RESEARCH REGARDING THE MODAL PARAMETERS IDENTIFICATION FOR METALLIC STRUCTURES (I)

[1]**Prof. phd. eng.** Dan **ILINCIOIU**, University of Craiova, Faculty of Mechanics, Department of Applied Mechanics and Civil Constructions, Calea Bucuresti Street, no. 107, Craiova,Code 200512, Romania, danilin@mecanica.ucv.ro
[2]**Eng.** Ion **TĂTARU,** University of Craiova, Faculty of Mechanics, Department of Applied Mechanics and Civil Constructions, Calea Bucuresti Street, no. 107, Craiova,Code 200512
[3]**Eng.** Cosmin-Mihai **MIRIŢOIU,** University of Craiova, Faculty of Mechanics, Department of Vehicles, Transports and Industrial Engineering, Calea Bucuresti Street, no. 107, Craiova,Code 200512, Romania, miritoiucosmin@yahoo.com

Abstract. *In this paper, starting from previous methods written by other authors, we present the theoretical background of the modal identification for hyperstatic metallic structures (in an own way). We present also the known methods which are used nowadays for modal parameters identification.*

Keywords: excitation, eigenmodes, eigenfrequency, admittance, modal parameters

Contents:

1. Introduction

The modal identification of three dimensional metallic structures consists in an assembly of theoretical and experimental procedures for determination of those parameters that characterize the system eigenmodes. The testing techniques for nowadays, for modal parameters identification can be broadly classified in two majour groups (according to Manea (2006)[17], Manea et al. (2007)[18], Miritoiu et al. (2011)[19] or Edwins (1987)[8]): <u>the multipoint-excitation</u> that involves the usage of multiple shakers located at various points on the structure and having controlled force amplitudes and phase relationships in order to nullify the damping forces presented in the structure and to drive the structure in an undamped mode; <u>the single point excitation</u> method used even if the structure is complicated and consists in applying a force in a given point and recording the vibratory response in all interest points, including the excitation point. The advantages of the first method are: the undamped normal frequencies and the corresponding mode shapes are immediately produced. The major disadvantage of the first method is: high complexity and expense in time and cost of installing multiple shakers. The main advantage of the second method is that it requires a minimum of equipment, but, as a disadvantage, it needs a laborious analysis to perform extensive processing of the result to interprete the dynamic behaviour of the structure under test. It also gives a good procedure and criteria to validate the mathematical model obtained by finite element analysis using specialised softwares. This article presents a theoretical background of single point excitation method from the authors point of view, a software for modal identification and an application for modal identification on a hyperstatic metallic structure.

Many papers present methods for eigenmodes calculus. For example, in Gomes et al. (2008)[10] the limit spectral problems are derived for the problem on oscillations of a solid with light inclusions. It is established that, for heavy inclusions, the limit problems are united into a more complex resultant problem describing the far action in the set of inclusions. In Mishakin and Samsonov (2011)[20] it is presented a method for calculating the dispersion characteristics of eigenmodes of metal waveguides with helical corrugations on the inner surface, which is based on the transition to a new nonortogonal system of coordinates. The analyzed problem is reduced to solving a generalized algebraic problem. Kairov (2001)[12] studies the effect of holes on the eigenmodes of reinforced shells of rotation. A solution is built on the basis of the linear theory of thin elastic shells using the Ritz method. The obtained numerical results are compared with experimental data. Tommaseo et al. study the subharmonic excitation of the eigenmodes of charged particles in a penning trap. Komarov (2011)[14] studies the eigenvalues and eigenfields of regular polygonal waveguides. The lowest and high-order TE- and TM-modes are identified on a basis of united classification scheme. Closed-form expressions for calculation of cutoff wavelengths of the lowest TE- and TM-modes are presented. Belousov et al. (2000)[2] study methods of spectrum calculation and parameter control of open-cavity eigenmodes. The potential of the proposed software and hardware is demonstrated on the basis of an orotron cavity model designed and examined for millimeter wavelengths. The numerical and experimental results are in good agreement. The developed methods and software can be used for designing open cavities in various frequency ranges. Vlasov (2006)[27] has determined the characteristics (eigenfrequencies and radiation Q-factors) of elastic oscillations existing at the boundary of a cylindrical cavity in a solid body. These oscillations become Rayleigh waves with increasing cavity radius. It was shown that such oscillations in bodies with moderate Poisson's ratios (about 0.2–0.3) can exist in the case of sufficiently large cavity diameters exceeding 100 Rayleigh wave lengths. In Schidt-Hattenberger (1992)[22] an important subclass of solutions has been analytically investigated for a non-linear three coupler fiber. These were the stationary solutions or nonlinear eigenmodes. Their stability is checked by using an exact method and numerical tests. In Stanescu et al. (2009)[23] it was made a study for modal identification for two bars from composite materials (*bar 1* made of phenolic fireproof resin reinforced with fiberglass; *bar 2* made of ortophtalic polyesteric resin reinforced with fiberglass).

