x}_B = -e \sin \phi \cdot \dot{\phi} \; ; \; \dot{y}_B = e \cos \phi \cdot \dot{\phi} \]  
\[ \dot{x}_E = \dot{x}_B - c \sin \alpha \cdot \dot{\alpha} \; ; \; \dot{y}_E = \dot{y}_B + c \cos \alpha \cdot \dot{\alpha} \]  
\[ \ddot{\alpha} = \dot{\phi} - \dot{\psi} \; ; \; a \dot{\phi} = b \dot{\psi} \]  
\[ \dot{x}_F = \dot{x}_E - d \sin \beta \cdot \ddot{\beta} = 0 \; ; \; \dot{y}_F = \dot{y}_E + d \cos \beta \cdot \ddot{\beta} = \dot{S}_4 \]

In fig. 12 are given the angular speeds in rad / s, of the port-satellite arm (Fi’), of wheel 2 (Alpha’), and of rod 3 (Beta ’). It appears that only the angular velocity of the rod varies, with some symmetries.
Piston velocity variation is shown in fig. 13. They are slight variations in the area of interest, at $\varphi = 60 \ldots 120$ degrees; in fig. 14 are observed in more detail the speed variation in this subinterval, ascertaining the values between $\pm 0,1 \text{ m/s}$.

![Fig. 13](image1)

![Fig. 14](image2)

4. Accelerations

By derivation of speeds from relations, with respect to time, are obtained the following relations:

\begin{align}
\ddot{x}_B &= -e \cos \varphi \cdot \dot{\varphi}^2 - e \sin \varphi \cdot \dot{\varphi} \\
\ddot{y}_B &= -e \sin \varphi \cdot \dot{\varphi}^2 + e \cos \varphi \cdot \dot{\varphi} \\
\ddot{x}_E &= \ddot{x}_B - c \cos \alpha \cdot \dot{\alpha}^2 - c \sin \alpha \cdot \ddot{\alpha} \\
\ddot{y}_E &= \ddot{y}_B - c \sin \alpha \cdot \dot{\alpha}^2 + c \cos \alpha \cdot \ddot{\alpha} \\
\ddot{x}_F &= \ddot{x}_E - d \cos \beta \cdot \dot{\beta}^2 - d \sin \beta \cdot \ddot{\beta} = 0 \\
\ddot{y}_F &= \ddot{y}_E - d \sin \beta \cdot \dot{\beta}^2 + d \cos \beta \cdot \ddot{\beta} = \ddot{S}_4
\end{align}

In fig. 15 is shown the variation of angular acceleration of the rod; jumps occur at the ends of hypocycloid branches.

Piston acceleration also has jumps, caused by hypocycloid branches, but in the field of interest variation is little, in the chart being an approximately linear portion.
5. Conclusions

The studied mechanism generates a hypocycloid with 6 branches, the useful branch being the one symmetrical to y axis, with values of $y > 0$.

The mechanism positions confirm that it provides a stationing of the final leading element, but with some errors caused by approximating the hypocycloid arc with a circle arc.

Piston velocities and accelerations have convenient values in the area of interest.

References

STUDY OF STATIC AND DYNAMIC STABILITY OF THIN-WALLED BARS EXCITED BY PERIODICAL AXIAL EXTERNAL FORCES.

Lecturer dr.eng. Minodora Maria PASĂRE, Lecturer dr.eng. Nicoleta-Maria MIHUȚ
Constantin Brancusi University of Târgu Jiu,

Abstract: In these paper, starting from the relations for the displacements and spinning the transversal section of a bar with thin walls of sections opened expressed by the corresponding influence functions and introducing the components of the exterior forces distributed and the moments of the exterior forces distributed due to the inertia forces, the exciting axial forces together with the following effect of these and of the reaction forces of the elastic environment for leaning it may reach to the system of the equations of parametric vibrations under the form of three integral equation These equations may serve for the study of vibrations of the bars, to study the static stability and to study the dynamic stability

Keywords: vibrations, transversal section, elasticity.

It is known the fact that in the case of spatial parameters vibrations of the bars with thin walls of opened sections acted by external periodical forces of fix direction, u displacements (z, t), v (z, t) and rotating ϕ (z, t) of the transversal section of the bar in the process of vibrations is expressed under the integral form

\[ u(z,t) = \int_0^l G_n(z,t) \left[ Q_n(\eta,t) + \int \frac{\partial p_z(\eta,t)}{\partial \eta} \right] d\eta \]

\[ v(z,t) = \int_0^l G_s(z,n) \left[ q_s(\eta,t) + \int \frac{\partial p_z(\eta,t)}{\partial \eta} \right] d\eta \]

\[ \varphi(z,t) = \int_0^l G_\varphi(z,\eta) \left[ m_z(\eta,t) + \int \frac{\partial p_z(\eta,t)}{\partial \eta} \right] d\eta \]

where \( q_s \) and \( q_y \) are transversal distributed charges, \( m_z \) the moment of turning distributed and \( p_z \) the distributed force per unit of length of bar and per unit of length of the outline in the direction of z axe of the bar, having the expressions

\[ q_s = -m \left( \frac{\partial^2 u}{\partial t^2} + y_0 \frac{\partial^2 \varphi}{\partial t^2} \right) - \frac{\partial}{\partial z} \left( N(z,t) \left[ \frac{\partial u}{\partial z} + (y_0 - e_z) \frac{\partial \varphi}{\partial z} \right] \right) \]

\[ q_y = -m \left( \frac{\partial^2 v}{\partial t^2} - x_0 \frac{\partial^2 \varphi}{\partial t^2} \right) - \frac{\partial}{\partial z} \left( N(z,t) \left[ \frac{\partial v}{\partial z} -(x_0 - e_x) \frac{\partial \varphi}{\partial z} \right] \right) \]

\[ m_z = m \left( \frac{\partial \varphi}{\partial t} - x_0 \frac{\partial u}{\partial t} - \frac{\partial^2 \varphi}{\partial t^2} \right) + \frac{\partial}{\partial z} \left( N(z,t) \left[ \frac{\partial v}{\partial z} -(y_0 - e_y) \frac{\partial \varphi}{\partial z} \right] \right) \]

\[ p_z = -m \left( \frac{\partial^2 v}{\partial t^2} - \frac{\partial^3 u}{\partial z \partial t^2} \right) - \frac{\partial}{\partial z} \left[ \frac{\partial^3 u}{\partial z \partial t^2} y - \frac{\partial^3 \varphi}{\partial z \partial t^2} \right] \]
At the same time, in the relations (1), \( N(z, t) \) is the axial force in the bar, function of the section abscissa and of time expressed under the form (3)

\[
N(z, t) = \alpha N_0(z) + \beta N_1(z) \phi(t)
\] (3)

Where \( \alpha \) and \( \beta \) are the constant parameters, \( N_0(z) \) and \( N_1(z) \) known functions of the section abscissa, characterized by the distribution along the axis of the static component and respectively of the periodic one following the charge, and \( \phi(t) \) is a periodical function of time of T period.

\[
\phi(t) = \phi(t + T)
\]

In the case of the following charge, upon the bar will action supplementary transversal charges on the account of rotating the forces once with the fibers applied (fig.1).

Admitting, for the generality, that the external charge, respectively the periodical variable component in time of this one is partially following and \( \Psi_u \) and \( \Psi_v \) are the spinning of the eccentric axis after which is applied the exterior charge in parallel planes with the main planes of inertia of the bar,

\[
\psi_n = \frac{\partial u}{\partial z} + (y_0 - e_y) \frac{\partial \phi}{\partial z}; \quad \psi_v = \frac{\partial v}{\partial z} - (x_0 - e_x) \frac{\partial \phi}{\partial z}
\]

the normal components at the bar axis of the following force is written

\[
p_x'(z, t) = -p_x(z, t) \psi_n = \epsilon \beta \frac{N(z)}{dz} \left[ \frac{\partial u}{\partial z} + (y_0 - e_y) \frac{\partial \phi}{\partial z} \right] \phi(t)
\]

\[
p_y'(z, t) = -p_y(z, t) \psi_v = \epsilon \beta \frac{dN(z)}{dz} \left[ \frac{\partial v}{\partial z} - (x_0 - e_x) \frac{\partial \phi}{\partial t} \right] \phi(t)
\] (4)

To the distribution forces (4), are corresponded also an exterior moment distributed towards the spinning centre.

\[
m_z' = -\epsilon \beta \frac{dN(z)}{dz} \left[ (x_0 - e_x) \frac{\partial v}{\partial z} - (y_0 - e_y) \frac{\partial u}{\partial t} - \left[ (y_0 - e_y) \right]^2 \frac{\partial \phi}{\partial z} \right] \phi(t)
\]

It is supposed at the same time that the bar is leaned elastically over the entire length, after a parallel line with the bar’s axis, ex centered towards the given axis by the distances \( h_x \) and \( h_y \) between the axes (fig.2), the elastic constants of the environment, defining the elasticity of elastic leaning at displacements and spinings, being \( k_x, k_y \) and \( k_y \). In this case, the displacements of the axe after which are distributed the reactions are.

\[
u_r = u + (y_0 - h_y) \phi; \quad v_r = v - (x_0 - h_x) \phi
\]

\[
p_{rx} = -k_x \left[ u + (y_0 - h_y) \phi \right]
\]

\[
p_{ry} = -k_y \left[ v - (x_0 - h_x) \phi \right]
\] (6)
\[ m_{r,z} = -k_\varphi \cdot \varphi \]

The reactions \( p_{r,x} \) and \( p_{r,y} \) action eccentrically upon the central axe of bending-spinning, and will action also upon the bar, so that the spinning moment per the unit of bar length of the reactive forces will be

\[ m_{r,z} = -k_x \left[ u + (y_0 - h_y) \varphi \right] [y_0 - h_y] + k_y \left[ v - (x_0 - h_z) \varphi \right] [x_0 - h_z] - k_\varphi \cdot \varphi \quad (7) \]

In the relations (1) where \( G_n, G_v, G_\varphi \) play a role of Green functions, and are developed the displacements \( u(z,t) \) and spinnings \( \varphi(z,t) \) in series of functions according to the fundamental complete system of functions \( Z^u_k(z), Z^v_k(z), Z^\varphi_k(z) \), which may satisfy the conditions to the limit under the form

\[ u(z,t) = \sum_{k=1}^{\infty} T^u_k(t) Z^u_k(z) \]

\[ v(z,t) = \sum_{k=1}^{\infty} T^v_k(t) Z^v_k(z) \quad (8) \]

\[ \varphi(z,t) = \sum_{k=1}^{\infty} T^\varphi_k Z^\varphi_k(z) \]

amplitude \( T^u_k(t), T^v_k(t), T^\varphi_k(t) \) being functions of time unknown.

