# SOME ASPECTS REGARDING COMPUTER AIDED DESIGN OF CAM MECHANISMS WITH FLAT FACE OSCILLATING FOLLOWER

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***Abstract:*** *The paper presents some aspects concerning the design of cam mechanisms with oscillating flat faced follower, using computer softwares. There are deduced the analytical equations for the profile of the cam, thus enabling the study of mechanism's reliability. There are emphasised the effects due to an inadequate choice of law of motion, mainly shocks during mechanism operation.*

**Keywords**: cam mechanism, oscillating flat face follower, FEM analysis

## 1. Introduction

Cam mechanisms are machines widely used in technical applications due to reduced constructive complexity, [1], [2], consisting of only two mobile elements, the cam as mechanically driver element and the follower as driven element. The main advantage of these mechanisms is that, by an appropriate cam design, the driven element can carry out any imposed motion. The major disadvantage of these mechanisms consists in the presence of the cam-follower higher pair. This kinematical joint is characterised by occurrence of contact pressures that subjects the materials to important stresses in the immediate vicinity of the contact. In addition, the vast majority of mechanisms have a periodic motion, conducting to periodic loading for both cam and follower and therefore to fatigue phenomenon.

In the case of mechanisms with flat face follower, the cam-follower interaction presents also a tangential component. This component is directly influenced by the contact pressure from the higher pair and can produce adverse effects such as wear of elements and from this, cam profile modification with direct consequence, the alteration of the law of motion. A key parameter for these mechanisms is the minimum radius of curvature allowed in the cam. This parameter presents a direct influence upon both contact pressure and friction force from the higher pair. The value of the minimum radius of curvature is directly ordered by the follower's law of motion and by the main geometrical parameters. The main geometrical parameters are constant characteristics that together with the law of motion control the cam's size and shape. For the cam mechanism with ***translating flat***-faced ***follower*** the relation is straightforward and is given in all monographies on theory of mechanisms, [3] and for the knife-edge follower, the authors found these relations, [4], [5].

For the mechanisms with oscillating flat faced follower and revolving cam, this approach is not possible as a very complicated expression results and therefore, it is more convenient to find the minimum radius of curvature after setting the cam profile.

## 2. Tracing the profile of the mechanism with rotating cam and oscillating flat-faced follower

The mechanism is presented in Figure 1 and the geometrical and kinematical parameters of the mechanism are shown in Figure 2. For the lowest position, the follower is tangent to the circle with minimum radius of curvature, in the point $T_0$. The main geometrical parameters of the mechanism are:

– the distance $d$ between the two joints, of the cam and of the follower;

– the angle $OA_0T_0 = \Psi_0$, produced by the follower direction in the lowest position and the axis of the joints.

The cam rotates with a constant angular velocity, $\omega$. At a certain moment, the cam's position is characterised by the angle of rotation, $\varphi$ while the position of the follower is given by the angle $\psi = \psi(\varphi)$.

