A SPECIAL SPIRAL: THE COCHLEOID

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Abstract. *They are shown geometrical considerations and properties of cochleoid, which is a flat curve from the spiral's family. Are established various mathematical relations, which are useful when drawing the curves. They are given examples of different drawn cochleoid.*

Keywords: cochleoid, flat curve, curve tracing

1. Introduction

The cochleoid (in English cochleoid, in Italian cocleoide, in Greek Kochlias) draws its name from the snail "shell". It is a flat curve, similar to the vertical projection of the spatial curve representing the line describing the snail shell (Fig. 1).

Fig. 1*. The snail shells*

The cochleoid was researched by J. Peck in 1700, by Bernoulli in 1726 and by Benthan and Falkenburg in 1884.

The cochleoid is part of the spiral's family.

2. Geometrical aspects

The BD cochleoid from Fig. 2 is described by B point on the AP radius, when P point goes on DP circle of radius AP = a = constant, with AB = ρ, the condition being that the PQ segment is equal to the BC arc, meaning:

$$\rho \varphi = a \sin\varphi \qquad (1)$$

For an arc of r radius and length of EF =a (Fig. 3), which is positioned symmetrically regarding y-axis, it is determined the ordinate of center of gravity G, with the relationship [1]:

$$OG = \frac{r}{a} \bullet E'F' \qquad (2)$$

where E'F 'is the projection of EF arc on the x-axis

Having: a = r. α , it is obtained:

$$OG = \frac{2r}{\alpha}\sin\frac{\alpha}{2} \qquad (3)$$

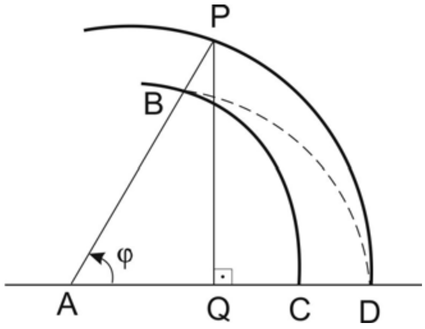


Fig. 2. Geometry of cochleoid

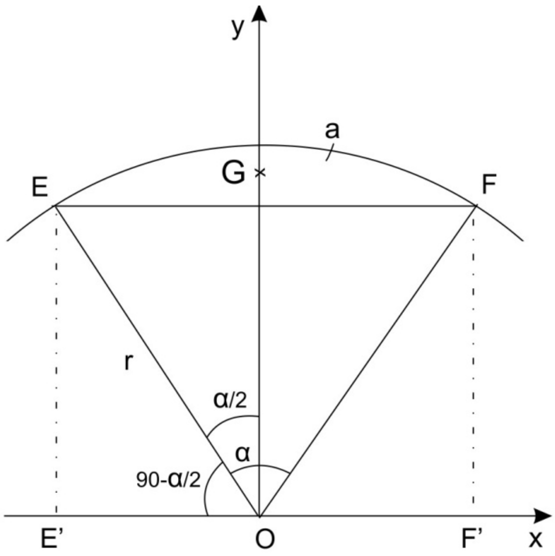


Fig. 3. The gravity center of the arc of a circle

If we consider an arc of constant length, but varying radius of the circle, then the geometrical locus of gravity center of the arcs with the same length but with variable rays is a cochleoid (Fig. 4) [1], with the equations:

- in polar coordinates:

$$r = a.\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \qquad (4)$$

- in cartesian coordinates:

$$(x^2 + y^2)arctg\frac{x}{y} - ay = 0 \cdot \qquad (5)$$

AB_i arc is shown in the figure and the center of gravity is C_i.
In [2] it is shown that (Fig. 5):

- the tangent in P to cochleoid passes through the A' symmetrical to OP, the one for A belonging to the curve, at its intersection with the x-axis;
- the curve points where the tangent is parallel to the x axis is on a circle with A center and radius a = arc OP.