It is taken into account the fact that the symmetrical waves \( G_u(z,\eta), G_v(z,\eta), G_\varphi(z,\eta) \) may discompose in square series after realizing the own functions under the form

\[ G_n(z,\eta) = \sum_{k=1}^{\infty} \frac{1}{\omega_{n,k}^2} Z^u_k(z) Z^u_k(\eta) \]

\[ G_v(z,\eta) = \sum_{k=1}^{\infty} \frac{1}{\omega_{v,k}^2} Z^v_k(z) Z^v_k(\eta) \quad (9) \]

\[ G_\varphi(z,\eta) = \sum_{k=1}^{\infty} \frac{1}{\omega_{\varphi,k}^2} Z^\varphi_k(z) Z^\varphi_k(\eta) \]

Replacing in the relations (1) the distributed forces \( q_u, q_p, p_z \) and \( m_z \) and performing all the calculations is obtained the system of differential equations

\[ C \frac{d^2 T}{dt^2} + [E + D - \alpha A - \beta \Phi(t) B] T = 0 \quad (10) \]

where \( T \) is the infinite hyper vector

\[ T = \{T_1, T_2, T_3, ..., T_n\} \quad (11) \]
In the equations (10) E is the infinite unit matrix and the system 10 represents the system of differential equations of second order of the linear homogenous and have variable coefficients, of the parametric special vibrations of the bars with thin walls of the opened sections, leaned on the elastic environment and following excited by periodical axial external forces. These equations may serve for the study of vibrations of the bars, to study the static stability and to study the dynamic stability.

References

THEORETICAL AND EXPERIMENTAL CONTRIBUTIONS CONCERNING THE PROPOSED MODEL FOR THE DISC-TYPED ROTARY ULTRASONIC MOTOR

Asist. dr.eng. Oana CHIVU, Polytechnic University of Bucharest, Prof. dr.eng. Dan Dobrota, "Constantin Brancusi" University of Targu-Jiu, Dr. eng. Gabriel Chirculescu, Rovinari Energy Complex

Abstract - In this work the proposed model for type-disk, ultrasonic motor rotating, elliptic movement to surface beam. A sinusoidal vibration of the vertical displacement in the z-direction. Assume that the vertical displacement of the neutral plane equals the product of the slope of the neutral plane and half of the beam height, the tangential velocity vs at the upper surface is given.

Keywords: elastic-body, sinusoidal vibration, ultrasonic.

1. Introduction

Employing the fundamental theories for ultrasonic motors, we will extend to applying them to rotary ultrasonic motors. Instead of a beam, we now consider an elastic-body ring shown in figure 1 below.

Fig. 1. Visualization from a beam to a ring.

Treating the beam as an infinite body that carries flexural waves. Cutting a length equal to the wavelength multiplied by m times and joining the two ends together forms a ring. A sinusoidal vibration of the vertical displacement in the z-direction can be expressed as:

\[ w_n = C_m \sin \left( \omega_n t - \frac{2m\pi}{l} x + \phi_m \right) + D_m \sin \left( \omega_m \frac{2m\pi}{l} x + \phi_m \right) \]  

where \( \omega_m \) and \( m \) are appropriate phase angles and l is equal to the wavelength multiply by \( m \). The relationship between the vibrational mode number \( n \), and the number of waves cycles present on the ring is:

\[ n = 2m \]  

2. Elliptical motion at the beam’s surface

Figure 2 below illustrates how points on a beam’s surface rotate in a counter-clockwise direction as the flexural wave travels from left to right. Assume that the vertical displacement of the neutral plane can be describe by:

\[ w = \xi_0 \sin( \omega t - kx ) \]
Fig. 2. Points at beam’s surface rotate counterclockwise in an elliptical motion.

As depicted in figure 2, the displacement \( u \) in the \( x \) direction equals the product of the slope of the neutral plane and half of the beam height.

\[
u = \left( k \xi oh/2 \right) \cos(\omega t - kx) \quad (4)
\]

Since \( k = 2\pi/\lambda \), we obtain,

\[
u = \left( \pi \xi oh/\lambda \right) \cos(\omega t - kx) \quad (5)
\]

We observe that the ratio of the minor to the major axis of an ellipse is given by \( \pi h/\lambda \). By differentiating equation (5) with respect to time, we will derive the tangential velocity of each point at the beam’s surface as follows,

\[
\frac{du}{dt} = -\frac{\omega \pi \xi h}{\lambda} \sin(\omega t - kx) \quad (6)
\]

We have assumed that phases A and B are identical except with a 90° phase difference in position and time. Let \( \xi_A \) and \( \xi_B \) be the amplitudes at the neutral plane of phases A and B respectively. Then we will have,

\[
w = \xi_A \sin \omega t \sin kx + \xi_B \cos \omega t \cos kx
\]

\[
\omega = \xi_A \sin \omega t \cos kx + \xi_B \cos \omega t \sin kx \quad (7)
\]

The tangential velocity \( v_s \) at the upper surface is given by

\[
v_s = \frac{\partial u}{\partial t} = -\frac{h \partial^2 w}{2\partial x \partial t} = \frac{-h \omega}{2} \left( \xi_A \cos \omega t \cos kx + \xi_B \sin \omega t \sin kx \right) \quad (8)
\]

To find the position of the crest, we let \( \partial w/\partial x = 0 \). Thus,

\[
\xi_A \sin \omega t \cos kx = \xi_B \cos \omega t \sin kx \quad (9)
\]

From this equation, we obtain the following,
Substituting these terms into Equation (8), we acquire the tangential velocity at the,

\[
\begin{align*}
[v_x]_{\text{top}} &= \frac{-\left(\frac{hk\omega}{2}\xi_A\xi_B\right)}{\sqrt{\xi_A^2\sin^2 wt + \xi_B^2\cos^2 wt}} \\
\end{align*}
\]  

Since \(k = 2\pi/\lambda\),

\[
\begin{align*}
[v_x]_{\text{top}} &= \frac{-\left(hk\omega\xi_A\xi_B\right)}{\lambda\sqrt{\xi_A^2\sin^2 wt + \xi_B^2\cos^2 wt}} \\
\end{align*}
\]  

When \(\xi_A = \xi_B = \xi_o\),

\[
\begin{align*}
[v_x]_{\text{top}} &= \frac{-\left(hk\omega\xi_B\right)}{\lambda} = \frac{-\left(h\xi_o\omega\right)}{\left(\lambda/2\pi\right)} \\
\end{align*}
\]  

According to Equation (14), we know that the velocity in the transverse direction is equal to \(\xi_o\omega\). By knowing this fact, the velocity of the crest is obtained at the end of the shorter arm. Taking moment about the fulcrum point at the neutral plane of the beam, we arrive with the conclusion that the longer arm of the lever has a length \(\lambda\) of divided by 2\(\pi\) while the shorter arm has a length \(h/2\). This analogy applied directly to the actual motor where the comb teeth are considered to be short arms of the lever. The comb teeth are regarded as a row of the levers as shown in Figure 3.

**Fig. 3.** The ‘lever’ principle modeling the comb teeth of an ultrasonic motor.

### 3. Conclusions

1° A rotary ultrasonic motor has a resolution limit. An ultrasonic motor can produce extremely fine position changes until the sub-nanometric field and small variations in operating voltage can be converted into continuous movements;

2° Piezo positioning systems can produce a force of tens of thousands of Newtons (units operating there that can support loads of several tons), making movements with nanometer resolution;

3° Experimentally it is found that ultrasonic motors have a response time of microseconds Odin;
4° Elements of an ultrasonic motor operates without wear. An engine displacement based on dynamic ultrasonic solid no-wear.

5° Experimentally been found not require lubrication. Ultrasonic motors do not need any lubricant and makes them ideal for high vacuum applications.

6° Ultrasonic motors can operate at cryogenic temperatures. Piezo effect is based on electric fields and produce up to near zero degrees Kelvin is only important point Curie temperature.

References


THERMO-MECHANICAL PROPERTIES OF FABRIC REINFORCED COMPOSITES WITH FILLED EPOXY MATRIX

Eng. Igor ROMAN, Dunărea de Jos University, Galați, România
Eng. Vasile BRIA, Dunărea de Jos University, Galați, România,
Eng. Ion POSTOLACHE, Dunărea de Jos University, Galați, România,
Dr. Eng. Adrian CÎRCIUMARU, Dunărea de Jos University, Galați, România,
Dr. Eng. Iulian-Gabriel BÎRSAN, Dunărea de Jos University, Galați, România

Abstract: While the design problem seems to be essential in order to form a high performance composite one may ask more: is it possible to form a material able to give information about its state? Is it possible to control the properties of a composite through alternation of its various layers? Is it possible, finally, to obtain a multifunctional material based on a right design, on a cheap forming technique, on accessible components? This study is about partially answering the above questions. Two types of fiber fabric were used to form composites with filled epoxy matrix and materials bending and thermo-mechanical properties were evaluated using appropriate recommended methods.

Keywords: carbon, aramide fiber, clay, talc, epoxy.

1.Introduction
While the design problem seems to be essential in order to form a high performance composite one may ask: is it possible to form a material able to give information about its state? Is it possible to control the properties of a composite through alternation of its various layers? Is it possible, finally, to obtain a multifunctional material based on a right design, on a cheap forming technique, on accessible components? This study is about partially answering the above questions.

Assuming that a composite material is a complex structure it is obvious that is hard to describe all its properties in terms of its parts properties. The properties of the composite depend not only on the properties of the components but on quality and nature of the interface between the components and its properties. It is obviously that changing the standard receipt of a composite its properties are modified. For example filling the polymer matrix of a reinforced composite not only the electro-magnetic properties will be affected but also the mechanical properties, due to the way in which the polymeric bonds are changed by the presence of the filler’s particles. One question is how long it may be increased the filler’s concentration such as basic mechanical properties to remain unchanged (or with negligible modifications).

Fabric reinforced or textile composites are increasingly used in aerospace, automotive, naval and other applications. They are convenient material forms providing adequate stiffness and strength in many structures. In such applications they are subjected to three-dimensional states of stress. The microstructure of composite laminates reinforced with woven or braided networks is significantly different from that of tape based laminates.

Powders are used as fillers in order to obtain bi-components composites. There is no structural order in such a filled composite, the most important aim being the uniform distribution of particles in matrix. If the fillers’ particles are arranged into the polymer volume is possible to change the electro-magnetic behavior of the obtained composite making this one to act as a meta-material [1]. The powders can be dielectric as talc, clay or ferrite can be magnetic active as ferrite, or electric active as CNT or carbon nano-fibers. All these powders, added to the polymeric matrix, have effects on the electro-magnetic, thermal and mechanical properties of the composite [2]. What about using all of them, based on partially changes induced by each one? There exist many models regarding the mathematic description of electromagnetic properties of the bi-component composites [3, 4].
This study is performed to point out the connection between thermo-mechanical properties of certain laminae and the thermo-mechanical properties of a composite formed with those laminae.

2. Materials

Two types of fabric were used during this study; first type is simple fabric made of untwisted tows of carbon fibers while the second is a simple fabric made of alternating untwisted tows of carbon and aramide fibers. From the beginning the two fabrics were choose because of the intrinsic properties of fibers (electromagnetic and mechanical in the case of carbon fiber, shock and thermal in the case of aramide fiber). Two problems had to be solved before use them to form reinforced composites: their stability – because during the cutting the tows are slipping one on each other leading to structural defects of fabric with consequences in mechanical properties of the composite; the second problem is about the low epoxy adhesion to the two types of fiber which leads to discontinuities at the interface level with consequences in all the composite’s properties. That is why the two fabrics were specially prepared.

