THE INFLUENCE OF TECHNICAL DEVIATIONS OVER EFFORTS FROM A 4R SPHERICAL QUADRILATERAL MECHANISM

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Abstract: In the present paper is mechanically defined the notion of technical deviation (geometrical) through a matrix of pluckerian coordinates and those influence over the efforts from the 4R spherical quadrilateral mechanism elements.

Keywords: elastic calculation, cardan transmission.

1. EXPRESSING TECHNICAL DEVIATIONS (GEOMETRICAL) USING THE PLUCKERIAN COORDINATES

It is considered the angle bar ABCD (see Figure 1.) made of the sections AB, BC, CD. As presented in papers [9], [11], the sections AB, BC, CD correspond to the flexibility matrices from the OXYZ reference system, noted with \( H_{AB}, H_{BC}, H_{CD} \) and their calculation program being presented in papers [3], [5], [12]. The flexibility, rigidity matrices \( H_{AB}, H_{BC}, H_{CD} \) are given [11] by:

\[
\begin{bmatrix}
H_{AB} & H_{BC} & H_{CD}
\end{bmatrix} \quad \text{(1)}
\]

Under the influence of the efforts \( \vec{F}_A, \vec{F}_D \) from the end of the angled bar, efforts given in pluckerian coordinates to the OXYZ reference system and being in balance:

\[
\vec{F}_A = \vec{F}_D \quad \text{(2)}
\]

The bar is deforming and the normal sections from A and D reach the position \( A', D' \). These displacements are small and defined by the low rotation vectors \( \vec{\theta}_A, \vec{\theta}_D \) and by the small displacements \( \vec{\delta}_A = AA'; \vec{\delta}_D = DD' \) of points A and D. In the local reference systems with the abscise along the bars AB, CD and with the other axes along the other main central inertia axes of the sections, the displacements are assigned the column matrixes of the pluckerian coordinates

\[
\begin{bmatrix}
\delta_A_y, \delta_A_x, \delta_A_z
\end{bmatrix} \quad \text{and}
\begin{bmatrix}
\delta_D_y, \delta_D_x, \delta_D_z
\end{bmatrix} \quad \text{(3)}
\]

If there are used the notations \( \mathbf{R}_{AB}, \mathbf{R}_{CD} \) for the rotation matrixes of the local reference system towards the system OXYZ, \( \mathbf{C}_A, Y_A, Z_A \), \( \mathbf{C}_D, Y_D, Z_D \), for the coordinates of points A, D; \( \mathbf{T}_{AB}, \mathbf{T}_{CD} \) for the translation matrixes.

\[
\begin{bmatrix}
0 & Z_A & Y_A \\
Z_A & 0 & -X_A \\
-Y_A & X_A & 0
\end{bmatrix} \quad \text{and}
\begin{bmatrix}
0 & -Z_D & Y_D \\
Z_D & 0 & -X_D \\
-Y_D & X_D & 0
\end{bmatrix} \quad \text{(4)}
\]
\[ \begin{bmatrix} I_{AB}^- \\ I_{CD}^- \end{bmatrix}, \] for the position matrices

\[ \begin{bmatrix} \frac{2}{3} I_{AB}^- \\ \frac{2}{3} I_{AB}^- - I_{AB}^- \end{bmatrix}; \quad \begin{bmatrix} \frac{2}{3} I_{CD}^- \\ \frac{2}{3} I_{CD}^- - I_{CD}^- \end{bmatrix} \quad (5) \]

then the displacements of the sections from A, D expressed in the OXYZ system, are:

\[ \begin{bmatrix} A_{4a} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (6) \]

and between efforts and displacements [9][11], with the notation \( A_{4d} \), \( A_{4a} \), \( A_{4d} \), there are the relations

\[ \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (7) \]

If the angled bar has technological deviations (geometrical) then those can be defined as a relative displacement:

\[ \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (8) \]

For example, in A there is a displaced fixed rotation cinematic couple and the bar has no deviations, then \( A_{4a} \), \( A_{4b} \), and results:

\[ \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (9) \]

If the displacements are on sections, then they are first expressed in the local reference systems and then based on the relation (6) in the general system then, in the end, they are summed:

\[ \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (10) \]

In the case where technical deviations exist, the relations (7) become:

\[ \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (11) \]

It is considered the 4R spherical quadrilateral mechanism (see Figure 2.) with the axes OA, OB, OC, OD concurrent in O and also considered that the parts AB, BC, CD have geometrical deviations and the bearings A, D are displaced with the displacements \( A_{4a} \), \( A_{4b} \).

