THE MATEMATICAL MODEL OF CLARIFYING PROCESSES FOR INDUSTRIAL WASTEWATERS

Prof. univ. dr. Miodrag **IOVANOV,** “Constantin Brâncuşi” University of Târgu-Jiu

Abstract: *To minimize the SS amounts in the effluent from the settling tanks, SS predictive methods are needed.By applying mathematical models, it is possible to predict the SS concentration from the settling tank; the predictions can be used in order to choose the best control action, that is whether changing the flow path is needed or not whether changing the flow path is needed or not; whether mixing caused by the sludge removal from the tank should be stopped or limited.From the presented model, as well as from the graph of Gauss bell, we notice that the SS concentrations in the effluent from the settling tanks grow from m to m+3T(/x-m/<3T); i.e. the SS concentrations in the effluent from the settling tanks are Gaussian.*

Keywords: mathematical model, clarifying processes, wastewaters

1.Introduction

Two layer model of settling in a sludge settling tank.

The two layer model of sludge settling in a tank is:

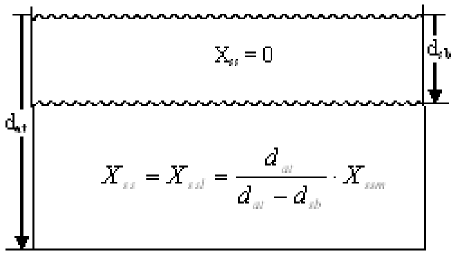

Fig. 1. *The mathematical model of clarifying processes for industrial Wastewaters.*

When the flow is stopped, the suspended solids settle. The water in the layer above the sludge blanket is assumed to be clear water, and the layer under the sludge blanket is assumed to contain all the SS fully mixed (in tank 1 the solution for the (α) – convex differential equation (12) and in tank 2 the solution of the(β) – convex differential equation (13).

The settling velocity is modelled according to veselind [1] according to the differential equation:

$$\frac{dd_{sb}}{dt} = V_0 e^{-n_v X_{sssl}} \qquad (1)$$

where d_{sb} denotes the sludge blanket depth, X_{sssl} is the SS concentration in the sludge layer (fig. 1) and V_0 and n_v are sludge volume index (SVI) dependent parameters. For simplicity, we use the expressions found by Härtel and Pöpel (1992)

$$\begin{cases} V_0 = (17.4e^{-0.0113SVI} + 3.931) \\ n_v = (-0.9834e^{-0.00581SVI} + 1.043) \end{cases} \qquad (2)$$

As the volume of the sludge layer is $\frac{(d_{at} - d_{sb})V_{at}}{d_{at}}$ the average SS concentration in the sludge layer is:

$$X_{sssl} = \frac{d_{at}}{d_{at} - d_{sb}} \cdot X_{ssm} \quad (3)$$

where X_{ssm} is the average SS concentration in the tank.

When the tank is fully mixed, the sludge blanket depth is $d_{sb} = 0$.

The following differential equation:

$$\frac{dd_{sb}}{dt} = -\frac{1}{\tau_{mix}} d_{sb} \quad (4)$$

show that when the flow is continuous, d_{sb} -> 0 ($d_{sb} = C \cdot e^{-\frac{t}{\tau_{mix}}}$) where τ_{mix} is a mixing capacity dependent time constant.

With m_1 and m_2 denoting the mixing signals for settling tank 1 and 2 respectively. The mixing signal is 1 when the corresponding tank 1 is mixed and 0 otherwise.

The signals can then be used to combine the settling equation (1) with the mixing equation (4).

We get for each of the two tanks, two (α) differential equations and (β) convex, respectively for each of the two sludge settling tanks.

$$\frac{ddsb_1}{dt} = l(m_1)(-\frac{1}{\tau_{mix}} dsb_1) + (1 - l(m_1))V_0 e^{-n_v X_{ssl_1}} \quad (5)$$

$$\frac{ddsb_2}{dt} = l(m_2)(-\frac{1}{\tau_{mix}} dsb_2) + (1 - l(m_2))V_0 e^{-n_v X_{ssl_2}} \quad (6)$$

where: dsb_1, dsb_2 = the sludge blanket depths

$Xsssl_1$, $Xsssl_2$ = SS concentrations in the sludge settling tank 1 & 2 respectively.