First steps are intended to clean the fiber’s surface and to increase its specific surface. After washing with detergent solution and distilled water rinsing, based on its oxidative properties, the fabric was treated with 50% hypochlorite aqueous solution. The last chemical step, after the second rinsing is the treatment with 20% NaOH aqueous solution to remove the residual hypochlorite and to attack fiber surfaces. After rinsing a film of Clay and Talc (5%) filled PNB rubber is deposed on the fabric surfaces.

The final step of the fabric preparation consists in deposing a film of clay and carbon black filled epoxy on the fabric. Once again the film was obtained by spraying A and B epoxy’s components solutions on the surfaces. The amounts of clay and carbon black were dispersed into the A component and then diluted with above mentioned diluent (20% A solution with 6% clay and 6% carbon black), after diluent vaporization the B component solution (5%) was sprayed on the fabric. There is a difference between the recommended amounts of A and B components of RE 4020-DE 4020 epoxy system allowing, at the end, a porous aspect of the fabric surface (Fig. 1).

![Fig. 1. Microscopic images before and after full treatment of Carbon Fiber Fabric (left) and Mixed Fabric (right)](image)

The RE 4020 – DE 4020 epoxy system was used as matrix for the studied materials. During this study many powder fillers were used in order to emphasize their influence on reinforced composites. Based on the two treated fabrics laminated-like materials were formed with: Clay (5%) and Talc (5%) filled matrix.

The forming technique is a layer-by-layer one. Each piece of fabric is imbued with filled pre-polymer and then placed into a mould. After all the reinforcements are placed the mould is closed and gases are extracted using the rubber bag technique.
The materials are extracted after 24 hours at room temperature. Ten different reinforcement structures were used to form the materials and these structures are presented in Table 1 where C denotes carbon fiber fabric and K denotes mixed fiber fabric and the angular values are relative orientations of yarn and fill to the mould edges.

<table>
<thead>
<tr>
<th>Pair</th>
<th>Reinforcement structure A</th>
<th>Reinforcement structure B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C(0°) C(45°) K(0°) K(45°) K(0°) C(45°) K(0°) C(45°)</td>
<td>K(0°) K(45°) C(0°) C(45°) C(45°) C(0°) K(45°) K(0°)</td>
</tr>
<tr>
<td>2</td>
<td>C(0°) C(45°) C(45°) K(0°) K(45°) K(0°) C(45°) C(0°)</td>
<td>K(0°) K(45°) K(0°) K(45°) C(0°) C(45°) C(0°) K(45°) K(0°)</td>
</tr>
<tr>
<td>3</td>
<td>C(0°) C(45°) C(0°) K(45°) K(0°) C(45°) C(0°)</td>
<td>K(0°) K(45°) K(0°) C(45°) K(0°) C(45°) C(45°)</td>
</tr>
<tr>
<td>4</td>
<td>C(0°) C(45°) C(45°) C(0°) C(45°) C(45°) C(0°)</td>
<td>K(0°) K(45°) K(45°) K(0°) K(45°) C(0°) K(45°) K(0°)</td>
</tr>
<tr>
<td>5</td>
<td>C(0°) K(45°) C(0°) K(45°) C(0°) K(45°) C(0°)</td>
<td>K(0°) K(45°) K(0°) C(45°) K(0°) C(45°) K(0°)</td>
</tr>
</tbody>
</table>

3. Measurements and Results

The thermo-mechanical properties are strongly influenced by the fabric due to the thermo-mechanical properties of carbon and aramid fibers. Coefficient of thermal expansion was evaluated both perpendicular on reinforcement and in plane of the reinforcement. Measurements were carried out using a TMA-SDTA 840 from Mettler Toledo.

The heating ratio was 10°C/min in both directions and interval temperature [50°C, 90°C]. The coefficient of thermal expansion was evaluated, using the same technique, for individual laminae and for filled epoxy matrix and the results are presented in Table 2. In the case of fabric laminae the CTE was measured only perpendicularly on the fabric.

Taking into account the ranges of coefficients it might be said that all the materials have the same thermal behavior. Regarding the parallel coefficient its values might be improved by using powders as carbon black or CNT which show negative values of the thermal expansion. It might be noticed that in the case of parallel with the reinforcement all the materials are stable. That is due to the presence of long carbon and aramide fibers which have negative values of CTE and compensate the matrix dilatation.

Using the mixing rule the values of CTE were evaluated based on above presented values of composites’ components. It is easily to notice, from Fig. 2, that there are significant differences between measured and evaluated values of CET but the magnitude of values is the same.

These differences may occur due to the different values of matrix layers (inevitable using the described forming technique) or due to the fact that during molding the reinforcement sheets may deviate from the parallel position.

Also, due to the forming technique there might appear gaseous intrusions inside the matrix with strong effects on CET, even the tests were performed on samples extracted from various areas of formed plates.
4. Conclusions

The composite’s design has to take into account all the aspects in order to offer right solutions for certain applications. Studies have to be carried out to find the influence of various polymers used in the same composite material at different levels – each one with a defined purpose – such as, at the end, the material to fit all the requirements. The use of carbon fibre allows even the opportunity to get information about material’s or structures’ state and integrity without using other devices which increase the costs and, not at least, the weight of applications.

The multi-component composites could represent the cheapest solution when controllable properties are required. A structural microscopically analysis is required in order to identify the quality of interfaces. In the case of a n-components composite there are n-1 interfaces each one of them having its own contribution at composites’ properties. The filler presence in the matrix produces discontinuities at the fibre-matrix interface with consequences regarding mechanical properties.

References

ULTRASONIC TREATMENT ENDODONTICE

Prof.Dr.Eng Gheorghe AMZA, Dr. Eng. Oana Elena AMZA, Dr.Eng. Zoia APOSTOLESCU
Department of Materials Technology and Welding - University Politehnica of Bucharest

Abstract: - The aim of this study is to demonstrate the ability of our ultrasonic endodontic dental device with rotary nickel-titanium alloy instruments for the root canal treatment. This study is aimed at investigating the cleaning and shaping efficacy with “Ultrasonic device for endodontic treatment of dental’s canals. The purpose of root canal treatment is to get rid of the damaged pulp and the bacteria that are causing the infection. It involves removing inflamed or dead nerves and blood vessels’ from the centre of the tooth with an ultrasonic endodontic device. This is done by drilling a hole through the top of the tooth to the root canal and removing the dead tissue. The empty root canal is then cleaned, filled and a permanent seal is put over the top of the tooth. After the main requirements for a successful endodontic treatment and after the elimination of all debris, the need for root canal preparation has long been accepted as an essential step in root canal therapy. Our ultrasonic endodontic device use rotary instrument of nickel-titanium alloy actionated by an ultrasonic device. The endodontic files from a nickel-titanium alloy with a very low modulus of elasticity which have been shown to have two to three times more elastic flexibility in bending and torsion, as well as superior resistance to torsional fracture.

Keywords: ultrasound, endodontic, root canal.

1. Introduction

Ultrasound is a sound energy with a frequency of more than 25 kHz. the application of ultrasound in dentistry has been limited mainly to periodontics, until Richman, in 1957, introduced this technique to endodontics[1]. Ultrasound is now used in endodontics for: improving root canal access (e.g. removal of pulp stones); irrigating root canals; removal of posts, broken instruments and other obstructions from the root canal; distribution of sealer around the root canal walls; condensing gutta-percha root fillings; periapical surgery; enhancing dentine permeability during bleaching. Root canal treatment aims to clean, shape and fill the entire root canal of a tooth. A root canal infection can be caused by several things, including decay, injury and possibly also gum disease. Root canal treatment can repair your damaged tooth without it having to be removed. The endodontist will take X-ray image of the tooth to check whether or not it definitely need root canal treatment. This can help to show how far any decay has spread, if there is an abscess and how many root canals the tooth has. If a dead tooth a one with severely damaged pulp, root canal treatment may be the only way to repair it. In the first step, the endodontist will make a local anesthetic. This completely blocks feeling from the area. The tooth will be separated from the rest of the mouth using a thin sheet of rubber called a dam. This keeps the tooth dry and protects the airways,[1],[2],[3],[5],[9]. It also allows effective cleaning of the root canal system and prevents it from becoming again, which can cause infection later. The endodontist will first make a hole in the top of the tooth through which the dead or diseased pulp is removed (Fig.1).

Fig. 1
The empty pulp cavity is then cleaned and the endodontist may also put in some medication to help get rid of bacteria. He will put a temporary filling on the tooth to keep it sealed until for further treatment. With a temporary filling, to the next visit, the endodontist will remove and fill the root canals with a suitable material. This is likely to be a putty-like substance called gutta-percha. A permanent filling or crown is then placed over the top of the tooth to protect the filled root canal and the vulnerable tooth structure. Teeth are hard calcified objects but their inner aspects are not completely solid. Inside every tooth there lies a hollow space which, when a tooth is healthy, contains the tooth’s nerve tissue. Each tooth’s nerve enters the tooth, in general, at the very tip of its root (s). From this entry point the nerve then runs through the center of the root in small “root canals” which subsequently join up with the tooth’s pulp chamber.[4],[7],[8],[10],[12].

2. The use and the design of the ultrasonic endodontic device

The design of the ultrasonic endodontic device is special made for treating the root canal by cleaning, introducing the medication through an irrigation instrument through ultrasonically vibrated endodontic files. For root canal treatment to be effective, all the canals within the tooth must be cleaned with an antiseptic solution, which helps to treat the source of the infection. The basic aim of the root canal is to clean the root and eliminate tissue debris and microorganisms with an ultrasonic endodontic apparatus. Furthermore instrumentation produces a smear layer and dentine debris in the root canal, which has to be removed by irrigation. Our ultrasonic endodontic device is working by converting electromagnetic energy to mechanical energy. A stack of magnetostrictive metal strips in a handpiece is subjected to a standing and alternating magnetic field, as a result of which vibrations are produced.[13],[14],[15],[16].

2.1. Irrigation of the root canals

Therefore irrigation remains an essential aspect of root canal treatment. During passive ultrasonic irrigation, a small ultrasonically oscillating file or smooth wire (e.g. size 15, 20) is placed in the centre of the root canal, following canal shaping, to transmit the energy of the file as efficiently as possible to the irrigant. As a result, acoustic micro-streaming and/or cavitation can occur(Fig. 2).
As the root canal has already been enlarged, the irrigant can flow through the canal and the file or wire can vibrate relatively freely. The file has a nodal and antinodal pattern, which also occurs on a pre-curved file, partly explaining the efficacy of passive ultrasonic irrigation in curved canals \(^{(6)}\) (See Fig. 3).