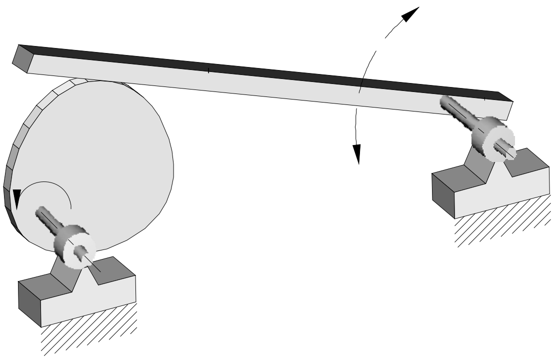

*Fig.1. Mechanism with rotating cam and oscillating flat-faced follower*

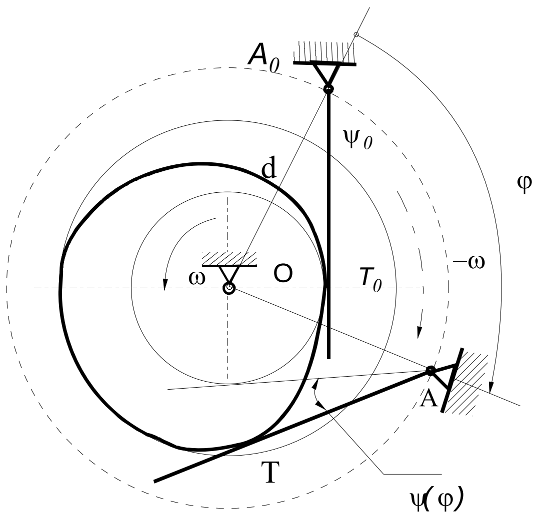


*Fig.2. Geometrical and kinematical parameters of the mechanism*

For a kinematical study of the motion, it is more convenient to perceive the motion of the entire mechanism to the plane of the cam. Consequently, the cam becomes fixed and the follower, besides its own motion, gets a rotation transport motion around the centre of the cam with the angular speed ($-\omega$).

The same method is used in finding the profile of the cam. The follower is supposed to perform two motions: rotation around the centre of the cam and relative oscillation against the frame. The cam's profile results as an envelope of the successive positions of the follower. From Figure 3 it can be noticed that for the same geometrical parameters and law of motion, two cam profiles can be obtained. The two positions corresponding to the follower, $\Delta_1$ and $\Delta_2$, can be regarded as two variable curves, having the position depending on the parameter $\varphi$. In the cam's co-ordinate system, each of these takes the form of general equation:

$$f(x, y, \varphi) = 0 \tag{1}$$

According to Ionescu, [6], in order to obtain the envelope of the curve family (1), the parameter $\varphi$ should be eliminated from the system:

$$\begin{cases} f(x,y,\varphi)=0 \\ \partial f(x,y,\varphi)/\partial\varphi=0 \end{cases}. \tag{2}$$

With the above system solved concerning variables $x$ and $y$, these would represent factual the parametric co-ordinates of the envelope. The equations of the two straight lines can be written in a concentrated manner:

$$y+d\sin\varphi-\tan\{(\pi-\varphi)\pm[(\psi_0+\psi(\varphi)]\}=0 \tag{3}$$

where the sign $''+''$ corresponds to the straight line $\Delta_1$ and „-" to $\Delta_2$, respectively.

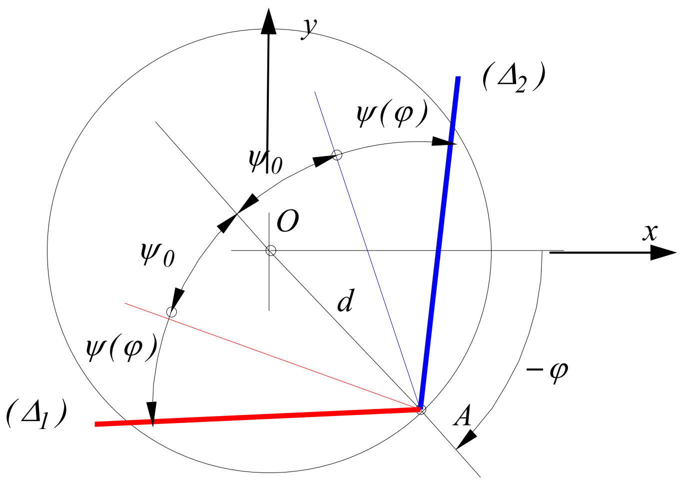


*Fig.3. The two possible cam profile generations*

For the particular case of system (2) for the equation (3), solving the system about $x$ and $y$, the equations of the cam profiles generated by the two straight lines are obtained as:

$$\begin{cases} \dfrac{x_{1,2}(\varphi)}{d}=\cos\varphi-\dfrac{\cos[\psi_0+\psi(\varphi)]\cos[-\varphi\pm(\psi_0+\psi(\varphi))]}{1\mp\dfrac{d\psi(\varphi)}{d\varphi}} \\ \dfrac{y_{1,2}(\varphi)}{d}=-\sin\varphi-\dfrac{\cos[\psi_0+\psi(\varphi)]\sin[-\varphi\pm(\psi_0+\psi(\varphi))]}{1\mp\dfrac{d\psi(\varphi)}{d\varphi}} \end{cases}; \tag{4}$$

