KINEMATICS OF MATERIAL REMOVAL
AND FORMING OF SURFACE AT GRINDING

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Abstract: The mathematical model of kinematics of material removal and a forming of surfaces is developed at grinding. Conditions of increase of productivity of processing are defined and new kinematic schemes of high-performance grinding are offered.

Keywords: grinding, wheel speed, processing productivity, thickness of cut

1. INTRODUCTION

Grinding is one of the main methods of machining of the materials, providing high rates of accuracy, quality and productivity of processing [1, 2]. At the same time, its technological capabilities fully aren't used due to the lack scientifically reasonable recommendations about a choice of optimum modes of cutting taking into account strength properties of a working surface of a wheel. In a special measure it belongs to diamond grinding which owing to unique cutting properties of diamond has considerable reserves of increase of productivity of processing when providing requirements for quality and accuracy of processed surfaces [3]. It is important to know first of all regularities material removal and formation of surfaces at grinding at the level of kinematics of microcuts for identification of realization of potential opportunities of process of grinding. It will allow to develop high-performance kinematic schemes of grinding and to define optimum modes of cutting. Therefore the purpose of work is development of mathematical model of kinematics of material removal and a forming of surfaces at grinding.

2. MATHEMATICAL MODEL AND THE SETTLEMENT SCHEME THE GRINDING PROCESS

The removed allowance is presented in the offered settlement scheme of process of grinding by a package of infinitely thin cylindrical covers which under different corners enter into a working surface of a wheel (fig. 1 [4]). As a result of interaction of cylindrical covers with grains, from each cover there is the cutting of metal described by probabilistic function – relative completeness of a profile of a wheel \( e_n y \). On construction structure this function corresponds to the classical relative basic length of a microprofile of the processed surface [5] and considers probabilistic imposing and overlapping of projections of grains on each of considered cylindrical covers:
where \( k \) - superficial concentration of grains of a wheel, piece/ \( m^2 \); \( b \) - the maximum height of a protrusion of tops of grains over level of a linking of a wheel, \( m \); \( 2\gamma \) - corner at top of cone-shaped cutting grain; \( V_c, V_{pr} \) - according to the speed of a wheel and preparation, \( m/s \); \( \rho = 1/R_c + 1/R_{pr} \); \( R_c, R_{pr} \) - respectively radiuses of a wheel and preparation, \( m \); \( t_{if} = t_T + i \cdot t \) - cover coordinate at \( i \) contact with a wheel, \( m \); \( t_{ni} = t_T + n - 1 \cdot t \) - cover coordinate at \( n \) contact with a wheel, \( m \); \( t_T \) - coordinate of the current infinitely thin cover by which the removed allowance is conditionally presented, to \( m \); \( n \) - number of passes of a wheel.

The basis of the developed mathematical model of grinding is made by the analytical decision on the description of border of completion of dispergating by cutting grains of the material brought in a zone of cutting, along an arch of contact of a wheel with preparation:

\[
H = \frac{3}{2} \cdot \frac{t_{nT}}{t_{nT}} \cdot \frac{H_{max}^3}{2 \cdot \sqrt{t_{ni_{ear}}}} + \sum_{i=0}^{n-2} \frac{t_{i_{ear}}^{5/2}}{t_{iT}} - \sum_{i=0}^{n-2} \frac{t_{i_{ear}}^{5/2}}{t_i},
\]

where \( H_{max} \) - the maximum thickness of a cut, \( m \); \( t_{ni_{ear}} \) - coordinate of extreme provision of a cover at which the full profile \( \varepsilon_n \phi = 0.95 \) is formed at level \( H_{max} \), \( m \); \( t_{iT_{ear}} \) - coordinate of extreme provision of a cover at \( i \) contact with a wheel, \( m \).

As shown in fig. 1, this line is drawn on tops of microroughnesses of a processed material, has a difficult configuration, connects a processed surface with processed and by analogy to blade processing determines the provision of a conditional probabilistic surface of cutting at grinding. Characteristic points of border are a basis for calculation of physical and technological parameters of grinding (the maximum thickness of a cut, a roughness of the processed surface, the actual length of contact of a wheel with preparation, etc.). It allows from uniform positions quite unambiguously analytically to describe regularities of process of grinding in all possible range of change of depth of grinding (including ranges of multipass and deep grinding).

\[\varepsilon_n \phi = 1 - \exp \left\{ \frac{\sqrt{2} \cdot tg\gamma \cdot k \cdot V_{cT}}{3 \cdot b \cdot V_{prT} \cdot \sqrt{\rho}} \left[ \frac{y^3}{2 \cdot \sqrt{t_{nT}}} + \sum_{i=0}^{n-2} \frac{t_{iT}^{5/2}}{t_i} \right] \right\}, \]
3. MAXIMUM THICKNESS OF A UNIT CUT AND MICROGEOMETRY OF GROUND SURFACE

By calculations it is established that the provision of border is defined by a ratio of two parameters – maximum (given probabilistic) thickness $H_{\text{max}}$ of a cut and grinding depth $t$. In a case $t < H_{\text{max}}$ (multipass grinding) the border accepts approximately a symmetric form of rather axial plane of grinding, in a $t > H_{\text{max}}$ (deep grinding) – an asymmetric form.