The S.S. concentrations in the effluent from a sludge settling tank is modeled as a function of the suction depth, d_{suct} and the SS concentration in the sludge layer x_{sssl}:

$$X_{ssoutat} = \begin{cases} \frac{d_{suct} - d_{sb}}{d_{suct}} X_{sssl} & for \quad d_{suct} \geq d_{sb} \\ 0 \quad for \quad all \quad remains \end{cases} \quad (7)$$

The suction depth is expected to depend on the flow and is modeled as:

$$d_{suct} = d_0 (\frac{Q_i + Q_r}{Q_0})^{b_{suct}} \quad (8)$$

where d_0 and b_{suct} are positive parameters and $Q_0 = 1000 m^3/h$. Combining (3) and (7) we have:

$$X_{ssoutat} = \frac{d_{suct} - d_{sb}}{d_{suct}} \cdot \frac{d_{at}}{d_{at} - d_{sb}} \cdot X_{ssm} = \frac{1 - \frac{d_{sb}}{d_{suct}}}{1 - \frac{d_{sb}}{d_{at}}} \cdot X_{ssm} \quad for \quad d_{suct} > d_{sb} \quad (9)$$

To study $X_{ssoutat}$ when the point $d_{suct} = d_{sb}$ we introduce a function which we will call logistic function.

Here, the logistic function:

$$l(x) = l(x,a,b) = \frac{1}{1 + e^{\frac{a-x}{b}}} \quad (10)$$

is used.

For x = a, l(a) = 0.5. In the below figure we done e(x) for $b \in \{0.2; 1.0; 2.0\}$ and a = 0.

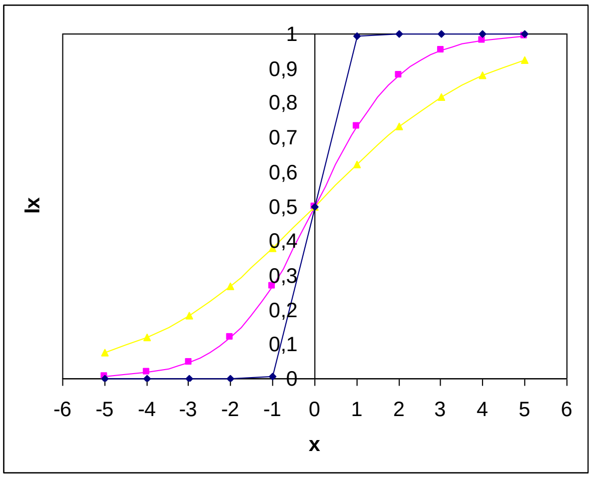

Fig. 2. *The logistic function for a = 0 and different b values.*

The logistic function (10) is used to calculate $x_{ssoutat}$ while the flow path variable is used to select the discharge tank.

$$X_{ssoutat} = f_p(l(d_{suct} - d_{sb2}) \cdot \frac{1 - d_{sb2}/d_{suct}}{1 - d_{sb2}/d_{at}} \cdot X_{ssm2}) + (1 - f_p)(l(d_{suct} - d_{sb1}) \cdot \frac{1 - d_{sb1}/d_{suct}}{1 - d_{sb1}/d_{at}} \cdot X_{ssm1}) \quad (11)$$

As d_{suct} is only dependent on the flow, there is no need to consider different suction depthes for eachof the sludge settling tanks.

