# STUDY OF STATIC AND DYNAMIC STABILITY OF THIN-WALLED BARS EXCITED BY PERIODICAL AXIAL EXTERNAL FORCES.

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***Abstract:*** *In these paper, starting from the relations for the displacements and spinning the transversal section of a bar with thin walls of sections opened expressed by the corresponding influence functions and introducing the components of the exterior forces distributed and the moments of the exterior forces distributed due to the inertia forces, the exciting axial forces together with the following effect of these and of the reaction forces of the elastic environment for leaning it may reach to the system of the equations of parametric vibrations under the form of three integral equation These equations may serve for the study of vibrations of the bars, to study the static stability and to study the dynamic stability*

**Keywords:** vibrations, transversal section, elasticity.

It is known the fact that in the case of spatial parameters vibrations of the bars with thin walls of opened sections acted by external periodical forces of fix direction, u displacements (z, t), v (z, t) and rotating $\varphi$(z, t) of the transversal section of the bar in the process of vibrations is expressed under the integral form

$$u(z,t) = \int_0^l G_n(z,t)\left[Q_n(\eta,t) + \int \frac{}{\partial\eta}\partial p_z(\eta,t)xds\right]d\eta$$

$$v(z,t) = \int_l^l G_v(z,n)\left[q_y(\eta,t) + \int \frac{\partial p_z(\eta,t)}{\partial\eta}yds\right]d\eta \qquad (1)$$

$$\varphi(z,t) = \int_0^l G_\varphi(z,\eta)\left[m_z(\eta,t) + \int \frac{\partial p_z(\eta,t)}{\partial\eta}\omega ds\right]d\eta$$

where $q_n$ and $q_y$ are transversal distributed charges, $m_z$ the moment of turning distributed and $p_z$ the distributed force per unit of length of bar and per unit of length of the outline in the direction of z axe of the bar, having the expressions

$$q_x = -m\left(\frac{\partial^2 u}{\partial t^2} + y_0\frac{\partial^2\varphi}{\partial t^2}\right) - \frac{\partial}{\partial z}\left\{N(z,t)\left[\frac{\partial u}{\partial z} + (y_0 - e_y)\frac{\partial\varphi}{\partial z}\right]\right\}$$

$$q_y = -m\left(\frac{\partial^2 v}{\partial t^2} - x_0\frac{\partial^2\varphi}{\partial t^2}\right) - \frac{\partial}{\partial z}\left\{N(z,t)\left[\frac{\partial v}{\partial z} - (x_0 - e\ )\frac{\partial\varphi}{\partial z}\right]\right\} \qquad (2)$$

$$m_z = m\left(x_0\frac{\partial^2 v}{\partial t^2} - y_0\frac{\partial^2 u}{\partial t^2} - r_0^2\frac{\partial^2\varphi}{\partial t^2}\right) + \frac{\partial}{\partial z}\left\{N(z,t)\left[(x_0 - e_n)\frac{\partial v}{\partial z} - (y_0 - e_y)\frac{\partial u}{\partial z} - (e_y\beta_1 + e_x\beta_2 + r_0^2)\frac{\partial\varphi}{\partial z}\right]\right\}$$

$$p_z = -m\left(\frac{\partial^2 w}{\partial t^2} - \frac{\partial^3 v}{\partial z\partial t^2}x - \frac{\partial^3 u}{\partial z\partial t^2}y - \frac{\partial^3\varphi}{\partial z\partial t^2}\omega\right)$$

At the same time, in the relations (1), N (z, t) is the axial force in the bar, function of the section abscissa and of time expressed under the form (3)

$$\mathrm{N(z,t)} = \alpha N_0(z) + \beta N_t(z)\phi(t) \qquad (3)$$

Where $\alpha$ and β are the constant parameters, $N_o$ (z) and $N_t$ (z) known functions of the section abscissa, characterized by the distribution along the axe bar of the static component and respectively of the periodic one following the charge, and $\phi(t)$ is a periodical function of time of T period.

$$\phi(t) = \phi(t+T)$$

In the case of the following charge, upon the bar will action supplementary transversal charges on the account of rotating the forces once with the fibers applied (fig.1).
Admitting, for the generality, that the external charge, respectively the periodical variable component in time of this one is partially following and $\Psi_u$ and $\Psi_v$ are the spinning of the eccentric axe after which is applied the exterior charge in parallel plans with the main plans of inertia of the bar,

$$\psi_n = \frac{\partial u}{\partial z} + (y_0 - e_y)\frac{\partial \varphi}{\partial z}; \; \psi_v = \frac{\partial v}{\partial z} - (x_0 - e_n)\frac{\partial \varphi}{\partial z}$$

the normal components at the bar axe of the following force is written

$$p_x^t(z,t) = -p_t(z,t)\varepsilon\psi_n = \varepsilon\beta\frac{Nt(z)}{dz}\left[\frac{\partial u}{\partial z} + (y_0 - e_y)\frac{\partial \varphi}{\partial z}\right]\phi(t) \qquad (4)$$

$$p_y^t(z,t) = -p_t(z,t)\varepsilon\psi_v = \varepsilon\beta\frac{dN_t(z)}{dz}\left[\frac{\partial v}{\partial z} - (x_0 - e_n)\frac{\partial \varphi}{\partial t}\right]\phi(t)$$