By calculations it is established that the percent of working grains for a case $t > H_{\text{max}}$ makes about 50%, and for a case $t < H_{\text{max}} - 5 \ldots 10\%$ (i.e. grains pass almost "a trace in a trace" that as it will be shown above, is an important reserve of increase in productivity of processing).

In a case $t > H_{\text{max}}$ analytical dependence for definition of provision of border assumes a simple air

$$H = H_{\text{max}} \sqrt{\frac{t_f}{t}},$$

where $t_f$ – the coordinate of the current elementary (infinitely thin) cylindrical cover by which the removed allowance is conditionally presented in the settlement scheme, m.

Respectively, parameters of border $H_{\text{max}}$ and $R_{\text{max}}$ (parameter of a roughness of processing, m) are described by analytical dependences

$$H_{\text{max}} = \left(630 \cdot \pi \cdot \overline{X}^3 \cdot V_{pr} \cdot t^{0.5} \cdot \rho^{0.5} \right)^{0.33} \cdot \frac{m \cdot V_c}{c},$$

$$R_{\text{max}} = 10 \cdot \left(\frac{\pi \cdot \overline{X}^3 \cdot V_{pr} \cdot \rho^{0.5}}{m \cdot V_c} \right)^{0.4},$$

where $\overline{X}$ - granularity of a wheel, m; $m$ - volume concentration of grains in a wheel (dimensionless size).

<table>
<thead>
<tr>
<th>Authors</th>
<th>F. Novikov</th>
<th>E. Maslov</th>
<th>G. Lurye</th>
<th>A. Reznikov</th>
<th>Experiment data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{\text{max}}$, МКМ</td>
<td>14,7</td>
<td>0,007</td>
<td>0,12</td>
<td>1,1</td>
<td>10,5</td>
</tr>
</tbody>
</table>

Table 1: Calculated values of thickness of a cut $H_{\text{max}}$ (basic data: $R_{pr} = 80 \cdot 10^{-3}$ m; $R_c = 150 \cdot 10^{-3}$ m; $\overline{X} = 0,225 \cdot 10^{-6}$ m; $m = 100$; $V_c = 30$ m/s; $V_{pr} = 1$ m/min; $t = 0,1 \cdot 10^{-3}$ m)

Shown in tab. 1 values of thickness of the cut given in tab., received on settlement dependences of a number of authors, show on a big divergence of settlement and experimental data. For example, for the settlement dependence offered by prof. E. Maslov this divergence exceeds 1000 times. The most correct result is received with use of dependence (4). The divergence of calculated and experimental values available
here $H_{\text{max}}$ (to 40%) is connected with that in kinematic model of process of grinding wear of grains of a wheel isn't considered. For the purpose of specification of the received results settlement dependences which contain the new dimensionless parameter $\eta = x / H$ defining degree of linear wear of grains and changing within $0 \ldots 1$ (for "sharp" grain $\eta \to 0$, for become blunted $\eta \to 1$) are established:

$$H_{\text{max}} = \left[ \frac{630 \pi \bar{X} V_{pr} \cdot t^{0.5} \cdot \rho^{0.5} \cdot \eta^{2} - \eta^{2}}{m \cdot V_{c} \cdot t^{0.5} \cdot \eta^{2}} \right]^{0.33}; \quad \text{(6)}$$

$$R_{\text{max}} = 10 \left[ \frac{\eta^{2} \pi X V_{pr} \cdot t^{0.5} \cdot \rho^{0.5} \cdot \eta^{2}}{m \cdot V_{c} \cdot t^{0.5} \cdot \eta^{2}} \right]^{0.4}. \quad \text{(7)}$$

Taking into account parameter $\eta$ ($\eta > 0$) values also decrease. Therefore, decreases (and even it is eliminated) a divergence between calculated and experimental values $H_{\text{max}}$ (tab. 1). Comparison of experimental values of the maximum thickness of chip with the corresponding calculated values of parameter $H_{\text{max}}$ showed their approximate coincidence at $\eta = 0, 2$ (tab. 2). It follows from this that the accounting of size of linear wear of grain $x$ (by means of parameter $\eta$) in settlement dependences allows to bring the theory and practice of grinding into accord.

### Table 2: Calculated values $H_{\text{max}}$ and experimental values of the maximum thickness of chips in mm ($m = 100$; $\bar{X} = 0.2 \cdot 10^{-6}$ m; $V_{c} = 30$ m/s; $R_{pr} = 0.02$ m; $R_{c} = 0.15$ m)

<table>
<thead>
<tr>
<th>No.</th>
<th>Grinding modes</th>
<th>$\eta$</th>
<th>Maximum thickness of chip, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t$, mm</td>
<td>$V_{pr}$, m/min</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.5</td>
<td>0.0130</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.8</td>
<td>0.0154</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>1.0</td>
<td>0.0167</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>2.0</td>
<td>0.021</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.0148</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>1.0</td>
<td>0.0187</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>2.0</td>
<td>0.0236</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>0.5</td>
<td>0.0167</td>
</tr>
<tr>
<td>9</td>
<td>0.2</td>
<td>1.0</td>
<td>0.0210</td>
</tr>
</tbody>
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4. STUDY, DISCUSSION AND PROSPECTS FOR THE IMPLEMENTATION OF GRINDING PRODUCTIVE POSSIBILITIES