2.Matrix form

In order to use a matrix notation, we introduce the state vector **X**, the input vector **U** and the observation vector **Y**.

$$\begin{cases} \mathbf{X} = [X_{ssm1}, X_{ssm2}, d_{sb1}, d_{sb2}]' \\ \mathbf{U} = [f_p, m_1, m_2, X_{ssr}, Q_i, Q_r]' \\ \mathbf{Y} = [X_{ssm2}, X_{soutat}]' \end{cases} \quad (12)$$

By use of the vector function **f(X,U**,t), the mass balances and sludge blanket depth equations can be expressed in an „a-convex” vector differential (a = α sau a = β) modeled as:

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{f}(\mathbf{X}, \mathbf{U}, t) \quad (13)$$

where f(X,U,t) is constructed from equations (1) – (13) and denotations (12).

The measurements are described by the observation equation:

$$\mathbf{Y}(t) = \mathbf{h}(\mathbf{X}, \mathbf{U}, t) \quad (14)$$

where **h(X,U**,t) is constructed from equations 11 and notations 12.

To count in uncertainties in the model formulation and to enable use of the maximum likelihood parameter estimation method, stochastic noise terms are introduced. Hence, the „a-convex” equation (a = α or a = β) (13) turns an „a-convex” differential equation (a = α or a = β) where the time equations describing the mass balances and the sludge blanket depths in the sludge settling tanks can be written as the no called differential equation „a-convex”(Oksendal, 1995).

$$d\mathbf{X}(t) = \mathbf{f}(\mathbf{X}, \mathbf{U}, t)dt + \mathbf{G}(\mathbf{X}, \mathbf{U}, t)d\mathbf{w}(t) = (\prod_{t=1}^{N} p(\boldsymbol{Y}(t)/\boldsymbol{Y}(t-1), \boldsymbol{\theta})\, p(\boldsymbol{Y}(0)/\boldsymbol{\theta}) \quad (15)$$

Where the stochastic process w(t) is assumed to be a standard vector wiener process [5]. The function **G(X,U**,t) describes any state, input or time dependent variation related to now the variation generated by the wiener process enters the system.

Here **G(X,U**,t) is assumed to be a constant diagonal matrix.

$$\mathbf{G}(\mathbf{X},\mathbf{U},t)=\mathbf{G}=\begin{bmatrix}\sigma_{ss} & 0 & 0 & 0\\ 0 & \sigma_{ss} & 0 & 0\\ 0 & 0 & \sigma_{sb} & 0\\ 0 & 0 & 0 & \sigma_{sb}\end{bmatrix} \tag{16}$$

The covariance of G dw(t) becomes

$$\sum=\mathbf{GG}'=\begin{bmatrix}\sigma_{ss}^2 & 0 & 0 & 0\\ 0 & \sigma_{ss}^2 & 0 & 0\\ 0 & 0 & \sigma_{sb}^2 & 0\\ 0 & 0 & 0 & \sigma_{sb}^2\end{bmatrix} \tag{17}$$

The observation uncertainties are included in the observation equation

$$\mathbf{Y}(t)=\mathbf{h}(\mathbf{X},\mathbf{U},t)+\mathbf{e}(t) \tag{18}$$

where the term $\mathbf{e}(t)$ is the measurement error, which is assumed to be a zero mean Gaussian white noise independent of $\mathbf{w}(t)$ and with covariance matrix:

$$\mathbf{V}(\mathbf{e}(t))=\begin{bmatrix}\sigma_{ss2}^2(t) & \mathbf{O}\\ \mathbf{O} & \sigma_{ssoutat}^2(t)\end{bmatrix} \tag{19}$$

3.Estimation models

The method used to estimate the parameter of the model (15), (16), (18) is a maximum likelihood method for estimating parameters in stochastic differential equations based on information in different times given by the observation equation [1].

All the unknown parameters, denoted by the vector $\boldsymbol{\theta}$, are embedding led in the continuous discrete time state model (15) and (17).