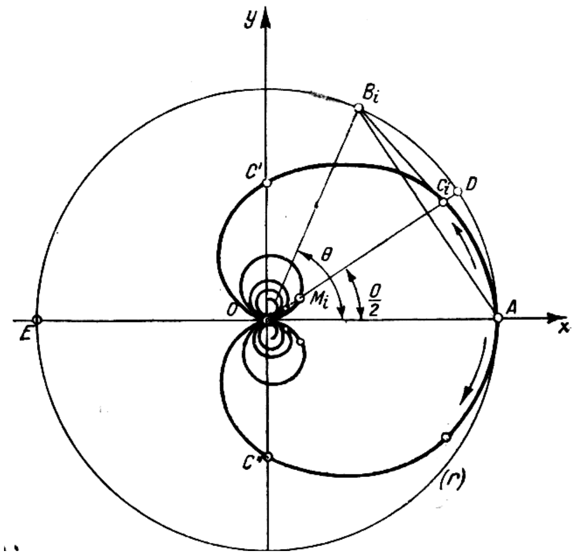


Fig. 4 *.Variable circle radius*

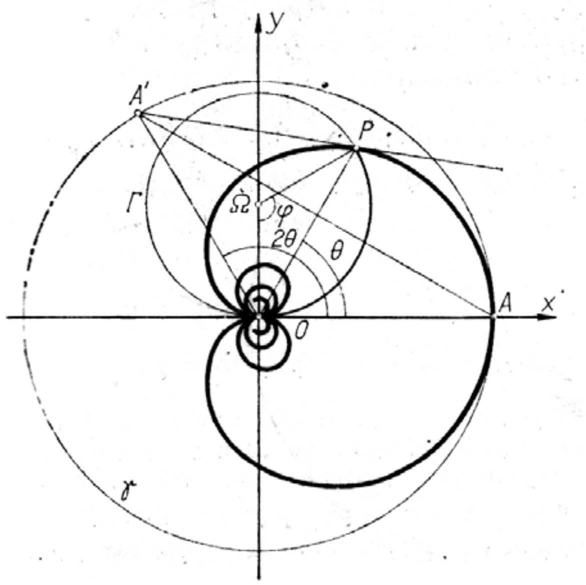


Fig. 5*. Geometric properties*

3. The cochleoid drawing

Fig. 6 shows the possibility of tracing the cochleoid: are plotted circles with OA_i rays and OP_i arcs are taken on this, with the same length. P_i points are on the cochleoid. Each of these points is also on the one ortocycloid which runs the length OM = constant.

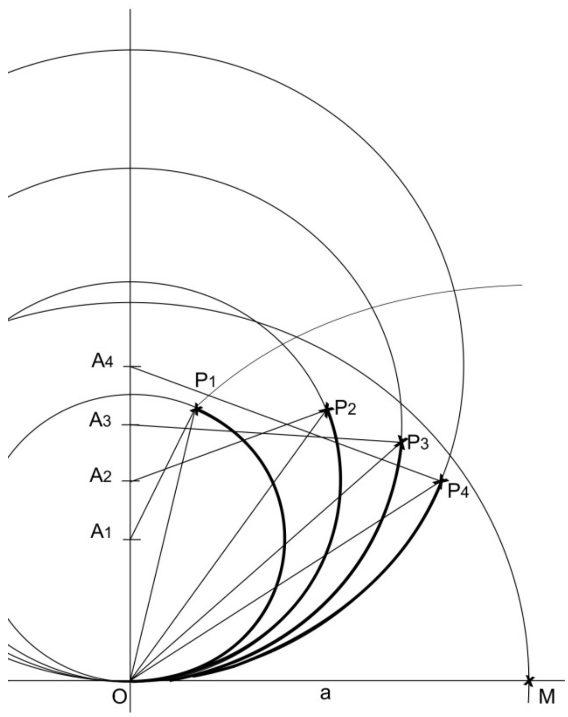


Fig. 6. *Geometrical drawing of cochleoid*

4. The obtained results

Considering the arc OP = a = constant (Fig. 7), it shows that OAP is an isosceles triangle. It results:

$$\text{arc } OP = r.\varphi = a = \text{constant}, \quad (6)$$

$$\varphi = 2.\theta. \quad (7)$$

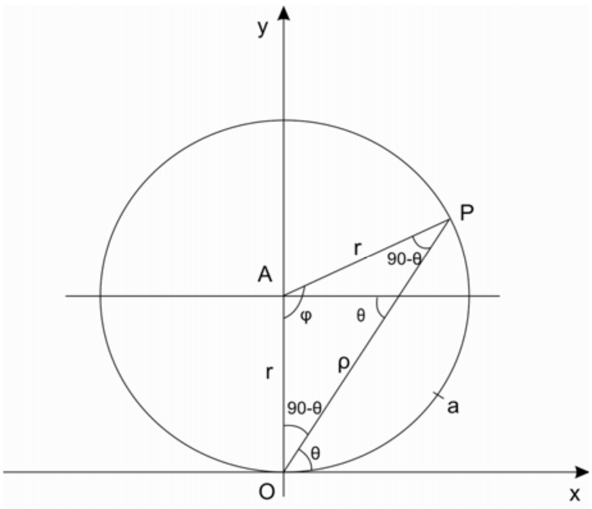


***Fig. 7**. Polar coordinates*

Radius r can be cycled, resulting in polar coordinates of cochleoid.