From all the entering into dependences (6) and (7) parameters, the greatest influence on \( H_{\text{max}} \) and \( R_{\text{max}} \) \( \eta \) renders. It indicates a prevailing role of size \( \eta \) in formation of the key physical and technological parameters of grinding and confirms the made hypothesis of effective management of grinding process on the basis of size regulation \( \eta \). For its realization the greatest possible productivity of processing caused by strength properties of a working surface of a wheel is determined, i.e. at the fixed (limit) area of cross section of a cut by separate grain \( S = 0.5 \cdot \psi \cdot H_{\text{max}}^2 \cdot \frac{4\eta_7 + \eta_8}{4\eta_7} \) and \( \eta = 0 \):

\[
Q = \frac{\sqrt{2} \cdot m \cdot V_c \cdot B}{450 \cdot \pi \cdot R^3} \cdot \left[ \frac{t}{\rho} \cdot \frac{H_{\text{max}}^3}{2 \cdot \sqrt{t_{1T}}} + \sum_{i=0}^{n-2} i^{5/2} \right],
\]

where \( B \) - width of a wheel, m.

Values of the parameter defining percent of working grains, are given in tab. 3.

**Table 3**: Calculated values \( \psi \) at \( H_{\text{max}}=10 \cdot 10^{-6} \) m

<table>
<thead>
<tr>
<th>( t \cdot 10^{-6}, \text{m} )</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi,% )</td>
<td>6.65</td>
<td>15.25</td>
<td>43.19</td>
<td>44.0</td>
<td>44.0</td>
</tr>
</tbody>
</table>

It is theoretically established that generally processing productivity \( Q \) with increase in depth of grinding \( t \) changes on extreme dependence, passing a point of a minimum (fig. 2). Preparation speed \( V_{pr} = Q / B \cdot t \) thus continuously decreases. It is proved that depth of grinding \( Q \) is equal in a minimum point \( t \) to parameter \( H_{\text{max}} \). It corresponds to transition from the scheme multipass \( (t < H_{\text{max}}) \) to the scheme of traditional deep grinding \( (t > H_{\text{max}}) \).

From the physical point of view the minimum of productivity of processing \( Q \) under a condition \( S = \text{const} \) is caused by existence of the shortest on the chip length \( \Delta l \) (considering productivity of processing proportional \( S \cdot \Delta l \)) as with increase and reduction of depth of grinding \( t \), since value \( t = H_{\text{max}}, \text{length of chip} \ \Delta l \) increases. In the first case – at the expense of increase in length of an arch of contact of a wheel with preparation, in the second case – at the expense of increase \( V_{pr} \).

The carried-out analysis of known methods of diamond and abrasive grinding showed that all of them, as a rule, realize a condition \( t = H_{\text{max}} \), i.e. a minimum of productivity of processing \( Q \).
Obviously, for abrasive grinding it is effective since at the expense of action on grains of big loadings the mode of intensive self-sharpening of a wheel is provided and its high cutting ability is maintained. For diamond grinding this condition results in the increased wear of a wheel that, actually, and predetermines low efficiency of application of diamond wheels at high-performance grinding and inexpediency of their use instead of usual abrasive wheels at removal of big allowances.

The received extreme dependence $Q - t$ defines kinematic conditions of essential increase of productivity the processings consisting in realization of new ratios between parameters $t$ and $H_{max}$ ($t < H_{max}, t > H_{max}$), i.e. in realization of the left and right branches of dependence (fig. 2).

On this basis the new ways of grinding realizing the left branch of dependence are developed $Q - t$. They are based on application of schemes multipass (fig. 3a) and deep (fig. 3c) round external grinding with rather high speed of the preparation close to speed of a wheel; schemes of deep round external grinding by the wheel periphery with rather small speed of preparation and big longitudinal giving (fig. 3e); schemes of deep round external grinding with rather small speed of preparation and additional tangential high-frequency movements of a wheel of big amplitude (fig. 3b); schemes of deep flat face grinding with use of additional high-frequency oscillating motions of a wheel or preparation in the direction, perpendicular to the direction of giving of a wheel (fig. 3d).

It is established that efficiency of grinding in this case is caused by passing of grains almost "a trace in a trace" and possibility of increase $H_{max}$ at the fixed value $S$ (i.e. the loading operating on grain) that allows message processing with a high speed of a wheel $V_c$ – to 600 m/s and above.
Application of such conditions will provide increase in productivity of processing by 10 times and more that will well be coordinated with experience of leading machine-tool constructing firms which came for creation of grinders with a speed of wheel up to 300 m/s. Implementation of the offered schemes of grinding assumes development of the new machines providing big speeds of a wheel and preparation. It will allow to change the content of grinding operations cardinally.

REFERENCES