Using the MATHCAD software, a programme was written with the relations (3) and (4) to obtain the profiles of the cams as envelopes of the straight lines (3). The result given by the programme is presented in Figures 4 and 5. From Figure 5 it results that the undercut occurs on the cam profile and therefore it cannot be used.

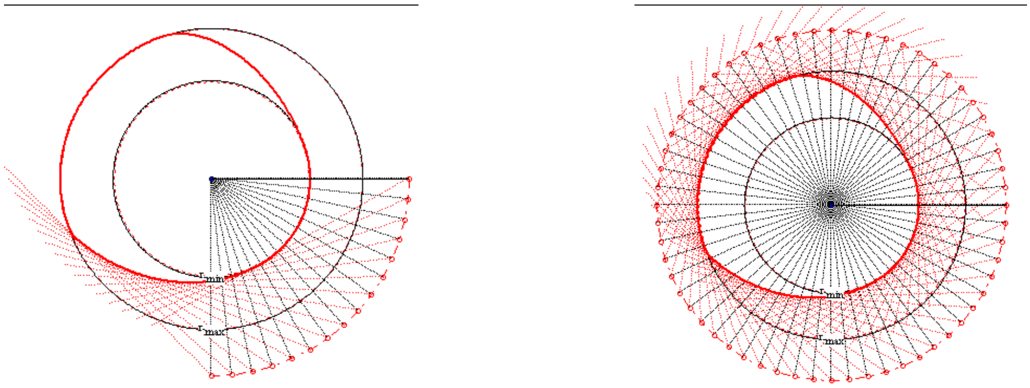

***Fig.4.** Tracing profile of the cam, "1"*

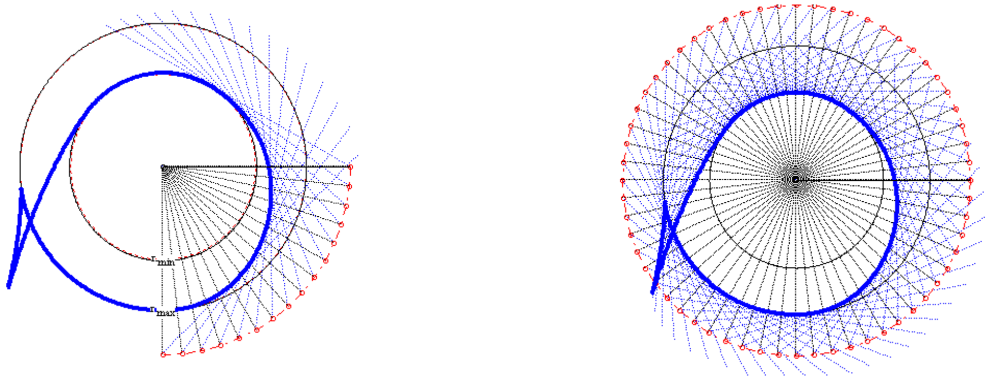

***Fig.5.** Tracing the profile of the cam, "2"*

**3. Finding the radius of curvature**

In order to use the cam in a mechanism with flat-faced follower, the obtained cam profile should look similar to the one obtained in Figure 4. To characterize unequivocal the concavity of the profile of the cam, the definition of concavity reported to the centre of the cam is requested, using as pole the rotation centre of the cam. In a point, the profile is concave if the pole and the curve are situated on the same side of the tangent in the considered point. As the profile of the cam can present straight line regions having the radius of curvature $\rho_c = \infty$, this aspect can be avoided by introducing the curvature, defined as the inverse of the radius of curvature, $k = 1/\rho_c$. In polar co-ordinates the relation for the curvature is given by:

$$k = \frac{r_t \varphi'_t r''_t + r_t^2 \varphi'^3_t + 2r'^2_t \varphi' + r'_t r_t \varphi''_t}{[r_t^2 \varphi'^2_t + r'^2_t]^{3/2}}, \tag{5}$$

where $r_t$ and $\varphi_t$ are the polar co-ordinates of the contact point between the profile of the cam and the follower and the symbols (‘) and (“) represent the first and the second derivative, correspondingly, as regards angle $\varphi$.

The complexity of the relation makes impossible to express the curvature as function of geometrical and kinematical parameters of the mechanism. For this reason, the equations (4) for the cam's profile are used and next, by applying the relation (5), the curvature is found.

As an example, it is considered a mechanism with continuous acceleration, on regions, for ascending phase and with sinusoidal acceleration for descending. The variation of the follower rotation, $\psi(\varphi)$, of reduced angular velocity $\omega_2(\varphi)/\omega_1$, and of reduced angular acceleration $\varepsilon_2(\varphi)/\omega_1^2$ are presented in Figure 6. In Figure 7 there are presented the variations of the curvatures for the two profiles, considered with changed sign.

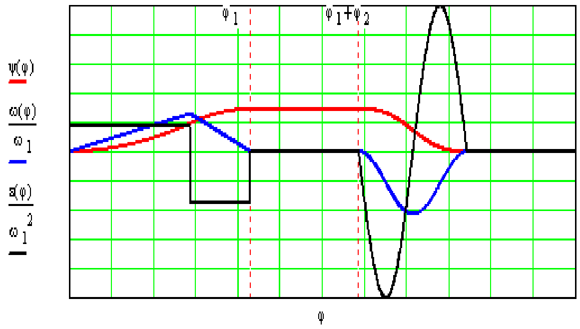

*Fig.6. Kinematical parameters of the follower for a complete cam rotation*

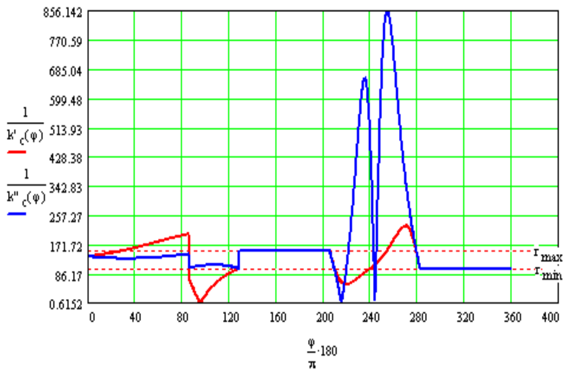


*Fig.7. Variation of radius of curvature for the two profiles*

From Figure 7 it can be observed that for finite discontinuities of the follower's acceleration, the radius of curvature will also present consequent discontinuities. In addition, the minimum radius of the cam represents the minimum radius of curvature. The presence of acceleration discontinuities will induce shocks in mechanism's operation and finite variations of the radius of curvature for the cam. This fact adds more complexity to the calculus for contact strength of the mechanism. In the theory of line Hertz contact, the contact pressure is calculated based on the assumption of continuous variation of the radius of curvature, [7]. Further more, for the deformation calculus, there are not available general analytical relations. The calculus of the deformations is made either via empirical relations or by relations applicable for narrow domains of the variables.