![Node- and antinode pattern of a sonically activated file.](image)

Fig. 3

Passive ultrasonic irrigation with sodium hypochlorite (NaOCl) as irrigant, removes more dentine debris, planktonic bacteria and pulp tissue from the root canal than manual syringe irrigation\(^{(7)}\). NaOCl is very effective in combination with passive ultrasonic irrigation and more effective than water \(^{(8)}\). Ultrasonic activation canals enhance the antibacterial and organic tissue dissolving potential of NaOCl through temperature increment \(^{(9)}\). The diameter of the root canal has an influence on the effectiveness of dentine debris removal during passive ultrasonic irrigation. For a root canal of size 20, taper 0.10, the dentine debris can be removed more easily than from a root canal size 20, taper 0.08 or 0.06 \(^{(10)}\). With careful maintenance and by using NaOCl at concentration no higher than 5%, most of the currently available equipment will function trouble free. Clinical advice on ultrasonic irrigation: use NaOCl as the irrigant; continuous flush method: 3 minutes of irrigation per canal, the canal flushing can be reduced to 15 ml/min; intermittent flush method: 1 minute of irrigation per canal with three ultrasonic activation sequences of 20 seconds each and refreshed with 2 ml of NaOCl three times; use a thin (size 15 or 20) non-cutting file or wire; in a curved canal, pre-bending of the instrument is advised.\(^{(15)}\),\(^{(16)}\),\(^{(17)}\),\(^{(18)}\).

2.2. Root canal obturation.

The obturation is realized in 3 steps. First, is the preparation: a X-ray action to assess the damage, Local anesthesia may or may not be needed depending on whether or not infection is present. To prepare the tooth, the area is first kept dry and saliva-free with a dental dam. A access hole is than drilled in the crown, or the top layer of the tooth. Root canal files, increasing in size, are than used to clean the inside of the tooth \(^{(11)}\). (Fig.4). The second step of the root canal obturation is sealing of the hole. This may be the decision if there is an infection present, and medication is usually given to speed the healing process. If the tooth is not sealed immediately, a temporary filling is necessary \(^{(12)}\). After the tooth is sealed the inside of the tooth must be filled.
The inside of the tooth is filled with a rubber substance called gutta-percha.[17],[18],[19].

**Fig. 4**

2.3. **Ultrasonic condensation of gutta-percha.**

Ultrasonically activated spreaders have been used to thermoplasticize gutta-percha in a warm lateral condensation technique. Initial placement of a gutta-percha cone to the working length is followed by lateral condensation of two or three accessory cones using a finger spreader (13). The ultrasonic spreader is then placed into the center of the gutta-percha mass 1 mm short of the working length and activated at intermediate power. After activation, the ultrasonic spreader is removed, and an additional accessory cone is placed, followed by energizing with the activated ultrasonic spreader. The spreader must be in a canal only 10 sec. Using a sealer with ultrasound gives the best results (14, 15). In Fig. 5 is illustrated an X-ray image of root canals filled with gutta-percha and sealer. [2],[3],[4].

**Fig. 5**

2.4. **Root canal preparation.**

Using rotary burs in a micro handpiece is faced with several problems, such as a cavity preparation not being parallel to the canal, difficult access to the root end, and risk of lingual perforation of the root. Furthermore, the inability to prepare to a sufficient depth “retention”. Several studies have shown that ultrasonically or sonically prepared teeth prepared by hand instruments. There is a relative inefficiency of ultrasounds in debridement of the canals, so we can use it not as initial instrument but with handle files to give the best results. Elimination of micro-organisms and their products form the root canal system and to shape it to receive an inert filling material. Micro-organisms are found to a variable degree up to the apical foramen in three modes: as a suspension in the root canal, colonizing the canal walls and colonizing the dentinal tubules. From the preoperative radiograph, estimate the average length of the tooth. Select a reproducible coronal reference point. It should not be part of a portion of tooth or restorative material that is likely to break off. It is chooses a file large enough to be visible on the radiograph (at least size 10). Insert a file into the root canal, 1-2 mm short of the estimated length and take a parallel view radiograph. An average distance of 1 mm short of the radiographic apex is widely accepted as a reasonable estimate of the terminal portion of the canal. Remember, at times, this may be inaccurate by up to 3 mm. (Fig. 6).
Using the electronic apex locator of the ultrasonic endodontic device, we will appreciate the working length. It is very important this measurement for chose the right taper. Mechanical preparation refers to controlled removal of dentin by manipulating root canal instruments \(^{(16)}\). To avoid procedural errors resulting in loss of working length, it is used smaller instruments (No. 20 or smaller) for sufficient time until the larger sizes pass in the canal without force. You can even create intermediate files as suggested by Weine, by trimming 1m from the tip of the file and rounding off sharp edges on a diamond nail file \(^{(17)}\). This way you can convert 10, 15 and 20 files to 12, 17 and 22.

2.5. Operating mode of the ultrasonic endodontic device.

An ultrasonic endodontic dental handpiece has an elongated housing supporting a coil connected for establishing an alternating magnetic field, the housing having a cooling fluid inlet at one end and being open at the other for receiving and supporting a removable insert. The insert includes an elongated hollow body having one end adapted to be insertably mounted in the open end of the housing in fluid communication with the interior of the handpiece, and an elongated tool support assembly telescoping received in the body. The tool support assembly includes an elongated shaft member having a vibrator rigidly mounted on one end in position to be vibrated by the electromagnetic field and a seal located between the body and shank outboard of the housing to prevent the flow of cooling liquid through the body past the seal. A cooling fluid outlet is provided in the body between the housing in the seal to permit cooling fluid to flow through the handpiece, and an irrigation fluid passage is provided in the shaft outboard of the housing to permit the flow of irrigation fluid along the shaft to the terminal end of the insert assembly \(^{(18)}\). A mounting head on the end of the shaft supports an endodontic instrument to be vibrated by the vibrator, with the head including a fluid flow passage for discharging irrigation fluid longitudinally of the endodontic instrument. An ultrasonic endodontic apparatus is presented in Fig. 7. Fig. 7a is a real view and Fig. 7b is a longitudinal section through our ultrasonic endodontic apparatus.\cite{16,17,18,19}.
Ultrasonic endodontic device illustrated in Fig. 7 utilizes ultrasonic energy and, preferably, includes a shaft assembly and a stainless steel hypodermic needle which directs and concentrates the ultrasonic energy at the tip of the needle. In the ultrasonic device including a handpiece having an elongated hollow housing, coil means in said housing adapted to be connected to an external energy source for establishing an alternating electromagnetic field within said housing, said housing having a cooling fluid inlet at one end and being open at its other end, and an insert adapted to be removable mounted on the other end of said handpiece, said insert comprising (19). An elongated hollow body having one end adapted to be insertably mounted in said open end of said being in fluid communication with the interior of the housing whereby cooling fluid admitted into said housing can flow into said body. A tool support assembly telescopingly received in said body, said tool support assembly including an elongated shank having vibrating means rigidly mounted on one end, said vibrating means projecting from the open end of the said body in position to be vibrated by the electromagnetic field when said body is mounted in said housing. Seal means provided within said insert between said body and said shank outboard of said housing for directing the flow of cooling fluid through said body between said seal means, cooling fluid outlet means in said body between said seal means and said one end of said body providing a flow path for cooling fluid from said handpiece, the another end of said shank projecting outward from said body and terminating in mounting means for supporting an endodontic instrument, said shank having a fluid flow passage extending longitudinally therein from said mounting means and terminating at a location within said body. Irrigation fluid inlet means for admitting an irrigation fluid into said fluid flow passage to be discharged from said mounting means in a direction substantially parallel to the endodontic instrument mounted in said mounting means, whereby said irrigation fluid and said cooling fluid may be separately controlled. [1],[2],[3].

2.6 Surgical endodontics.

A root canal surgery, also known as apicectomy is an endodontic surgical procedure whereby a tooth’s root tip is removed and a root canal cavity is prepared and filled with a biocompatible material. State of the art procedures make use of microsurgical techniques, such as a dental operating microscope, micro instruments, ultrasonic preparation tips and calcium-silicate based filling materials. A conventional endodontic treatment is indicated if the dental pulp (nerve) of a tooth becomes non-vital (dies) or is likely to be put at risk due to the type or size of restoration needed to repair the tooth. During endodontic treatment, it is removed the dead remnants of the dental pulp and replaces it with an inert filling material which is visible on an X-ray. A tooth that must be assumed to an endodontic surgical procedure is given in Fig. 8. In this procedure, the endodontist opens the gum tissue near the tooth to see the underlying bone and to remove any inflamed or infected tissue. The very end of the root is also removed. (See Fig. 9)
A small filling may be placed in the root to seal the end of the root canal, and a few stitches or sutures are placed in the gingiva to help the tissue heal properly. (see Fig. 10). Over a period of months, the bone heals around the end of the root. (See Fig. 11)

3. Problem Solution: FEM

Ultraacoustic system is the most important unit of an ultrasound processing plant because it made acoustic parameters (sound intensity, sound energy density, the amplitude of oscillation, wave type, frequency of oscillation) and mechanical parameters (static pressure and pressing force). The ultraacoustic system used in this study consists of a piezoceramic set: 1 – reflector, 2 – piezoceramics pills, 3 – speaker, 4 – booster (intermediate item), 5 – concentrator. (See Fig. 12). Ensemble piezoceramic components, given the frequency of 20 kHz, generates ultrasonic oscillations (mainly longitudinal). Ultrasonic energy concentrator is coupled with all piezoceramic components through a booster (intermediate element). Booster is a ultraacoustic system which is interposed between the transducer and the concentrator and a certain factor increases mechanical vibration amplitude transmitted by the transducer supported within the concentrator. [4], [5], [6].

All active elements of the ultraacoustic system are piezoceramic’s pills PZT4 with geometric dimensions shown in Fig. 13.

Meshing (Fig. 14) is done with SOLID 98 item (3D with 10 tetrahedron solid knots).
Volumes of two piezoceramic pills coupled with opposite polarization directions are represented in fig. 15. In Fig. 16 is given the meshing of pills with selected element mesh. Respecting the physical reality, nodes placed in common areas of the volumes they are taken all the degrees of freedom, corresponding symbols are represented in Fig. 17. Symbols corresponding to electrical charges applied to nodes in areas that are deposited electrodes with values in the range allowed (0-2000v) is illustrated in Fig 18.

In Fig. 19 are represented two piezoceramic pills deformed and non-deformed in frontal position and in Fig. 20 the same thing in isometric situation where the external electrodes appear with negative charge and the mass focal plane.

In Fig. 21 (geometry volumes piezoceramic assembly) and Fig. 22 (finite element mesh volumes piezoceramic assembly) are represented all stages of construction of piezoceramic model. In Fig. 23 is represented deformed/non-deformed state in isometric and vertical view of piezoceramic assembly for resonant frequency of 20kHz. In Fig. 24 is represented the deformed/non-deformed state in vertical view of an piezoceramic assembly for resonant frequency of 20k Hz.[16],[17].
Graphical representation of the variation in amplitude across an piezoceramic assembly is given in Fig. 25.