The measurements are given in discrete time, and, in order to simplify the notation, it is assumed that the time index T belongs to the set {0,1,2….,N} where N is the number of observations. Introducing:

$$\boldsymbol{Y}(t)=[\mathbf{Y}(t),\mathbf{Y}(t-1),...,\mathbf{Y}(1),\mathbf{Y}(0)]' \tag{20}$$

where y(t) is a vector containing all the observations up to and including time T, T…{0,1,2,…N} the likelihood function is the joint probability density of all the observations, assuming that the parameters are known:

$$L'(\boldsymbol{\theta},\boldsymbol{Y}(N))=p(\boldsymbol{Y}(N)/\boldsymbol{\theta})=p(\boldsymbol{Y}(N)/\boldsymbol{Y}(N-1),\boldsymbol{\theta})p(\boldsymbol{Y}(N-1)/\boldsymbol{\theta})=$$

$$=(\prod_{t=1}^{N}p(\boldsymbol{Y}(t)/\boldsymbol{Y}(t-1),\boldsymbol{\theta})\,p(\boldsymbol{Y}(0)/\boldsymbol{\theta}) \tag{21}$$

where successive applications of the rule $P(A\cap B)=P(A/B)\cdot P(B)$ are used to express the likelihood function as a product of conditional densities. In order to evaluate the likelihood function it is assumed that all the conditional densities are Gaussian. In the case of a linear state apace model, it is easily shown that the conditional densities actually are Gaussian [13].

In a non-linear case, as described by (15) and (17) the Gaussian assumption is an approximation.

The Gaussian distribution is completely characterized by the mean and covariance. Hence, to parameterize the conditional densities in (21), we introduce the conditional mean and the conditional covariance as

$$\hat{\mathbf{Y}}(t/t-1) = E[\mathbf{Y}(t)/\mathbf{Y}(t-1),\boldsymbol{\theta}] \text{ and } \mathbf{R}(t/t-1) = V[\mathbf{Y}(t)/\mathbf{Y}(t-1),\boldsymbol{\theta}] \quad (22)$$

It should be mentioned that these correspond to the one step prediction and the associated covariance, respectively. Furthermore it is convenient to introduce the one step prediction error

$$\boldsymbol{\varepsilon}(t) = \mathbf{Y}(t) - \hat{\mathbf{Y}}(t/t-1) \quad (23)$$

The conditional likelihood function (conditioned or $\mathbf{Y}(0)$) becomes

$$L(\boldsymbol{\theta},\boldsymbol{Y}(N)) = \prod_{t=1}^{N}((2\pi)^{-m/2}\det\mathbf{R}(t/t-1))^{-1/2}\cdot\cdot\exp(-\frac{1}{2}\varepsilon(t)'\mathbf{R}(t/t-1)^{-1}\varepsilon(t)) \quad (24)$$

where m is the conditional likelihood function is given by the following relation:

$$\log L(\boldsymbol{\theta},\boldsymbol{Y}(N)) = -\frac{1}{2}\sum_{t=1}^{N}\log\det\mathbf{R}(t/t-1) + \boldsymbol{\varepsilon}(t)'\mathbf{R}(t/t-1)^{-1}\boldsymbol{\varepsilon}(t)) + const \quad (25)$$

maximum likelihood estimate (ML- estimate) is the set $\hat{\boldsymbol{\theta}}$, which maximizes the likelihood function. Since it is not, in general, possible to optimize the likelihood function, a numerical method has to be used. A reasonable method is the quasi – Newton method.

An estimate of the uncertainty of the parameters is obtained by the fact that the ML – estimator is normally distributed with mean $\boldsymbol{\theta}$ and covariance

$$\mathbf{D} = \mathbf{H}^{-1} \quad (26)$$

where the matrix H is given by

$$\{h_{lk}\} = -E[\frac{\partial^2}{\partial\theta_l\partial\theta_k}\log L(\boldsymbol{\theta},\boldsymbol{Y}(N))] \quad (27)$$

An estimate of **D** is obtained by equating the observed value with its expectation and applying

$$\{h_{lk}\} \approx -E(\frac{\partial^2}{\partial\theta_l\partial\theta_k}\log L(\boldsymbol{\theta},\boldsymbol{Y}(N))_{|\theta=\hat{\theta}} \quad (28)$$

The above equation can be used for estimating the variances of the parameter estimates. The variance serves as a basis for calculating t- test values, for tests under the hypothesis that the parameter is equal to zero. Finally, the correlation between the parameter, estimates is readily found based on the covariance matrix D from (26).

