MV-ALGEBRAS AND BL-ALGEBRAS

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ABSTRACT. This material is studying MV-algebras and BL-algebras in terms of applicability in Boole.Algebrele Boole algebra expanded use in fuzzy logic.

MV-algebras

Definition *1*
An MV algebra is an algebra $\Re(A,\oplus,*,0)$ *of type (2, 1,0)* which satisfies the following equation*:*

$(MV_1)\, x\oplus(y\oplus z)=(x\oplus y)\oplus z$

$(MV_2)\, x\oplus y=y\oplus x;$

$(MV_3)\, x\oplus 0=x;$

$(MV_4)\, x^{**}=x;$

$(MV_5)\, x\oplus 0^{*}=0^{*};$

$(MV_6)\, (x^{*}\oplus y)^{*}\oplus y=(y^{*}\oplus x)^{*}\oplus x$

for all x,y,z ⍰ A.

Notice that the axioms MV_1 ⍰MV_3 establish that $(A,\oplus,0)$ abelian monoid is a.

Remark 1 If we put y = 0 MV $_6$ we obtain
$x^{**}=0^{**}\oplus x,$ so, if $0^{**}=0$ then $x^{**}=x$ for all $x\in A$. So, the axiom MV_4
s equivalent to $(MV_4^{'})\ 0^{**}=0$.

To simplify the observations, an MV algebra of type $\Re(A,\oplus,*,0)$ the crowd will refer only to support , A.
n MV algebra is regular if it is supported only by a unit.

In an MV algebra of type A we define a constant and auxiliary operation $\otimes$, $\odot$ and $\to$ as follows:

$1 = 0^*$, $x \odot y = \left(x^* \oplus y^*\right)^*$, $x \otimes y = x \odot y^* = \left(x^* \oplus y\right)^*$, $x \to y = x^* \oplus y$, for all x, y A
.
We consider the operation * more about than any other operation and $\odot$ more about / more connected than $\oplus$ and a.

Remark 2 Let A be an MV algebra of type $\Re\left(A, \oplus, *, 0\right)$ *andi x,y,e* $\in A$ we have*:*

$$(e^{**} \odot x^*) \oplus e \oplus y = (e^* \odot x) \oplus e \oplus y \qquad (1)$$

$$= (e^* \odot x) \oplus (e^{**} \oplus y^{**}) = (e^* \odot x) \oplus (e^* \odot y^*) \qquad (2)$$

$$= e^* \odot (x \oplus y^*) \oplus 1 = e^* \odot (x \oplus y^*) \oplus (e^* \oplus e) \qquad (3)$$

$$= e^* \odot [(x \oplus y^*) \oplus e] = e^* \odot [(x^{**} \odot y^*) \oplus e] \qquad (4)$$

$$= e^* \odot [(x^* \odot y)^* \oplus e] = e \wedge (x^* \odot y) \qquad (5)$$

Customize

Let y= $e \oplus x^*$

$$(e \oplus x^*)^* \oplus e \oplus (e \oplus x^*) = 1 \oplus e = e. \qquad (6)$$

Remark 3 Let A be an MV algebra *x,y,e* $\in A$ we have*:*

$$0=0 \odot (x \oplus y^*) = (e^* \odot e) \odot (x \odot y^*) \qquad (7)$$

$$= (e^* \odot x) \odot e \odot y^* = (e^* \odot x) \odot [o \oplus (e \odot y^*)] \qquad (8)$$

$$=(e^{*}\odot x)\odot[(e\odot e^{*})\oplus(e\odot y^{*})] \quad (9)$$

$$=(e^{*}\odot x)\odot e\odot e^{*}\oplus[(e^{*}\odot x)\odot(e\odot y^{*})] \quad (10)$$

$$=(e^{*}\odot x)\odot e\odot(e^{*}\oplus y^{*})=e^{*}\odot[(e\odot x)^{*}\oplus(e\odot y)] \quad (11)$$

Theorem 4 Let A be an MV algebra and $x,y \in A$ $,e \in B(A)$ then we have*:*

$$(e\odot x^{*})^{*}\oplus e\oplus y=0 \quad \textit{(12)}$$

$\Pr oof$.