![Graphical representation of the variation in amplitude across an piezoceramic assembly](image)

**Fig. 25**

In this study it has also been proposed to use ultrasonically vibrated endodontic files in the performance of root canal therapy by supporting the file for axial or longitudinally ultrasonic vibrations and mounting the file in a rigid clamping means and ultrasonically vibrating the file in a transverse, wave-like motion to enhance the debriding action. It is also known to provide an irrigating fluid directed from an ultrasonically vibrated endodontic file longitudinally of the axis of the file to enable irrigation of the root canal while the debriding action is proceeding; however the known ultrasonically vibrated ndodontic file supports and root canal irrigating devices have not generally been capable of selectively separately and controllably applying fluid medication or other irrigating liquid other than the cooling liquid circulated through the dental handpiece. Finally, integration of new technologies such as ultrasound, leading to improved techniques and use of materials, has changed the way endodontic is being practiced today.[12],[13],[14].

4. Conclusion

The ultrasound was introduced in dentistry about 50 years ago, when it was discovered that it was able to cut the dental material with the aid of an abrasive slurry composed of aluminum oxide. During the decade of 1950 dentists uncovered the following advantages of the dental cavity preparation with ultrasound: cutting of the tooth with very light pressure; absence of pain in the majority of the procedures; absence of harmful effect on the pulp; precision of cut and excellent quality of finishing. The most important advantages are: minimum noise; absence of pain in the great majority of the cases, preventing the use of anesthesia; total visibility and access; cutting precision; minimally invasive preparation; selective cutting of hard materials; it does not cut soft materials, as gingive or tongue; it minimizes the bleeding; it does not add residues on the tip, facilitating the cleaness; it minimizes the smear layer. is the modern age of the dental cavity preparation with ultrasound, which gives superior quality and efficiency. Disadvantages: it cannot be used in patients who use pace-maker, handpiece is large and heavy.

References:
LEARNING TOPSOLID SOFTWARE FOR INDUSTRIAL AND EDUCATIONAL PURPOSES – A COLLABORATIVE PROGRAM DEVELOPED BY THE UNIVERSITY OF CRAIOVA AND DICO ROMANIA COMPANY

Alexandru STANIMIR, Dumitru PANDURU, Cătălin VĂDUVA,
Andrei Onu NEACȘU, Bogdan MĂȘU
University of Craiova

Abstract: In this paper we present a learning program of TopSolid software and some results obtained on Computer Aided Programming of machine tools acquired by a group of students that followed an introductory course in TopSolid 2009 at the Faculty of Mechanics from Craiova and a traineeship in DiCo Romania company. This program took place outside school hours and it was attended by students who have voluntarily enrolled. After completing the program learning of TopSolid CAD and CAM modules, two students of the group, who are in the final year of study at the Faculty of Mechanics from Craiova, have applied for employment in the mentioned company. Currently, until completion of their undergraduate, they are working part time at DiCo Romania.

Keywords: TopSolid, CAD/CAM, learning

1. Introduction

Designing products that can be easily manufactured is one of the key challenges of companies nowadays. As a result, a large game of computer-aided design and computer-aided manufacturing software (CAD/CAM) was developed [7,9]. There are several types of CAD/CAM software that are listed as CAD programs. Often, capabilities differ by application [3,14,15]. TopSolid is a contemporary CAD product developed by Missler Software that offer a global and integrated general mechanical solution for both design and manufacturing [10, 11, 12, 13].

With increasing complexity of shape parts, the factories were forced to increasingly use CAD/CAM systems and in many cases they are interested in collaboration with universities to ensure its specialists in the field. DiCo Romania S.R.L. is an example of factory in which our students carry out practical internships focused mainly on computer assisted programming machine tools, using TopSolid. TopSolid’Cam is recognized today as a CAD/CAM software leader thanks to its ability to manage 2 axes milling, 3 axis milling, 4 & 5 axis continued milling, 4 & 5 axis continued turning, synchronization and complex simulation [17].

The role of the ‘typical’ university engineering program is to produce graduate engineers prepared to enter engineering practice, and to conduct, applied and fundamental research in the engineering disciplines. Because of the rapid development of certain technologies by industrial research organizations and software developers, the engineering program may find itself lagging in the integration of current engineering practice and technology. In the CAM application, details on lines and surfaces joining properly in the solid model are critical to creation of the NC code [2, 6, 16].

Engineering students rarely have the opportunity to experience the entire product realization process, from designing a product to developing a manufacturing plan for it and subsequently producing it in volume. In this conditions, a lot of courses are implemented in the engineering curriculum, with the objective to improve manufacturing engineering education by providing students with manufacturing and production experiences [1, 4, 5, 8].

In this paper are presented some results, obtained on Computer Aided Programming of machine tools, acquired by a group of students that followed an introductory course in TopSolid 2009 and a traineeship in DiCo Romania S.R.L. and in our own laboratory on a YMC 1050 machining center [18,19].
2. Manufacturing laboratory facilities used in the learning program

In order to promote production techniques on CNC machines and create an appropriate framework for their learning, at the Faculty of Mechanics from Craiova was developed an intense activity of endowment of laboratories as required.

Currently, the manufacturing laboratory of the faculty has, among others, a 3 axis machining center (fig.1), a TESA measuring arm (fig.2) and a Mitutoyo SJ201 rugozimeter (fig.3), that ensure parts fabrication and their control. Also there are available three Pentium computers and a lot of others devices and apparatus (fig.4).

3. Brief presentation of DiCo Romania company

Dico Company chain has been created as a result of association between a couple of Italian companies. Dico Romania company belongs to this network and has been created in the summer of 2004, being already operational in November, the same year.

The main activity domains of this company are: milling, electric automation panels, sales and service for cutting tools. It is foreseen that by the beginning of the next year to launch production in the mounting department, where will be made assembly units and assembly machine. Currently the company has 65 employees, and the forecast is this number will be increased to 100..120 employees by 2011. The company turnover for this year is estimated at 1.500.000 euros. The company is structured on departments and we can emphasize three of them in cutting zone: traditional machines (miller, lathes), NC machines, vertical and horizontal machining centers.
The Dico Company promotes the development of educational system in order to train youngsters using a structured approach based on practical and theoretical experience. In this respect the Dico Romania Company has decided to begin collaboration with University of Craiova by offering students with the opportunity of practical exercise in this company. Simultaneously, the University has been provided with a platform of CAD / CAM program.

By this approach, it is aimed to discover skilled students and graduates, who can contribute at enhancing and developing Dico service.

4. The learning program

In 2009, Dico Romania company from Craiova, which is specialised in manufacturing on numerical control machines, and which has old links with our faculty, proposed us a collaboration with regards to using Top Solid software in both industrial and educational purposes. Following this, in 2010, when the above mentioned company provided us with three licenses of TopSolid software, we started an optional training course in Top Solid programming.

This learning program initially started with three students who offered to voluntarily undertake this course. Subsequently, three other students (of different specializations, other than industrial engineering) were asked to participate in the program's learning CAM module and they have been accepted. Because participating students came from different specializations and they had no the same ability to use computer, we started from a level that allows everyone a proper assimilation of new information. It should also be mentioned that, due to the urgent need to introduce the TopSolid at Dico company, which need to realize at that time a variety of products, we could not make a proper publicity of this learning program.

The schedule has been established function of participant’s options.

The training involved an introductory course regarding the numeric command machine programming and presentation of some practical elements of operation on YMC 1050 machining center (fig.7), belonging to our faculty (1 week), a traineeship at Dico Romania (1 week), and the learning to use of CAD and CAM modules of TopSolid software (approx. 1 month for each).
The learning phase of each module was completed with a test which, depending on each situation, involves the creation of the 3D model or the creation of the numerical command program for a medium complexity part. Figure 9 presents an example of 3D part model, which was the object of such test, and into the figures 8, 10, 11 and 12 are some steps taken to create the NC program.
Finally, the students have developed a representative part for manufacturing in 3 axis machining centre.

Acknowledgement: The authors thank Ms. Vittorio Grandi, General Manager of DiCo Romania for his involvement in the developing of this collaborative program. Also, we thank Ms. Michele Luppi, a specialist in programming CNC machines and an accomplished practitioner, who supported the practical training of our students.

5. Conclusions

Based on the student’s results analyze concerning the programming of machine tools with Top solid software, we can conclude:
- The TopSolid interface is friendly, easy to use and well accepted by students due to its suggestive icons;
- Although the amount of information necessary for the proper use of the TopSolid Cad and Cam modules is large compared to the time that was provided for study, each student who attended the course, proved to be able to create a 3D model and a NC programe for parts of a medium complexity;
- the final phase of the learning program, which consisted in modeling, programming and execution of a representative part for 2 axis machining, pointed out the quality of the work in group of students - dimensional and geometrical deviations of the part made by them on YMC 1050 machining center not exceeding 0.03 mm.

In terms of learning program structure and its development, we can conclude:
- the introductory course regarding the NC programming and the presentation of some practical elements of operation on YMC 1050 machining center, belonging to our faculty, provided the smoothing of student's knowledge on these issues;
- the traineeship in DiCo Romania company was essential for students to understand ISO programs for a big variety of parts, the correct interpretation of technological documents and the operation on machining centers;
- the beginning of TopSolid’Cad learning was compromised by the fact that the software was available only in demo version and this was one of the causes of the extension of this phase.

Finally, we can conclude that by this program DiCo Company had the possibility to select students who like its working style and the industrial engineering specialisation of our faculty demonstrates that it is able to produce students prepared under industrial environment requirements.
References


[10] TopSolid Quick references, Missler software
[12] Top Tool, Missler software
STRENGTH ANALYSIS METHODS OF CIRCULAR PULL BROACH COGS

Eng. Cosmin MIRITOIU, University of Craiova, Faculty of Mechanics

Abstract: A very big importance in a pull broach designing is represented by its mechanic computation, which trots out the pull broach resistance on various blank tooling, pull broach productivity and also the loadings which is subdued to and the stresses that appear during the chipping process. The pull broach geometric complexity leads to one difficulty concerning the resistance computing methods application (and implicitly, simplifying assumptions application). This present study presents a resistance computing of pull broach cogs, which dresses a circular hole trotting out more methods which can be used in this computing, and the theoretic aspects are then trotted out by an example of a numerical computation for a particular case.

Keywords: pull broaches, mechanical resistance, mechanical stress, finite-elements analysis, circular plate

1. Introduction

The pull broaches are tools of high-leveled productivity, which are used at chipping processing of circular holes, various inner channels, and also at simple or profiled plane outer surface processing. The pull-broaches are high-leveled, constructive and operational complexity tools which leads to a high-cost price. From this reason, it is only used in big production or in operations found in lots of adjusting strips (eg: for quoin channel tooling). An important chapter in pull broach designing is represented by its resistance computing. The high geometrical complexity of the pull broach leads to difficulties in picking the best computing method which has to ensure its resistance during the chipping operation, and to trot out the best phenomenon which takes place during the too ling process. The present study presents a strength computing of a circular pull broach cogs which processes a circular hole, possible stresses on which the pull broach is subdued during the too ling process, being studied. There are approached more mathematics computing simplifying methods and also finite elements analysis methods. The pull broaches are generally made of high steels. For this present study the high steel HS-18-01 STAS SR EN ISO 4957 is chosen as material. It is considered that the pull broach works on a draggin broached machine. In a general case, a circular pull broach which processes a circular hole has a form presented in figure 1.