4.Conclusions

- A two layer model of settling in a settling tank has been established, by using settling and mixing differential equations and concluding that when the tank is fully mixed, there is no sludge blanket, and when the flow is continuous, the sludge blanket depth tends to zero.
- Further mixing signals m_1 and m_2 of settling tanks 1 and 2 have been used, combined with mixing and settling equations, to establish the SS concentration in the sludge layer from tank 1 and 2.
- The SS concentration in the effluent from the settling tanks has been established, depending on the suction depth and on the SS concentration in the sludge layer by using the logistic function l(x)=l(x,a,b) – where from we can conclude that the suction depth only depends on the flow, so there haven't been considered different suction depths for each of the suction tanks.
- The study of the flow process, obtained from „a- convexity" has been processed in a computer programme. By processing the information, different values of $\alpha \in [0,1]$ and β

$\in [0,1]$ have been obtained as well as different values of the sludge blanket depth dsb1 and dsb2.

- The results from the elaborated programme have been interpreted as discrete likely variables. By using x likely variable, a likelihood study and a maximum likelihood one of the modelled process, have been established.
- We have calculated the mathematical expectation, denoted by m and the mathematical dispersal denoted by σ, and by using „α - convex" and „β.- convex" equations presented in the model, we obtained the values m_α, m_β, σ_α, σ_β. By using m and σ. We designated Laplace – Gauss curve.
- Gauss' bell helped us establish the SS concentrations ($X_{ssoutat}$) in the effluent from the settling tanks (the maximum values). Thus, it is noticed that $X_{ssoutat}$ = m is maximum point of n(x,m, σ) its maximum value being $\frac{1}{\sigma\sqrt{2\pi}}$.

References

1.**Bechmann, H.**, *Control of Sewer Systems and Wastewater Treatment Plants Using Pollutant Concentration Profiles*, Water Science and Technology nr. 37, 1998;

2.**Bechmann, H., Grey Box,***Modelling of First Flush and Incoming Wastewater at a Wastewater Treatment Plant*, Environmetrics, nr. 11, 2000;

3.**Bechmann, H.,***Modelling of Wastewater Systems,* Lyngby 1999, ATV Erhervskerprojekt EF 623 IMM-PHD, 1999-69

4.**Bennet, R. Edwin., and other.,***Optimize solid control in Water and wastes engineering*, S.U.A., vol. 8, nr. 5, may, 1991, p. 46-48

5.**Besik, F.,***Waste water reclamation in a closed system in Water & Sewage Works*, S.U.A., 118, nr. 7, iulie 1991, p. 213-219.

6.**Iovanov V.V.,***Wastewater Clarifying in the fluid – bedaled Clarifier,* Universitaria International Symposium, "I.M.P.", 1981.

7.**Iovanov V.V.,***Modern Automatic Systems for Sand removing and oil separating used for environmental protection in our country*, Universitaria International Symposium, "ROPET", 2001.

8.**Iovanov V.V.,***The Actual Stage of Technologies and Equipments for Environmental Protection and wastewater treating Abroad*, Universitaria International Symposium, "ROPET", 2001.

9.**Iovanov V.V.,***Technologies and Equipments for environmental protection and wastwater Treating in our country*, Universitaria International Symposium, "ROPET", 2001.

10.**Iovanov V.V.,***Automatic Plants for Concentrating and Dehydrating Mud for environmental Protection in our country*, Universitaria International Symposium, "ROPET", 2001.

11.**Iovanov V.V.,***The most modern sand renovers and oil separators for environmental protection in our country,* Universitaria International Symposium, "ROPET", 2002.

12.**Iovanov V.V.,***Systems of Concentrating and Dehydrating Mud for wastewater treatment in our country,* Universitaria International Symposium, "ROPET", 2002.

13.**Iovanov V.V.,***Modelling of Industrial Wastewater Clarifying Process in sludge settling tanks,* Universitaria International Symposium, "ROPET", 2003.