Be solved using Remark 4.

Corollary 5 Let A be an MV algebra and x,y ⍰ A $,e \in B(A)$.*If* x=y=$e \in B(A)$ relationship to Theorem 5 is true.

Proof.

Suppose it were true, then

$$(e\odot e^{*})^{*}\oplus e\oplus y=0 \Leftrightarrow 0^{*}\oplus e\oplus e=0 \Leftrightarrow 1\oplus e\oplus e=0 \quad (13)$$

$$\Leftrightarrow 1\oplus e=0 \Leftrightarrow 1=0 \quad (14)$$

Contradiction.

Remark 6 Let A be an MV algebra $x,y,e \in A$ we have*:*

$$e\odot(x^{*}\vee y)=e\odot(x^{*}\odot y^{*})\oplus y=e\odot(x\oplus y)\oplus y \quad (15)$$

$$=(e\odot x)\oplus(e\odot y)\oplus y=(e\odot x)\oplus(e\oplus y)\odot(y\oplus y).$$

Remark 7 Let A be an MV algebra $x,y,e \in A$ we have:

$$e\odot[(e\odot x)^{*}\vee(e\odot y)]=e\odot\{[((e\odot x)^{*}]^{*}\odot[(e\odot y)^{*}\oplus(e\odot x)^{*}] \qquad (16)$$

$$=e\odot(e\odot x)\odot(e\odot y\odot e\odot x) \qquad (17)$$

$$=\mathrm{e}\odot e\odot x\odot e\odot e\odot y\odot x=e\odot x\odot e\odot y\odot x=e\odot x^{2}\odot y. \qquad (18)$$

Theorem 8 Let A be an MV algebra $x,y,e \in A$ $,e \in B(A)$ we have:

$$(e\odot x)\oplus(e\oplus y)\odot(y\oplus y)=e\odot x^{2}\odot y. \qquad (19)$$

Proof.
result from Remarca 7 and Remarca 8.
Corollary 9 Let A be an MV algebra $x,y,e \in A$ putting $x=y=e$.
In Theorem 9 it remains true.
Proof.
evaluate

$$(\mathrm{e}\odot e)\oplus(e\oplus e)\odot(e\oplus e)=e\oplus e\odot e=e\odot e=e. \qquad (20)$$

The other member of the equality of Theorem 9 is

$$\mathrm{e}\odot e^{2}\oplus e=e\odot e\oplus e=e\oplus e=e. \qquad (21)$$

Corollary 10 Let A be an MV algebra $x,e \in B(A)$,$x=e$ *si* $y \in A$ *then*

$$(e\odot y)\oplus y\oplus y=e\oplus y. \qquad (22)$$

Proof.

(e $\odot e)\oplus(e\odot y)\odot(y\oplus y)=e\oplus(e\odot y)\odot(y\oplus y)=e\oplus e\odot(e\odot y)\oplus y\oplus y=$ (23)

e $\odot(e\odot y)\oplus y\oplus y=(e\odot y)\oplus y\oplus y.$ (24)

So ,

e $\odot e^2\oplus y=e\odot e\oplus y=e\oplus y.$ (25)

Theorem 11 Let A be an MV algebra , $e\in B(A)$,$x,y\in A$

$\Leftrightarrow e\odot(x^*\oplus y)=e\odot[(e\odot x)^*\oplus(e\odot y)]$ *(26)*

Proof.Show that

$x^*\oplus y=(x\odot z)^*\oplus(y\odot z)$, $\forall z\in A$ (27)

I think z $\leq x^*\oplus y\Leftrightarrow x\odot z\odot(x^*\oplus y)\leq y\odot z$ (28)

$x\odot z\odot x^*\oplus(z\odot y)\leq y\odot z\Leftrightarrow 0\oplus z\odot y\leq y\odot z$ (29)

$z\odot y\leq y\odot z$ (30)

On the other hand, we have

(e $\odot x)\odot[(e\odot x)^*\vee(e\odot y)]\leq e\odot y\leq y\Leftrightarrow$ (31)

e $\odot[(e\odot x)^*\vee(e\odot y)]\leq x^*\vee y.$ (32)

Corollary 12 Let A be an MV algebra ,$e\in B(A)$ *si y=e* then check Theorem 12.