Fig.1. circular pull broach

2. The strength computing of the pull broach cogs

The circular pool borach is a tool which has, in general, its cutting part made of three parts: roughing, finishing and calibrating parts. The strength computing will be made for the first roughing cog that contacts the material and parts the addition processing, which is the most intensively loaded.
2.1 The cog is subdued to bending
Two cases are taken into consideration: variant 1 - the cog is a bar with a fixed end; variant 2 - the cog is a circular plate.

2.1.1 Variant 1
This method considers the pull broach cog as a bar with a fixed end and is actuated at the other end by the chipping force according to the figure 2, using the notations: h - the height of the cog, F - the chipping force, M₂ and V₂ reactions (the unknown introduced by constraint).

![Fig. 2: bar with a fixed end](image)

According to the upper schematization, the cog is subdued only to bending. The bending stress is determined by the relation (1):

\[ \sigma_{ii} = \frac{M_{\text{max}}}{W}, M_{\text{max}} = F \cdot h \]  

(1)

where \( M_{\text{max}} \) – maximum bending moment, \( W \) - axial strength modulus.

So that the pull broach would not be broken during the tooling process, the condition \( \sigma_{ii} < \sigma_{ai} \) has to be fulfilled, where \( \sigma_{ai} \) is the stress admissible to the bending of the material of which is made the pull broach (in our case there is HS-18-01 STAS SR EN ISO 4957).

To demonstrate the upper announced theoretical relations, this method is used for a chosen particular case, in this way:

- \( F = 2400 \text{N}; \) \( h = 3,5 \text{ mm}; \) \( f_1 = 3 \text{mm}; \) \( W = 4,5 \text{mm}^3, \) \( \sigma_{ai} = 1000 \text{ N/mm}^2 \) (3), where \( f_1 \) is the width of the cog.

The results are: \( M_{\text{max}} = 8400 \text{ N·mm}, \) \( \sigma_{ii} = 1867 \text{ N/mm}^2 \) (4)

We observe that \( \sigma_{ii} > \sigma_{ai} \) which means that the cog doesn’t resist at the bending stress. In this case, so that the cog would resist, the next measures have to be taken: changing the chosen material with a more resistant one to bending; increasing the axial strength modulus which leads to a smaller bending stress; decreasing the height of the cog which leads to the decreasing of the bending moment.

2.1.2 Variant 2
In this case, the cog is also subdued to bending, but it is taken into consideration the fact that the cog is a plate, the action of the chipping force won’t be in a point, but on the all exterior contour. Because of the geometrical complexity the cog has, it is considered that the pull broach cog has the same width on all its height (practically, the most inimical case is considered because the section of the cog base decreases). This is presented in figures 3,4 and 5. According to the considerations that were made, axial chipping force, actuates the cog just like in figure 5. To make this computing, the cog will be considered as a circular plate, constrained on the interior contour and actuated on the exterior contour. According to [5], the maximum stress that makes the bending is computed with the relation:

\[ \sigma_{\text{max}} = (2 \cdot F \cdot k)/(f_1)^2 \]  

(5)

where \( k \) is a factor that heeds the a/b ratio with the values given in tabel 1.
If the a/b ratio values is found in table 1, \( \sigma_{\text{imax}} \) is directly computed and the next condition must be fulfilled: \( \sigma_{\text{imax}} < \sigma_{\text{ai}} \) (6), so that the cog resists bending. If the a/b ratio is not found in table 1, the interpolation is used to find the value of \( k \), following next steps: we create the function \( y(x) = mx^2 + nx + p \), the values are consecutively given to \( x \) and three unknown equations system is made; the unknown 3 equations system is solved and then the values \( m, n, p \) are obtained resulting that all the parameters of the function \( y(x) \) are known; \( x = a/b \) is inserted in the function \( y(x) \) resulting \( k = y(a/b) \); \( \sigma_{\text{imax}} \) will result by computing the value of \( k \); the strength condition (6) is checked. To demonstrate the upper theoretical relations, this method is used for a particular case, the same from 2.1.1 subitem:

\[
F=2400\text{N}; h=3,5\text{mm}; f_1=3\text{mm}; \sigma_{\text{ai}}=1000\text{N/mm}^2; D=45\text{mm}; d=44,5\text{mm}
\]

(7)

The steps showed at 2.1.2 subitem are followed and there are obtained next results (it is mentioned that is the case where \( k \) is not directly chosen from the tabel):

\[
a=22,5\text{ mm}; b=19,25\text{mm}; m=1,253; n=-3,311; p=2,437; y(x)=1,235x^2-3,311x+2,437; (8)
\]

\[
x=a/b=1,184; k=y(1,184)=0,247; \sigma_{\text{imax}}=131,8\text{N/mm}^2<\sigma_{\text{ai}}.
\]

The strength condition at the bending of the cog is checked by having:

\[
\sigma_{\text{imax}}=131,8\text{N/mm}^2<\sigma_{\text{ai}}=1000\text{N/mm}^2.
\]

| Table 1 |
|-------|-------|-------|-------|
| a/b   | 1,25  | 1,5   | 2     |
| K     | 0,227 | 0,428 | 0,753 |

**Fig.3.** Real pull broach cog [4]

**Fig.4.** Simplified pull broach cog; 3D sectional representation

**Fig.5.** Chipping force action

**Fig.6.** The cog is a circular plate constrained on the interior contour and actuated on the exterior contour

**Fig.7.** Cog loading schematization [5]
2.2 The cog is subdued to bending impact

It is considered that the cog enters the material with impact. There are considered two cases: variant 3 - the cog is a bar with a fixed end and actuated at the other end by chipping force; variant 4 - the cog is a circular plate actuated by the chipping force on all the exterior contour.

2.2.1 Variant 3

It is considered the same case as the one at the subitem 2.1.1. The static displacement is computed with the relation (9):

\[ v = \frac{(F \cdot h^3)}{(3 \cdot E \cdot I_z)} \tag{9} \]

Fig. 8: variant 3 computing scheme

where \( E \) - longitudinal elastic modulus, \( I_z \) - inertia moment.

Then, the impact intensifier \( \psi \) is computed with relation (10):

\[ \psi = 1 + \sqrt{1 + \frac{2 \cdot h_c}{v}} \tag{10} \]

where \( h_c \) - the height between the force \( F \) and the cog until they make contact. If the chipping force is suddenly applied, then \( h_c = 0 \). The maximum stress produced to impact will be: \( \sigma_{soc} = \sigma_{st} \cdot \psi \) (11), where \( \sigma_{st} \) - the maximum stress produced at the statical application of the chipping force (it is identical with the stress \( \sigma_{i1} \) produced at the bending case presented at the subitem 2.1.1). The condition \( \sigma_{soc} < \sigma_a \) (12) must be fulfilled, where \( \sigma_a \) is the impact admissible stress of the material of which the pull broach is made. The dinamic displacement is computed with the relation:

\[ f = \psi \cdot v \tag{13} \]

To demonstrate the upper defined theoretical relations, this method is used for a particular case (the same treated as the one from the item 2.1) defined this way: \( F = 2400 \text{N}; h = 3.5 \text{mm}; f_1 = 3 \text{mm}; I_z = 6.75 \text{mm}^3; \sigma_{a} = 1000 \text{N/mm}^2; \sigma_{i1} = 2.1 \cdot 10^6 \text{N/mm}^2; h_c = 0 \).

Numerically replacing, we obtain:

\[ v = 2.42 \cdot 10^{-3} \text{mm}; \psi = 2; \sigma_{soc} = 3733 \text{N/mm}^2; f = 4.84 \cdot 10^{-3} \text{mm} \] (15)

It is noticed that \( \sigma_{soc} > \sigma_a \), the condition (12) is not respected and the cog does not resist to impact.

2.2.2 Variant 4

It is used the scheme from figure 8. According to [5], the displacement is computed with the next relation:

\[ w = k_1 \cdot \frac{F \cdot a^2}{E \cdot f_1^3} \tag{16} \]

where \( k_1 \) is a factor that caters for \( a/b \) ratio with the values given in the table 2.

\[ \text{Table 2} \]
It the a/b value is found in tabel 2, w is directly computed and then the impact intensifier, $\psi$, is computed with relation (10). The maximum stress produced at impact is computed with relation (11), the strength condition (12) is checked and the dynamic displacement is computed with relation (13). If the a/b value is not found in tabel 2, the interpolation is used to find out the value of $k_1$ following the methodology described at 2.1.2 subitem. To demonstrate the upper defined theoretical relations, this method is used for a particular case (the same treated at subitem 2.2.1). There are obtained the next results (it is mentioned that it is the case where $k_1$ is not directly chosen from the tabel 2):

$$a=22.5\text{mm}; b=19.25\text{mm}; m=0.062; n=-0.091; p=0.022; y(x)=0.062x^2-0.091x+0.022;$$

$$x=a/b=1.184; k_1=y(1.184)=1.175\times10^{-3}.$$ 

(17)

The displacement is computed with relation (16): $w=2.517\times10^{-5}\text{mm}$.

(18)

Then, the other elements are computed, obtaining: $\Psi=2; \sigma_{soc}=263.6\text{N/mm}^2; f=5.035\times10^{-5}\text{mm}$.

It is noticed that $\sigma_{soc}<\sigma_a$, the strength condition is fulfilled and the cog resists at impact.

### 2.3 Finite element method

It is considered that the pull broach cog, which according to figure 10, is fixed on the interior contour and actuated on the exterior contour by the force $F$, the chopping force. It is used the same particular case defined at the previous items. After the finite element analysis, the stress map from figure 10 resulted. It is noticed that the maximum stress produce by force $F$ is $\sigma_{max}=34.14\text{N/mm}^2<\sigma_{ai}=1000\text{N/mm}^2$, so the cog resist to bending. The finite element analysis is made only for the static case.

**Fig.9. stress distribution in the cog.**

### 3. Conclusions

We can extract the following conclusions:

- the using of the first method (the one from 2.1 subitem) leads to implausible results because it is noticed that the cog does not resist to the bending solicitation, the bending stress being too big than the one really produced; same conclusions can be made from the impact loading too.
- the results obtained with the second assumption are sustained by the finite element
analysis (in this cases, similar values are obtained), the bigger bending stress obtained with the second method resulted from the fact that the most inimical case was chosen (which is: the width of the cog is constant on all its height)

- the second variant gives satisfying results also for bending impact computing, a solicitation to which the cog resists
- for a complete strength computing, it is recommended to use the methods presented at the subitems 2.1.2, 2.2.2 and 2.3.

References
NUMERICAL VERSUS ANALYTICAL METHOD IN FINDING STRESS STATE IN MECHANISMS ELEMENTS

Assoc. Prof. Stelian ALACI, Suceava University,
Lecturer Florina Carmen CIORNEI, Suceava University,
Professor Dumitru AMARANDEI, Suceava University,
Assoc. Prof. Constantin FILOTE, Suceava University,
Assoc. Prof. Delia-Aurora CERLINCA, Suceava University,
Lecturer Luminița IRIMESCU, Suceava University.