Many other vibration studies exist in the engineering literature. For example, in Hu (2011)[7] the vibration mode of the constrained damping cantilever is built up according to the mode superposition of the elastic cantilever beam. The control equation of the constrained damping cantilever beam is then derived using Lagrange's equation. There is made a comparison between analytical and experimental methods. Xinong and Zinghui (1998)[31] studied the active and passive control of vibration of the thin plate with Local Active Constrained Damping Layer. The governing equations of system were formulated based on the constitutive equations of elastic, viscoelastic, piezoelectric materials. Cao et al. (2011)[3] studied the free vibration characteristics of circular cylindrical shell with passive constrained layer damping (PCLD). Wave propagation approach rather than finite element method, transfer matrix method, and Rayleigh-Ritz method was used to solve the problem of vibration of PCLD circular cylindrical shell under a simply supported boundary condition at two ends. Numerical results show that the presented method was more effective in comparison with other methods. Xia and Lukasiewicz (1995)[29] studied the nonlinear, forced, damped vibrations of simply-supported rectangular sandwhich plates with a viscoelastic core. Damping was taken into account by modelling the viscoelastic core as a Voigt-Kelvin solid. It was also studied the influence of the thickness of the layers and the material properties on the nonlinear response of the plates. Lee and Han (2006)[11] studied the free and forced vibration of laminated composite plates and shells using a 9-node strain shell element. The natural frequencies of isotropic and composite laminates were presented. The forced vibration analysis of laminated composite plates and shells subjected to arbitrary loading was investigated. Karnopp et al. (1970)[13] studied the problem of determining the natural frequencies and modes of a statically indeterminant Timoshenko beam. By lumping the beam properties of linear and rotary inertia at discrete points along the length of the beam and by employing the complementary, variational principle, an approximate solution was obtained by using simple matrix iteration.

2. Theoretical background for the modal parameters identification

Any system can be modeled by n concentrated mass points jointed by elastic elements having k_k rigidity and elements having c_k damping. If this damped system, having n degrees of freedom, is loaded by an external excitation system marked with {Q(t)}, the motion equations are given in relation (1).

$$[M]\cdot\left\{\ddot{x}(t)\right\}+[C]\cdot\left\{\dot{x}(t)\right\}+[k]\cdot\{x(t)\}=\{Q(t)\} \tag{1}$$

where [M], [C] and [K] represent the matrices of mass, damping and rigidity; {x(t)} with the first and second derivatives are the vectors of displacements, velocity and acceleration; {Q(t)} is the generalised forces vector.

The system response at external excitation is calculated with (2), where it is processed as a sum of 'n' modal contributions due to each separate degree of freedom.

$$\{X(\omega)\}=\sum_{k=1}^{N}\left[\frac{\{\psi^k\}\cdot\{\psi^k\}^T\cdot\{Q(\omega)\}}{a_k\cdot(-\mu_k+i(\omega-\nu_k))}+\frac{\left\{\overline{\psi^k}\right\}\cdot\left\{\overline{\psi^k}\right\}^T\cdot\{Q(\omega)\}}{\overline{a_k}\cdot(-\mu_k+i(\omega-\nu_k))}\right] \tag{2}$$

where {X(ω)} is the Fourier transform of displacement; $\{\xi^k\}$ and $\{\overline{\xi^k}\}$ represent the k order eigenvector and its complex conjugate; μ_k is the k order of damping ratio; ν_k is the k order of damped natural frequency; a_k and $\overline{a_k}$ are the norm constants of eigenvector; ω is the external excitation frequency.

In practical applications in mechanical engineering, we usually replace the modal vectors with two constants that are determined with (3).

$$\begin{cases} U_{ij}^k+iV_{ij}^k=\dfrac{\xi_i^k\,\xi_j^k}{a_k} \\ U_{ij}^k-iV_{ij}^k=\dfrac{\xi_i^k\,\xi_j^k}{\overline{a_k}} \end{cases} \tag{3}$$

The system admittance, defined as the ratio between the displacement response and the force excitation is calculated with (4).

$$\alpha_{ij}(\omega)=\sum_{k=1}^{n}\left[\frac{U_{ij}^{k}+iV_{ij}^{k}}{-\mu_k+i(\omega-\nu_k)}+\frac{U_{ij}^{k}-iV_{ij}^{k}}{-\mu_k+i(\omega-\nu_k)}\right] \quad (4)$$