It is obtain $\varphi = 2.\theta = a / r$, $\theta = a / (2.r)$, $\rho = 2.\ r \cos(90 - \theta) = 2.\ r.\ \sin\theta$.

It was developed a calculation program which drew more cochleoids. Was taken a= 20 mm and r was cycled from 0.01 to 600 mm

In Fig. 8 is shown the obtained cochleoid for these data.

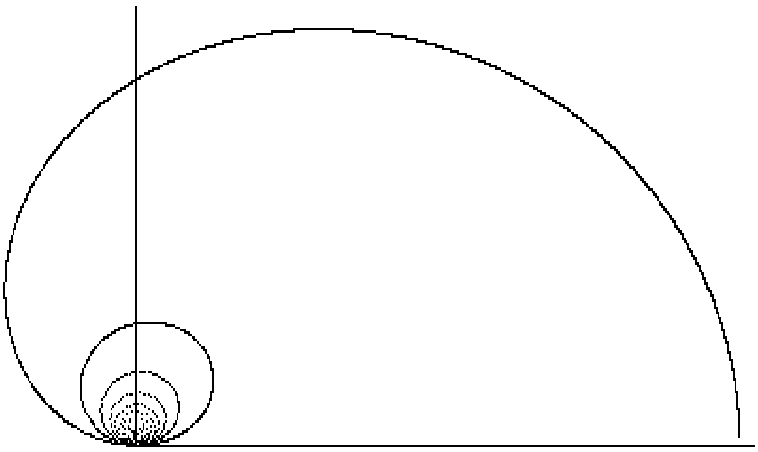
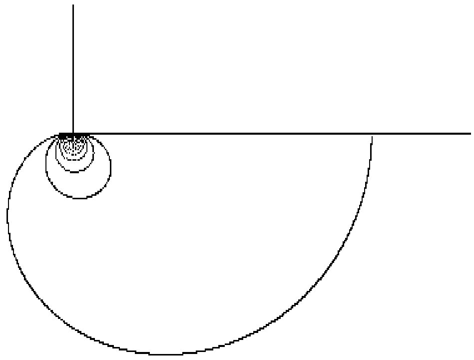


***Fig. 8.** Cochleoid with r> 0*

If the r radius is negative, it is obtained the cochleoid from Fig. 9.

In Fig. 10 is shown the complete cochleoid.



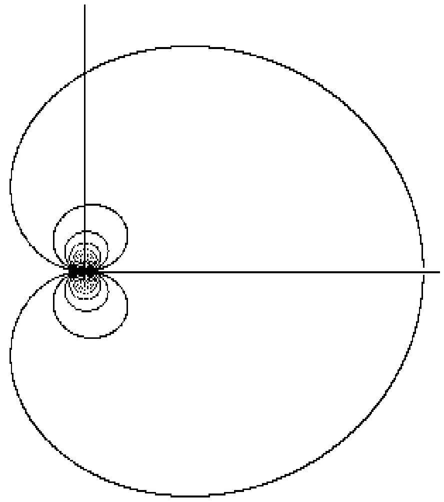
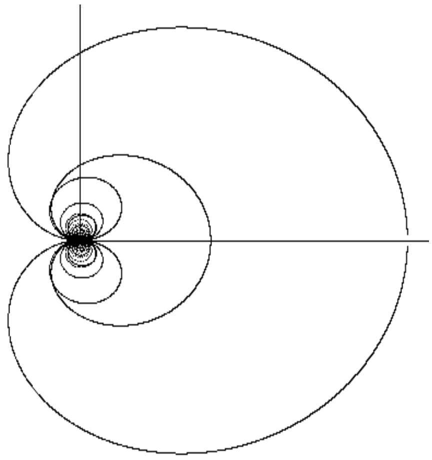


Fig. 9. *Cochleoid with r <0*

Fig. 10*. The complete cochleoid*

If it is changed the arc length (a = 10, a = 25), they are obtained two cochleoids, which are given in Fig. 11 (the inner one corresponding to a= 10). r was cycled between 0.01 and 600, with step of 0.005.

If the ray size is limited, they are obtained incomplete cochleoids (Fig. 10).



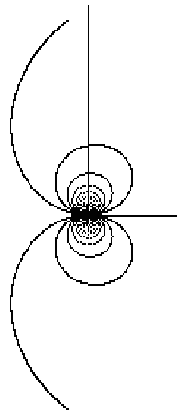


Fig. 11. *Two cochleoids*

Fig. 12. *Incomplete cochleoids*

5. Conclusions

- The cochleoid is a flat curve of spiral's type.
- This curve has interesting geometric properties.
- It has applications in the center of gravity of circle arcs.
- They drew different variants of cochleoids.

References

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