Abstract: The finite element method is applied to find the stress field from the parts of a wobble plate mechanism. There are thus identified the most stressed component elements and the respective regions. For the two contact neighbourhoods where stress concentrators occur, the maximum contact pressure is found by analytical methods and afterwards compared to the numerical results.

Keywords: finite element, gradients, derivatives equations.

1. Introduction

The elements of a mechanical structure work together, most of them either by direct contact or by means of electric or magnetic fields, cases that are seldom met and especially in electric drives, armature cores etc. For the parts from the first category, the contact can be accomplished in two ways, [1], namely contact characterized by broad surfaces or conforming contact, as in the case of sliding bearings, or, secondly, Hertzian contact, where there is no initial contact surface, for instance, as in ball bearings, cams and gears. These Hertzian contacts are characterized by strong stress gradients and solutions are required to diminish the maximum contact pressure in order to avoid material failure.

The stress state for a certain part, with précised loads and under the assumptions of elastic strains is found using the relations from theory of elasticity. These equations, which are partial derivatives equations, need finding a particular solution to satisfy the imposed boundary and initial conditions, [2]. Alas, due to the form taken by the boundary conditions, only some elasticity problems present analytical solutions.

The necessity of stress field estimation in mechanical parts for an optimum design imposed at a large scale level the numerical methods, [3]. These methods proved to be an extremely efficient but discrimination should be considered when using them. It is recommended that the numerical results of a certain problem to be compared, when possible, to the analytical result when the last exists, or to apply the numerical model to a simplified problem which presents a known analytical solution. The present paper aims to apply and emphasize the above aspects by using the exposed methodology for an actual case, namely the wobble plate mechanism.

2. Stress evaluation from mechanism finite element modelling

The swash plate mechanism is in fact a spatial mechanism, [4], with frontal cam, for which a scheme is presented in Figure 1. The mechanism consists of a disk, 1, assembled on a shaft so as between the normal to the disk plane and the shaft axis the $\alpha$ angle appears. A plunger linked to the ground through the prismatic joint, contacts the disk. The stresses in the plate-follower contact region are infinite and decreasing them requires increasing the curvature radius, and the mechanism appears as shown in Figure 2. In this new representation the contact stresses between sphere and plate are particularly high, yet.
In order to diminish the contact stresses between the sphere and the plate, a new part, having a spherical cavity of the same radius as the sphere and a plane surface in contact with the plate surface, is interposed between the two parts. The two plunger mechanism was considered for a comparative study, as shown in Figure 3, where a plunger has a spherical head and the other one contacts the interposed part. By using the Finite Element Analysis module of CATIA software, the $\sigma_z$ stresses in the parts of the mechanism were found and a component is shown in Figure 4.

One can notice that the most stressed region of the mechanism occurs in the neighbourhood of the ball-plane Hertzian contact, and this being the reason for introducing the intermediate part. Next, numerical results for different contacts that appear in the structure of the mechanism will be analyzed.
3. **Sphere-plane contact**

To model the sphere-plane contact, it was considered the contact between a steel ball with radius $R = 40\text{mm}$ and a steel plane plate, $20\text{mm}$ thick, having the same Young modulus $E_1 = E_2 = 2.1 \cdot 10^{11} \text{Pa}$ and Poisson coefficient, $\nu = 0.3$, pressed by a force $Q = 100\text{N}$, as seen in Figure 5. Due to geometrical and loading symmetry, the points from any axial plane will remain in the same plane after the parts deform. Considering these, the analysis was performed on a narrow slice containing the symmetry axis, as seen in Figure 5.b. The same figure also illustrates the meshing with variable parameters, a refined meshing being applied in the contact regions.

![Fig. 5a. Sphere–plate contact model](image)

![Fig. 5b. Meshing of the model](image)

As it can be seen from Figure 6, the maximum value of $\sigma_z$ stress, which has to match the admissible stress, explicitly $p_0 = 3.39\text{GPa}$. A precise calculus for maximum contact pressure made using relations from contact mechanics theory, [1], leads to a value $p_0 = 2.68\text{GPa}$ for maximum contact pressure, the numerical result being with 26% greater than the analytical result. The cause of this discrepancy is the fact that the minimum mesh size is $0.2\text{mm}$ while the contact radius found analytically is $a = 1.461\text{mm}$. Therefore, the meshing on the contact region is too rough to characterise the contact pressure profile.

![Fig. 6. Detail for finding maximum of $\sigma_z$ stress](image)
4. **The contact sphere-intermediate part-plate**

In this case, the meshing was also completed with a variable element size, the parameters of the netting being reduced in the regions with high stresses.

![Fig. 7. Meshing of the assembly](image1)

![Fig. 8. Von Mises equivalent stress](image2)

4.1. **Contact on spherical surface**

As it can be seen from Figure 7, on the contact region from the spherical surface there are not stress concentrators, even though the intermediate element material is highly stressed at its centre. Figure 9 presents the $\sigma_z$ stresses in this element and the von Mises equivalent stress, too.

![Fig. 9. Stresses in intermediate element: a) $\sigma_z$ stress; b) Von Mises stress](image3)

4.2. **Contact on plane surface**

The stresses in the plate are presented in Figure 10 and a stress concentrator is highlighted, occurring in the region where the contact between the plane face of the plate and the passing region from plane to filleted region of the intermediate part.
On the contact surface, the maximum of $\sigma_z$ stress is the maximum contact pressure and it takes the value 4.49 MPa. In order to estimate this numerical result, the analytical relations given by Shtaerman, [5], are applied, which refer to the calculus of contact pressure between an axially symmetrical indenter, with the profile given by a generator curve with two regions, more precise, a straight one connected to a parabolic one, as shown in Figure 11. In order to apply the relations given by Shtaerman, in the vicinity of the point of transition from straight line to parabolic profile, the parabola is replaced by its osculating circle, [6].

Allowing for the contact stresses have a strictly local effect, it is accepted the hypothesis that in the vicinity of change point from linear profile to circular profile, the circle has the same curvature radius as the parabola and the Shtaerman equations can be applied to find the contact stresses under the indenter. The problem occurring is to find the equation of the parabola having as osculating circle the same circle as the filleting circle for the profile of intermediate part. It must be found the parabola with vertical axis, with the tip in the connecting point of linear profile and having at tip the same curvature radius as the filleting radius of the intermediate part profile. The solution for this problem is presented in Figure 11.b, where there are shown together the actual profile of the intermediary part and the necessary profile used in applying Shtaerman method, and also, the osculating circle, common to both profiles. The pressure profile and its three-dimensional aspects are presented in Figure 12.
One can notice that the pressure under the indenter for Shtaerman’s problem has the minimum value in the centre of the indenter, $p_{min} = 1.7\text{MPa}$, a value different compared to the numerically found pressure, of $2.33\text{Pa}$ and the maximum pressure has the value $p_{max} = p(r_0) = 3.78\text{MPa}$, compared to the numerically found $4.49\text{MPa}$. One can conclude that the numerically found pressures are higher than the analytically found ones, but, we must remember that simplifying assumptions were made for the analytical calculus and, though, the results have the same magnitude order.

5. Conclusions

The paper presents the stresses from the wobble plate mechanism, numerically found using the finite element analysis performed in CATIA, and emphasising the most stresses regions of the mechanism. The mechanism was considered for two design solutions: first, considering the Hertzian contact between the plate and the plunger and for the second case, an equivalent variant presenting a newly introduced element aiming to reduce the contact stresses. For both cases the contact pressure was evaluated using analytical methods in order to estimate the accuracy of numerical method. For the first mechanism, the maximum Hertzian pressure was found and for the second one, it was estimated the contact pressure between the intermediate part and the swash plate. For both cases, the numerical results exceeded the analytical values. For the analytical calculus, simplifying assumptions were made and yet, the results are validated by the numerical ones. The main conclusion traced is that the numerical methods applied for stress and strain state evaluation are extremely efficient but the results must be cautiously analyzed. In the zones presenting high stress gradients, leading to stress concentrators, it is recommended to proceed in order to diminish these. The most common operation is increasing the curvature radii of the elements in the closed neighborhood zones where the elements are contacting.

References
METHODS FOR PROMOTING INNOVATION IN THE UNIVERSITY INDUSTRIAL DESIGN STUDIO

Lecturer dr.eng. Florin CIOFU, “Constantin Brâncuşi” University of Targu Jiu

Abstract: Flexible and original thinking is the key to outstanding design. In order to teach design effectively we must therefore offer students a framework that isn’t restrictive and allows them room to develop their own individual design process. This doesn’t mean that we can’t offer our students technical advice or suggestions that might influence their process. Creativity blossoms and a student’s creative process is discovered when the individual is immersed in an environment conducive to innovative thinking. Promoting mentally active drawing, encouraging students to identify user problems, perform in-depth research, and make discoveries through hands-on model-making are some of the areas elaborated upon in this paper.

Keywords: design program, design project, ownership.

1. Introduction

Most students that enter our industrial design program have a mainstream understanding of the education process. Much if not all their educational life they have been given material to learn and then asked to study and show what they have learned in the form of a test. Problem solving, individual understanding, and original thinking are not emphasized in most high school curriculums. In order for design students to succeed, we must emphasize and promote innovative and creative thinking. This is the most valuable skill a student will carry with them for the remainder of his or her design career.

2. Identifying design problems.

A good starting point for a project is to give the class a topic or product category and ask them to identify user problems that can be associated with it. The topic should not be too broad. A degree of class commonality is a good thing, in order to give students enough room for exploration.

An example of a topic that has been effective is the product category “hand tools.” Students are asked to investigate tool use through user observation and one-on-one user inquiry. They are expected to each come to the next design class ready to present three user problems associated with the assigned topic. Amazingly enough, students don’t have difficulty identifying problems. There are enough problems to go around. Problems that are inspired from first-hand experience should be encouraged.

Students should be asked to think back over their past experiences with the idea of finding a problem they can identify with, thereby imbuing the problem with meaning. For the student this activity starts the process of project ownership. Ownership is a very important aspect of a successful design project. It can be the fuel that enables a project to go forward or get through a difficult stage. Starting with identifying a problem as a jumping-off point has proven to work well over the years. It has the advantage of getting the students to take initiative and feel ownership, and allows them to find issues that can be used for further brainstorming and research.

3. The Sketchbook Requirement

Our students are required to make active use of a sketchbook to record visually all information relevant to their current design experience. We let them know how important drawing is to a designer. We tell them that designers have an ongoing internal monologue of images, ideas, and solutions to problems.
We explain that drawing helps define the thoughts that otherwise might never be realized—that without drawing, design communication suffers, not only between designers but also within oneself. We ask our students to use their sketchbook on a daily basis, and let the pages become a visual diary.

Students enter five types of information:
1. Information, images, and ideas about their current project.
2. The drawing problems that we assign.
3. Design insights. These are inspired solutions to design problems. Sketches, images, and descriptive words.
4. Design problems that have been noticed either through one’s own experience or by observing others. Sketches, images and descriptive words.
5. Thoughts or images that are important to the designer for almost any reason should find their way into the sketchbook.