The concept of discrete system with concentrated mass in *n* material points was used in the approximations adopted during the mathematical model. In order to obtain an accurate approximation of the real system by the discrete one, *n* must converge to infinity. Because of experimental and processing technique and of the necessary time for data processing, this is impossible. The frequencies domain is limited to a reasonable width in practical applications, which is obtained by the major resonances of the analyzed equipment and the frequency domain of the application goal. In these conditions, the sum from relation (4) is reduced to several components marked in the following with *n*. The contribution of superior and inferior modes are included in two corrections factors known as *residual flexibility* S'_{ij} (for superior modes) and *inferior modal addmitance* $(-1)/(M'_{ij}\cdot\omega^2)$ (for inferior modes). The system admittance will be calculated with (5).

$$\alpha_{ij}(\omega)=\frac{-1}{M'_{ij}\cdot\omega^2}+\sum_{k=1}^{n}\left[\frac{U_{ij}^{k}+iV_{ij}^{k}}{-\mu_k+i(\omega-\nu_k)}+\frac{U_{ij}^{k}-iV_{ij}^{k}}{-\mu_k+i(\omega-\nu_k)}\right]+S'_{ij} \quad (5)$$

where i is the excitation point and j is the measuring point.

The modal identification of a system with *n* degrees of freedom assumes determination of *4n* modal parameters: μ_k, ν_k, U'_{ij}, V'_{ij}. These are the intrinsic characteristics of the system, independent of the external conditions. The system response to different excitations (like: seismic motion applied to base, concentrated electrodynamics forces due to the switching phenomena, distributed forces due to wind actions and so on) can be calculated with relation (2). The modal parameters are determined from experimental tests performed on the system brought into a controlled vibrations state with simultaneous measurement of the applied excitation and structure response. The controlled vibration state can be made using the following low-level excitation methods (according to Manea (2007)[18] or Miritoiu (2011)[19]): the relaxed step force, the one-point sinusoidal or large band steady-state vibration excitation and the impact force. In this paper, in order to bring the metallic structure into a controlled vibrations state, there will be used the impact force.

3. Modal parameters identification steps

To determine de modal parameters, we follow the next steps:

- We determine the frequency response characteristics by calculating the admittance $\alpha_{ij}(\omega)$ for all the pairs excitation/vibratory response points
- Preliminary resonances localization in the initial approximation of μ_k and ν_k (k=1,2,..,n) modal parameters
- The first stage identification of modal parameters μ_k, ν_k, U^k_{ij}, V^k_{ij}, S'_{ij} and $(-1/M'_{ij})$ (where k=1,2,..,n) on limited frequency domains. The identification is made by using linear procedures, determining those parameters which inserted in relation (5) generate theoretical characteristics that approximate with minimal error the experimental calculated frequency response function.

•The final identification of modal parameters μ_k, ν_k, U^k_{ij}, V^k_{ij}, $S^{'}_{ij}$ and $(-1/M^{'}_{ij})$ (where k=1,2,..,n) on the entire frequency domain. The identification is made similar with the above step.

4. Conclusions

In this paper we have presented the theoretical background for the modal parameters identification. The modal parameters can be used for structural changes analysis and for assesment of structure response to given excitations concentrated in distinct points or distributed on the structure.

Applied on a new equipment in the prototype stage, modal identification gives informations about the corectitude of design conception, construction, and it may give informations concerning the improvement of the vibration response of the equipment.

This method can be used in parallel with a finite element software. A very good finite element software can be a certain error source if it is used by an analyst that mindless of the fact that the material characteristics are only approximate known, even if the geometrical model is very good.

Applied on a recent mounted equipment, or on a working equipment, the modal identification gives informations concerning the quality of the mounting process, the weariness of material, possible cracks or the whickness of some parts.

5. Acknowledgement

This work was partially supported by the strategic grant POSDRU/88/1.5/S/50783 (2009), co-financed by the European Social Fund – Investing in People, within the Sectoral Operational Programme Human Resources Development 2007-2013.

References

1. Adam, C., (2001) Eigenstrain induced vibration of composite plates, Acta Mechanica 148, 35-53
2. Belousov, V., I., et al., (2000) Methods of calculation and parameter control of the eigenmodes of a simple two-mirror cavity, Radiophysics and Quantum electronics, 43(8), 663-670
3. Cao, X.,et al., (2011) Free vibration of cylindrical shell with constrained layer damping, Appl. Math. Mech.-Engl. Ed., 32(4), 495-506, DOI: 10.1007/s10483-011-1433-7
4. Chapman, C., L., et al., (2002) Spatial eigenmodes and synchronous oscillation: Co-incidence detection in simulated cerebral cortex, J. Math. Biol. 45, 57-78
5. Esmailzadeh, E., Jalali, M., A., (1999) Nonlinear oscilations of viscoelastic rectangular plates, Nonlinear Dynamics, 18:311-319
6. Hu, M-y, Wang, A-w., (2009) Free vibration and transverse stresses of viscoelastic laminated plates, Appl. Math. Mech.-Engl. Ed., 30(1), 101-108, DOI: 10.1007/s10483-009-0111-y
7. Hu, M-y, et al., (2011) Approximate analytical soluctions and experimental analysis for transient response of constrained damping cantilever beam, Appl. Math. Mech.-Engl. Ed., 31(11), 1359-1370
8. Edwins, D., J., (1987) Modal testing, Bruel&Kjaer
9. Jianxin, G., Yapeng, S., (1999) Vibration and damping analysis of a composite plate with active and passive damping layer, Appl. Math. Mech.-Engl. Ed., 20(10) 10-1075-12
10. Gomez., D., et al., (2008) Fromal asymptotics of eigenmodes of oscillation spatial bodies with concentrated masses, Journal of Mathematical Sciences, 148(5), 650-674

