KINEMATICS OF A MECHANISM WITH STOPPINGS, BASED ON A HYPOCYCLOID WITH 6 BRANCHES

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Abstract. They are studied the positions, velocities and accelerations of a mechanism based on planetary gear and a RRP dyad. The mechanism ensures the plunger stop on a certain subinterval of the cycle, although the leading element is rotating. Because of the approximation of the hypocycloid shortened arc with six branches, by a circle arc, errors occur.

Keywords: planetary mechanism, stoppings, hypocycloid

1. Introduction
In [1] is given a mechanism used to press for plastics, consisting of a planetary gear and an RRP type dyad. Points on the satellite wheel generate hypocycloids with 5 branches. Such a shortened hypocycloid is approximated with an arc, thus ensuring the temporarily stop of the final leading element, even though the port satellite arm rotates further. Below are studied the cinematic possibilities of such a mechanism, using a shortened hypocycloid, with 6 branches.

2. Positions mechanism
In fig. 1 is given the cinematic scheme of the planetary mechanism, and in fig. 2 is showed the equivalent mechanism, including also a RRP type dyad. The mechanism has a leading element with rotating movement 1, and a second leading element but with flat moving, 2 (his movement is dependent on the movement of 1) and EF dyad.

From fig. 1 it results: e=a-b, c<b.
Based on fig. 2 are written the following relations:

\[ x_B = e \cdot \cos \varphi \quad ; \quad y_B = e \cdot \sin \varphi \quad (1) \]
\[ x_E = x_B + c \cdot \cos \alpha \; ; \; y_E = y_B + c \cdot \sin \alpha \] 
\[ x_F = x_E + d \cos \beta = 0 \; ; \; y_F = y_E + d \sin \beta = S_4 \]

Provided that the circle of the wheel 2 rolls without slipping on the running circle of wheel 3, it results:
\[ \alpha = 2\pi - (\psi - \phi) \] 
\[ a\phi = b\psi \]

where \( \phi \) is the angle of rotation of the conveyor, and \( \psi \) is the arc described on base, equal to the one from the conveyor.

Were established the following dimensions: \( b = 25 \) mm, \( c = 20 \) mm, \( a = 150 \) mm, \( e = 125 \) mm, speed of port-satellite arm 60 rpm.

In fig. 3 is shown the generated hypocycloid. EF rod length can be mathematically determined based on the coordinates of three points on the hypocycloid branch, approximated by an arc. This solution, however, does not allow that the port-satellite arm perform a complete rotation, when ABE is in extension on x axis. Because of this, additional condition should be fulfilled:
\[ d > x_{E_{\text{max}}} \]

Based on the diagram from fig.4 it results:
\[ d = 150 \text{ mm.} \]

This new approximation causes larger errors at the end of the race.

\[ \text{Fig. 3} \] 
\[ \text{Fig. 4} \]

From fig. 5, where are presented successive positions of 1 and 2 elements, it appears that item 2 is positioned at different angles relative to 1, as the 2 wheel turns more rotations at one rotation of the port-satellite arm.
In fig. 6 are showed successive positions of the EF rod, observing variable distances of the piston in relation with x axis, which shows great variations in its speed. Fig. 7 shows successive positions of the entire mechanism, for the entire cycle, and in fig. 8 the positions only for useful branch of the hypocycloid.

From fig.9 it can be observed the generating of the useful hypocycloid branch.
Law of motion of the piston 4 can be traced in fig. 10. Large oscillations are found at $\varphi > 180$ degrees, and also small variations in the range $\varphi = 60 \ldots 120$ degrees.
Fig. 11 states only the useful area of the chart; deviations appear large due to selected scale, but they do not exceed 2.9 mm.

3. Speeds

They are derived, in relation with time, the relations from positions, resulting:

\[ \dot{x}_B = -e \sin \phi \cdot \dot{\phi} ; \quad \dot{y}_B = e \cos \phi \cdot \dot{\phi} \quad (6) \]

\[ \dot{x}_E = \dot{x}_B - c \sin \alpha \cdot \dot{\alpha} ; \quad \dot{y}_E = \dot{y}_B + c \cos \alpha \cdot \dot{\alpha} \quad (7) \]

\[ \dot{\alpha} = \dot{\phi} - \dot{\psi} ; \quad \phi = b \psi \quad (8) \]

\[ \dot{x}_F = \dot{x}_E - d \sin \beta \cdot \dot{\beta} = 0 ; \quad \dot{y}_F = \dot{y}_E + d \cos \beta \cdot \dot{\beta} = \dot{S}_4 \quad (9) \]

In fig. 12 are given the angular speeds in rad / s, of the port-satellite arm (Fi '), of wheel 2 (Alpha'), and of rod 3 (Beta'). It appears that only the angular velocity of the rod varies, with some symmetries.
Piston velocity variation is shown in fig. 13. They are slight variations in the area of interest, at $\phi = 60 \ldots 120$ degrees; in fig. 14 are observed in more detail the speed variation in this subinterval, ascertaining the values between ±0.1 m/s.

![Fig. 13](image1.png) ![Fig. 14](image2.png)

4. Accelerations

By derivation of speeds from relations, with respect to time, are obtained the following relations:

\[
\begin{align*}
\ddot{x}_B &= -e \cos \phi \cdot \dot{\phi}^2 - e \sin \phi \cdot \ddot{\phi} & (10) \\
\ddot{y}_B &= -e \sin \phi \cdot \dot{\phi}^2 + e \cos \phi \cdot \ddot{\phi} & (11) \\
\ddot{x}_E &= \ddot{x}_B - c \cos \alpha \cdot \dot{\alpha}^2 - c \sin \alpha \cdot \ddot{\alpha} & (12) \\
\ddot{y}_E &= \ddot{y}_B - c \sin \alpha \cdot \dot{\alpha}^2 + c \cos \alpha \cdot \ddot{\alpha} & (13) \\
\ddot{x}_F &= \ddot{x}_E - d \cos \beta \cdot \dot{\beta}^2 - d \sin \beta \cdot \ddot{\beta} = 0 & (14) \\
\ddot{y}_F &= \ddot{y}_E - d \sin \beta \cdot \dot{\beta}^2 + d \cos \beta \cdot \ddot{\beta} = \ddot{S}_4 & (15)
\end{align*}
\]

In fig. 15 is shown the variation of angular acceleration of the rod; jumps occur at the ends of hypocycloid branches.

Piston acceleration also has jumps, caused by hypocycloid branches, but in the field of interest variation is little, in the chart being an approximately linear portion.
5. Conclusions

The studied mechanism generates a hypocycloid with 6 branches, the useful branch being the one symmetrical to y axis, with values of $y > 0$.

The mechanism positions confirm that it provides a stationing of the final leading element, but with some errors caused by approximating the hypocycloid arc with a circle arc.

Piston velocities and accelerations have convenient values in the area of interest.

References