We are very open as to what and how images and information are entered into the sketchbook. Pen, pencil, paint, collage, poetry, descriptions, and diagrams are all very acceptable. We explain to our students that the sketchbook should become part of their body, and they should get used to having it with them at all times. The minimum requirement is two full pages a week.

4. Mentally Active Drawing

It is very important for a designer to be able to convey his or her thoughts and ideas in a drawing. In fact, there are not a lot of reasons for designers to draw if it’s not to communicate or resolve thoughts. It is possible to have good observational drawing ability without the ability to communicate thoughts and ideas well. Over the years of observing students we have found that the most informed drawings are those drawn to clarify thoughts, issues, or problems.

When a project is drawn for the purposes of illustration or appearance, rather than to resolve a problem, it will no longer convey active ideas. We differentiate between these two types of drawing by calling the problem-solving drawing “active” and the other, where the drawing becomes an illustration, “passive.” An active drawing occurs when we are either trying to explain something, understand something, or resolve a problem either with ourselves or with others. A passive drawing is a drawing in which problems and resolving design issues are not being considered. Instead it is a drawing that shows us what the project looks like. Clearly, these two types of drawings serve two very different purposes.
Active drawing, as we have defined it, is an extremely important design tool. It is the method that designers use to resolve problems, an important communication skill that is often the catalyst for further design solutions. We’ve all witnessed wonderfully informed, thoughtful drawings that are drawn on napkins or scraps of paper, because there was an urgency to get an idea across either to oneself or to others.

5. Brainstorming

An exercise at the very beginning of a design project is to break up into small groups and brainstorm. There is a chapter that explicitly discusses successful brainstorming sessions. It gives our students a first-hand experience at what a good brainstorming session is like. By the time they have their first brainstorming assignment they have a pretty good idea of what brainstorming is about.

Three of the most important rules are: one, do not judge other students’ ideas; two, a large quantity (maybe as many as 100 per hr.) of diverse, seemingly disconnected (anything goes, no matter how crazy) ideas are preferable to a few well grounded thoughts; and three, build on other peoples ideas.

These rules create an environment that is excellent for active drawing. We supply the class with boards to tack ideas onto, sticky notes, colored markers, and plenty of paper, and tell them to break up into groups and get to work.

The board they create from their brainstorming session becomes a collection of ideas that might be interconnected or hardly connected, depending on the dynamics of the group.

It is a visual dump sheet that represents the thought interactions of the group. The next assignment is to take this dump sheet, prioritize the information relative to its importance to their project, organize the information, and then create a board that substantiates a direction or communicates ideas to the rest of the class.

6. Research

Research is another area of design that can’t be overemphasized. We let our students know that good research is an essential ingredient to successful design. We ask our students to keep research binders in which they keep an organized record of their research findings. This documentation is information that backs up their design project.

When they get questioned about why they made a particular decision, their collected research can be of great value. Our students also gain a lot from organizing their research. It is
another way of becoming familiar with information that might ordinarily be ignored.

Another research concept we introduce to our students is the idea that research doesn’t have to follow a linear thought process. The more research the better, and going off on seemingly irrational tangents is one way to make unexpected discoveries. The following is a simplified list of research pointers we give our students:

<table>
<thead>
<tr>
<th>Research sources:</th>
<th>Research mindset:</th>
<th>Research techniques:</th>
<th>Research areas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Library</td>
<td>• Be organized</td>
<td>• Information</td>
<td>• History</td>
</tr>
<tr>
<td>• Internet</td>
<td>• Set goals</td>
<td>gathering, user, other,</td>
<td>• Function</td>
</tr>
<tr>
<td>• Contact experts</td>
<td>• Write everything</td>
<td>vicarious or actual</td>
<td>• Structure</td>
</tr>
<tr>
<td>• Contact companies</td>
<td>down and/or draw</td>
<td>experience</td>
<td>• Material factors</td>
</tr>
<tr>
<td>• User observation</td>
<td>your ideas</td>
<td>• Brainstorming</td>
<td>• Ergonomic factors</td>
</tr>
<tr>
<td>• User questionnaire</td>
<td>• Consider more than</td>
<td></td>
<td>• Environmental concerns</td>
</tr>
<tr>
<td></td>
<td>the obvious</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Conclusion

Students come to our studio with a virtually empty toolbox. Our responsibility as teachers is to fill up that box with design tools and create an environment where they develop their own personal methods as to how to use these tools. Students need to be given permission to discover and develop design methods for themselves. They need to be encouraged to play directly with materials without the worry of making mistakes. Fortunately there is no perfect formula for good design. The solution is to create or design an environment that is rigorous, open, and nurturing, that gives students adequate facilities, materials, and tools and, lastly, hands students the freedom to develop their own personal approach to design. It is in this type of environment where design innovation flourishes.

References

TRANSITION IN THE FRETTING PHENOMENON BASED ON THE VARIABILE COEFFICIENT OF FRICTION

Prof.dr.eng. Stefan GHIMİŞI, University Constantin Brancuşi of Targu Jiu

Abstract: Fretting is now fully identified as a small amplitude oscillatory motion which induces a harmonic tangential force between two surfaces in contact. It is related to three main loadings, i.e. fretting-wear, fretting-fatigue and fretting corrosion. Fretting regimes were first mapped by Vingsbo. In a similar way, three fretting regimes will be considered: stick regime, slip regime and mixed regime. The mixed regime was made up of initial gross slip followed by partial slip condition after a few hundred cycles. Obviously the partial slip transition develops the highest stress levels which can induce fatigue crack nucleation depending on the fatigue properties of the two contacting first bodies. Therefore prediction of the frontier between partial slip and gross slip is required.

Keywords: Fretting, transition, variable friction coefficient.

1. Introduction
Fretting is now fully identified as a small amplitude oscillatory motion which induces a harmonic tangential force between two surfaces in contact. It is related to three main loadings, i.e. fretting-wear, fretting-fatigue and fretting corrosion.

More recently fretting has been discussed using the third-body concept and using the means of the velocity accommodation mechanisms introduced by Godet et al.[1,2].

Fretting regimes were first mapped by Vingsbo [3]. In a similar way, three fretting regimes will be considered: stick regime, slip regime and mixed regime. The mixed regime was made up of initial gross slip followed by partial slip condition after a few hundred cycles. Obviously the partial slip transition develops the highest stress levels which can induce fatigue crack nucleation depending on the fatigue properties of the two contacting first bodies. Therefore prediction of the frontier between partial slip and gross slip is required.

This paper proposes several criteria to determine the transition between partial slip and gross slip. A theoretical expression of the transition depending on the applied normal force and the tangential displacement will be introduced in order to plot fretting maps. All the relations exposed in the present paper obey the restrictive conditions exposed by Mindlin[4]. A ball on flat contact will be considered with a constant normal force P and a varying tangential force Q. All the relations were written using Johnson’s notation [5].

2. Transition criteria
From this rapid description of the sliding behaviour, several criteria have been introduced that allow for a quantitative determination of the transition between a partial and gross slip behaviour for alternated loadings.

2.1. The energy ratio
The energy ratio A between dissipated energy Wd and the total energy Wt was introduced to normalise the energy evolution as a function of the loading conditions.

In this case we analysed the transition criteria for the case of one variable friction coefficient between surfaces.

In this case the energy ratio is calculated with the relation:

$$A_{\alpha\beta}(r_0, \beta, k_{\alpha\beta}, r_0, k_{\alpha\beta}, \alpha) = \frac{\Delta E_{\alpha\beta}(r_0, \beta, k_{\alpha\beta}, r_0, k_{\alpha\beta}, \alpha)}{4k_{\alpha\beta}\delta_{\beta}(r_0, \beta, k_{\alpha\beta}, r_0, k_{\alpha\beta}, \alpha)}$$  (1)
The condition of the partial sliding can be written: 

$$A_{adcr}(A_{adcr})$$

Where: $$A_{adcr}$$ represent the critical energy ratio in the case of one variable friction coefficient, being dependent by the contact conditions and by the material characteristics.

$$A_{adcr}$$ - has values corresponding with the parameters who satisfied the existence conditions previous specified ($$C_e = 0$$) and ratio $$\frac{r_{ad}}{\sqrt[3]{C_e}}$$

Thus, solving the equation (2) results the first existence solutions for the fretting contact:

$$C_e(\tau_0, \beta, k_{ad}, k_{ass}) = 1 - \frac{k_{ass}}{\beta k_{ad}^{3/2} (\tau_0, \beta, k_{ad})} = 0$$

(2)

The solutions presented in the Fig.1 and Fig.2 depend on contact conditions and materials characteristics.

$$\text{Fig.1. Solutions of the existence condition, } k_{ass}(\tau_0, \beta, k_{ad})$$

$$\text{Fig.2. Solutions of the existence condition, } k_{ass}(\tau_0, \beta, k_{ad})$$

The second condition can be written:

$$\frac{r_{ad}}{\sqrt[3]{C_e}}(1)$$

$$\frac{r_{ad}}{\sqrt[3]{C_e}} - 1 = \frac{r_a - \sqrt[3]{C_e}}{\sqrt[3]{C_e}} = C_{cr}(0)$$

(3)

Solving the equation:

$$C_{cr} = r_a - \sqrt[3]{C_e}$$

(4)

results the maximum radius of the adhesion circle.

In fig.3 and fig.4 we represented the dependence of the equation solutions (4) by the materials properties materiaelor ($$\beta$$ și $$\tau_a$$).

The graphic representation of the energy ratio in the case of variable friction coefficient is in fig.5.
The critical values for $A_{ad}$ is determined by graphic method or numerical calculation for the existence conditions previous specified.

3.2. The sliding ratio

The evolution of the aperture of fretting cycles was also used as a transition criterion. The sliding ratio was defined as:

$$D = \frac{\delta_0}{\delta}$$

In this case we analysed the transition criterions for the case of one variable friction coefficient between surfaces.

If we consider the friction coefficient being variable, the sliding ratio will be:

$$D_{ad}(\tau_0, \beta, k_{ad}, r_0, k_{as}, \alpha) = \frac{\delta_{00}(\tau_0, \beta, k_{ad}, r_0, k_{as}, \alpha)}{\delta_{as}(\tau_0, \beta, k_{ad}, r_0, k_{as}, \alpha)}$$

The critical values of the sliding ratio is determined by graphic method or by numerical calculus.
In fig.6 we represented the dependence of the sliding ratio $D_{ad}(\tau_0, \beta, k_{ad}, r_a, k_{as}, \alpha)$ by the materials characteristics and by the loadings of the contact, respectively.

4. Conclusion

The analytical development for both the energy and the sliding ratio permitted better identification of the fretting conditions and an expression of the theoretical expressions for the boundary between fretting partial and gross slip conditions. The calculations were complemented by experiments for two different tribosystems which present different coefficients of friction. Comparison between the theoretical and the experimental computations appears to be possible if the tangential compliance of the system is taken into account.

References