Proof.

$e\odot(e^*\oplus e)=e\odot[(e\odot e)^*\oplus(e\odot e)].$ (33)

Corollary 13 Let A be an MV algebra ,$e\in B(A)$ *si* $e^*\oplus e=1.$

If y=e then check Theorem 12.

Proof.

$e\odot(e^*\oplus e)=e\odot 1=e.$ (34)

On the other hand, we have

$e\odot[(e\odot e)^*\oplus(e\odot e)]=e\odot 1=e.$ (35)

Theorem 14 Let A be an BL algebra . If $e\in B(A)$ *then*

[$e\rightarrow(e\wedge e^*)]^*\rightarrow(e\wedge e^*)=e\wedge e^*$ *(36)*

Proof.

If L $\in B(A) \quad \Rightarrow L^* \to L = L$

putting L=e $\to (e \wedge e^*)$ achieve the desired result following Remark 14.

Remark 15 et A be an BL algebra *.If* $e \in B(A)$ *then*

$$e \to (e \wedge e^*) = (e \to e) \wedge (e \to e^*) \qquad (37)$$

Proof.

Let y= $e \wedge e^*$ (38)

$$\begin{aligned} y \leq e &\Leftrightarrow e \to y \leq e \to e \\ y \leq e^* &\Leftrightarrow e \to y \leq e \to e^* \end{aligned} \qquad (39)$$

It follows that

$$e \to y \leq (e \to e) \wedge (e \to e^*) \qquad (40)$$

On the other hand

$$(e \to e) \wedge (e \to e^*) \leq e \to y. \qquad (41)$$

show that e $\to e = 1$

We know that e $\to e \leq 1$. Show that

$$1 \leq e \to e \Leftrightarrow e \to 1 \leq e \Leftrightarrow e \leq e. \qquad (42)$$

Show that

$$e^* \to e = e \Leftrightarrow e \to e^* = e^* \Leftrightarrow (e^* \to e) \to e = 1. \qquad (43)$$

If

$$e \in B(A) \Rightarrow 1 = e \vee e^* \leq [(e \to e^*) \to e^*] \wedge [(e^* \to e) \to e] \leq 1 \qquad (44)$$

It follows that

$$(e \to e^*) \to e^* = 1 \quad \Leftrightarrow (e \to e^*) \leq e^* \qquad (45)$$

and

$$(e^* \to e) \to e = 1 \Leftrightarrow e^* \to e \leq e. \qquad (46)$$

Also as e $^*\odot e \leq$ e 👎 si e ⊙ e 👎 ✧e It follows that

e $^* \leq (e \to e^*)$ (47)

and

$e \le e^{*} \to e.$ (48)

So $e^{*} = (e \to e^{*})$ and $e = e^{*} \to e.$

Corollary 16 If A is a BL algebra and $e \in B(A)$ *then*

$e^{*} = e \wedge e^{\text{👎}} \cdot$ *(49)*

Theorem 17

Let A be an algebra and BL $e \in B(A)$ *then*

$[e \to (e \wedge e^{*})]^{*} = (e \to e) \wedge (e \to e^{*})$ *(50)*

Proof.

If $L \in B(A) \Rightarrow L^2 = L.$ Putting $L = e \to (e \wedge e^{*})$ and considering the first part of Theorem 15 we obtain the desired result.

CONCLUSIONS

The material has great resonance in Fuzzy systems used in robotics.

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