To the distribution forces (4), are corresponded also an exterior moment distributed towards the spinning centre.

$$m_z^t = -\varepsilon\beta\frac{dN_t(z)}{dz}\left\{(x_o - e_x)\frac{\partial v}{\partial z} - (y_0 - e_y)\frac{\partial u}{\partial t} - \left[(y_0 - e_y)^2 + (x_0 - e_x)^2\right]\frac{\partial \varphi}{\partial z}\right\}\phi(t) \qquad (5)$$

It is supposed at the same time that the bar is leaned elastically over the entire length, after a parallel line with the bar's axe, ex centered towards the given axe by the distances $h_x$ and $h_y$ between the axes (fig.2), the elastic constants of the environment, defining the elasticity of elastic leaning at displacements and spinnings, being $k_x$, $k_y$ and $k_y$. In this case, the displacements of the axe after which are distributed the reactions are.

$$u_r = u + (y_0 - hy)\varphi; \; v_r = v - (x_0 - h_x)\varphi$$

$$p_{r,n} = -k_x\left[u + (y_0 - h_y)\varphi\right]$$

$$p_{r,y} = -k_y\left[v - (x_0 - h_x)\varphi\right] \qquad (6)$$

$$m_{r,z} = -k_{\varphi} \cdot \varphi$$

The reactions $p_{r,x}$ and $p_{r,y}$ action eccentrically upon the central axe of bending-spinning, and will action also upon the bar, so that the spinning moment per the unit of bar length of the reactive forces will be

$$m_{r,z} = -k_x\left[u + (y_0 - hy)\varphi\right](y_0 - h_y) + k_y\left[v - (x_0 - h_x)\varphi\right](x_0 - h_x) - k_{\varphi} \cdot \varphi \qquad (7)$$

In the relations (1) where $G_n$ , $G_v$ $G\varphi$ play a role of Green functions, and are developed the displacements u (z, t) and spinnings $\varphi$ (z, t) in series of functions according to the fundamental complete system of functions $Z_k^u(z), Z_k^v(z), Z_k^{\varphi}(z)$, which may satisfy the conditions to the limit under the form

$$u(z,t)= \sum_{k=1}^{\infty} T_k^u(t) Z_k^u(z)$$

$$v(z,t)= \sum_{k=1}^{\infty} T_k^v(t) Z_k^v(z) \qquad (8)$$

$$\varphi(z,t) = \sum_{k=1}^{\infty} T_k^{\varphi} Z_k^{\varphi}(z)$$

amplitude $T_k^n(t), T_k^v(t), T_k^{\varphi}(t)$ being functions of time unknown.

It is taken into account the fact that the symmetrical waves $G_u$ (z, η), $G_v$ (z,η), $G\varphi$ (z,η) may discompose in square series after realizing the own functions under the form

$$G_n(z,\eta) = \sum_{k=1}^{\infty} \frac{1}{\omega_{u,k}^2} Z_k^u(z) Z_k^u(\eta)$$

$$G_v(z,\eta) = \sum_{k=1}^{\infty} \frac{1}{\omega_{v,k}^2} Z_k^v(z) Z_k^v(\eta) \qquad (9)$$

$$G_{\varphi}(z,\eta) = \sum_{k=1}^{\infty} \frac{1}{\omega_{\varphi,k}^2} Z_k^{\varphi}(z) Z_k^{\varphi}(\eta)$$

Replacing in the relations (1) the distributed forces $q_u$ $q_p$ $p_z$ and $m_z$ and performing all the calculations is obtained the system of differential equations

$$C\frac{d^2T}{dt^2} + [E + D - \alpha A - \beta\Phi(t)B]T = 0 \qquad (10)$$

where T is the infinite hyper vector

$$T = \{T_1, T_2, T_3, ..., T_n\} \qquad (11)$$

In the equations (10) E is the infinite unit matrix and the system 10 represents the system of differential equations of second order of the linear homogenous and have variable coefficients, of the parametric special vibrations of the bars with thin walls of the opened sections, leaned on the elastic environment and following excited by periodical axial external forces. These equations may serve for the study of vibrations of the bars, to study the static stability and to study the dynamic stability.

**References**


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