11. Lee, W-H., Han S-C.,(2006) Free and forced vibration analysis of laminated composite plates and shells using a 9-node Assumed strain shell element, Comput Mech, 39:41-58, DOI: 10.1007/s00466-005-0007-8
12. Kairov, A., S.,(2001) Effect of holes on the eigenmodes of reinforces shells of rotation, Journal of Mathematical Sciences, 103(3), 393-397
13. Karnopp, B., H., et. al.,(1970) On the Oscillations of Statically Indeterminate Beams, Acta Mechanica 9, 121-129
14. Komarov, V., V.,(2011) Eigenmodes of regular polygonal waveguides, J. Infrared. Milli. Terahz. Waves., 32:40-46, DOI 10.1007/s10762-010-9755-4
15. Kulkarni, S., D., Kapuria, S., (2008) Free vibration analysis of composites and sandwhich plates using an improved discrete Kirchoff quadrilateral element based in third-order zig-zag theory, Comput. Mech., 42:803-824, DOI 10.1007/s00466-008-0285-z
16. Maheri, M., R., et al.,(2008) Vibration damping in sandwhich panels, J Mater Sci, 43: 6604-6618, DOI 10.1007/s10853-008-2694-y
17. Manea, I.,(2006) Experimental Modal Analysis, Universitaria Publishing, Craiova
18. Manea, I., et al., (2007) Software for mechanical systems modal identification, Annals of the University of Craiova, Mechanical Series, 1:191-198
19. Miritoiu, et al., (2011) A comparison between the modal parameters obtained by two different accelerometers, The 5TH International Conference on Manufacturing Science and Education – MSE 2011,1:39-43,Sibiu, Romania
20. Mishakin, S., V., Samsonov, S., V., (2011) Method for calculation of helical-waveguide eigenmodes of the basis of solving the equivalent two-dimensional problem by field expansion in circular wave-guides modes, Radiophysics and Quantum Electronics, 54(3), 174-184
21. Park, J-T., Choi, N-S., (2004) Flexural vibration analysis of a sandwhich beam specimen with a partially inserted viscoelastic layer, KSME International Journal, 18(3), 247-356
22. Schimidt-Hattenberger, C., et al., (1992) Nonlinear eigenmodes of a three-core fiber coupler, Optical and Quantum Electronics, 24:691-701
23. Stanescu, M., M., et al., (2009) Experimental researches concerning the properties of composite materials with random distribution of reinforcement, Materiale Plastice, 46(2), 73-78
24. Sudahar, R., M., Kant, T., (2007) On the angle ply higher order beam vibrations, Comput Mech, 40:25-33, DOI 10.1007/s00466-006-0079-0
25. Tommaseo, G., et al., (2004) Subharmonic excitation of the eigenmodes of charged particles in a Penning trap, Eur. Phys. J. D., 28: 39-48, DOI 10.1140/epjd/e2003-00296-0
26. Trompette, P., Fatemi, J.,(1997) Damping of beams, Optimal distribution of cuts in the viscoelastic constrained layer, Structural Optimization, 13: 167-171
27. Vlasov, S., N.,(2006) Eigenmodes of a cylindrical cavity in a solid body, Radiophysics and Quantum Electronics, 49(12), 956-960
28. Wang, J., M., Guo, B., Z., (2008) Analyticity and dynamic behaviour of a damped three layer sandwhich beam, J Optim Theory Appl, 137:675-689, DOI 10.1007/s10957-007-9341-7
29. Xia., Z., Lukasiewicz, S., (1995) Nonlinear damped vibrations of simply-supported rectangular sandwich plates, Nonlinear Dynamics, 8: 417-433
30. Xiang, Y., et al.,(2008) New matrix method for analyzing vibration and damping effect of sandwhich circular cylindrical shell with viscoelastic core, Appl. Math. Mech.-Engl. Ed., 29(12), 1587-1600, DOI: 10.1007/s10483-008-1207-x
31. Xinong, Z., Jinghui, Z., (1998) The hybrid control of vibration of thin plate with active constrained damping layer, Applied Mathematics and Mechanics, English Edition, 19(12), 1119-1134