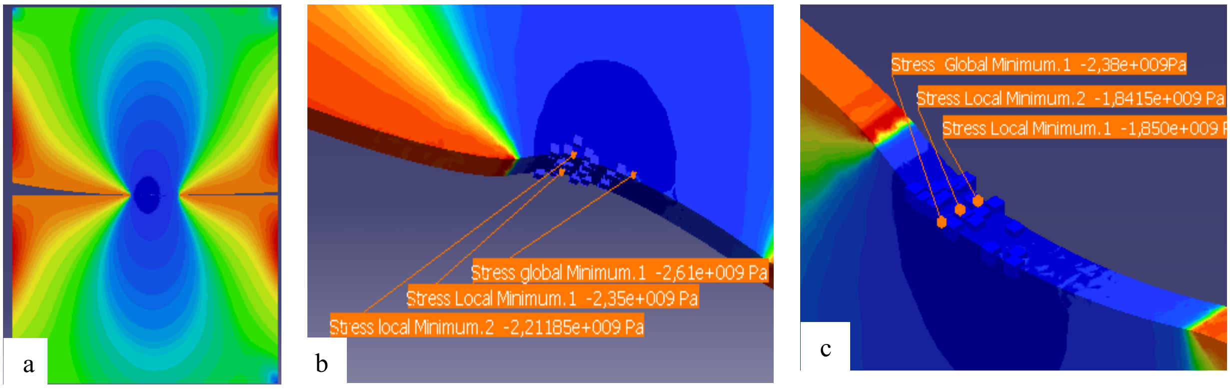


***Fig.8.** Contact normal stress variation from FE analysis*
*a) cam – follower contact; b) extreme stresses in cam; c) extreme stresses in follower*

With the purpose of illustrating the effect of finite variation of the curvature radius of the cam, it was considered the contact between an elastic half-plane and an indenter with radii $R_1 = 50mm$ and $R_2 = 200mm$, loaded by a normal force $Q = 1000N$.

The thickness of the two elements is the same, $0.4mm$.

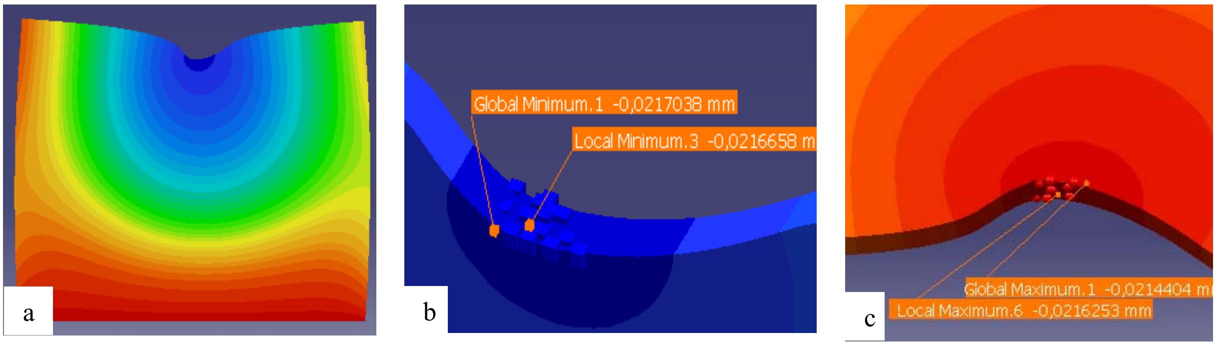


***Fig.9.*** *a) deformed shape of the follower; b) local deformation of follower; c) local deformation of cam*

## 4. Conclusions

The present work presents the methodology of obtaining the profile of the cam from mechanisms with flat-faced oscillating follower. The profile of the cam results as the envelope of the positions of the follower performing relative motion with respect to the cam. The parametric Cartesian equations of the cam are obtained. The relation for calculus of radius of curvature is presented, expressed in polar co-ordinates.

The relation is applied for a given case, when for rising phase, the acceleration presents finite discontinuities. This fact is reflected into geometry as finite discontinuities for the radius of curvature of the cam and by dynamic point of view, into shocks generated during mechanism's operation. The effect of finite variation of the contact radius upon deformations and stresses from the cam-follower contact region represents a special case, analyzed via FEM. The analysis is especially useful for Hertzian line contact since there are no analytical relations for the characterization of contact deformations.

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