The deformation of part AB, indexed with the number 1 is defined by the displacement of the section noted with \( A_{4b} \) and then:

\[ \begin{bmatrix} A_{4a} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (12) \]

The deformation of part BC, indexed with number 2 is \( A_{4b} \) and the deformation of part CD, indexed with 3 is \( A_{4b} \), and to that is added the displacement \( A_{4d} \) of the bearing, so results:

\[ \begin{bmatrix} A_{4b} \\ \frac{2}{3} I_{AB}^- - A_{4d} \end{bmatrix}; \quad \begin{bmatrix} A_{4d} \\ \frac{2}{3} I_{CD}^- - A_{4d} \end{bmatrix} \quad (13) \]
2. EFFORTS CALCULATION

In the elastic calculation it is considered that the joint from A is blocked (see Figure 3.). If both the elements and the cinematic couples are isolated, a formal diagram is obtained by taking into account balance condition (2) of every isolated bar. From the cinematic couples balance results:

\[
\mathbf{R}_B \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} \mathbf{R}_C \begin{bmatrix} \frac{3}{3} \end{bmatrix} \mathbf{R}_D.
\]

If from the second relation (11) written for every isolated element, the equations are obtained:

\[
A_{ab} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{ab} \begin{bmatrix} \frac{2}{3} \end{bmatrix} ; \quad A_{ac} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{ac} \begin{bmatrix} \frac{2}{3} \end{bmatrix} ; \quad A_{bc} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{bc} \begin{bmatrix} \frac{2}{3} \end{bmatrix} ; \quad A_{bc} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{bc} \begin{bmatrix} \frac{2}{3} \end{bmatrix}.
\]

Notating with \( \xi_B, \xi_C, \xi_D \) the displacements from the cinematic couples, with \( \mathbf{q}_b, \mathbf{q}_c, \mathbf{q}_d \) the pluckerian column matrices attached to [11]. the cinematic couples, and by assigning the indices 1,2,3 to the elements AB, BC, CD, the relations can be written:

\[
A_{ab} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{ab} \begin{bmatrix} \frac{2}{3} \end{bmatrix} ; \quad A_{ac} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{ac} \begin{bmatrix} \frac{2}{3} \end{bmatrix} ; \quad A_{bc} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{bc} \begin{bmatrix} \frac{2}{3} \end{bmatrix} ; \quad A_{bc} \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix} A_{bc} \begin{bmatrix} \frac{2}{3} \end{bmatrix}.
\]

by summing the relations (15) and by using the notations:

\[
\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} ; \quad [D_{ab} D_{ac} D_{bc}] ; \quad [D_{ab} D_{ac} D_{bc}].
\]

the equation is obtained

\[
R_B \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} R_C \begin{bmatrix} \frac{3}{3} \end{bmatrix} R_D.
\]

The column matrix \( \mathbf{d} \) is defined [11]. by the relation

\[
\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\]

The matrices \( \mathbf{d}_b, \mathbf{d}_c, \mathbf{d}_d \) are fulfilling the conditions [11].

\[
\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad [d_{ab} d_{ac} d_{bc}] ; \quad [d_{ab} d_{ac} d_{bc}].
\]

and by taking into account [14]. and the notation

\[
\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
\]

The equations (21) is combined in the equation matrix

\[
[\mathbf{d} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
\]

From the conditions (20) is deduced the unknown \( \mathbf{d} \) and then from (19) the reaction \( R_B \) will be determined, according to relation (14) that is equal with the efforts from the elements AB, BC, CD.

If we consider the form of the matrices \( \mathbf{k}_{ad} \), \( \mathbf{f}_{ad} \) like:
and it is noticed that the matrixes $\tilde{F}_{AD}$ have the form

$$\tilde{F}_{AD} = \begin{bmatrix} \tilde{F}_{11} & \tilde{F}_{12} \\ \tilde{F}_{21} & \tilde{F}_{22} \end{bmatrix}$$

Then, by using the notations

$$A = \frac{1}{2} \begin{bmatrix} \tilde{\omega}_{X} & \tilde{\omega}_{Y} & \tilde{\omega}_{Z} \\ \tilde{\omega}_{1} & \tilde{\omega}_{2} & \tilde{\omega}_{3} \end{bmatrix}$$

$$R_{A} = \frac{1}{3} \begin{bmatrix} F_{AX}^{0} & F_{AY}^{0} & M_{AX}^{0} + M_{AY}^{0} \\ F_{AZ}^{0} & F_{AZ}^{0} & M_{AZ}^{0} \end{bmatrix}$$

are obtained [5],[6]. The results

$$\tilde{F}_{11} = \begin{bmatrix} \tilde{F}_{11} \tilde{F}_{12} \\ \tilde{F}_{21} \tilde{F}_{22} \end{bmatrix}$$

$$A_{R} = \begin{bmatrix} \tilde{\omega}_{1} & \tilde{\omega}_{2} & \tilde{\omega}_{3} \end{bmatrix}$$

$$R_{A} = \begin{bmatrix} F_{AX}^{0} & F_{AY}^{0} & M_{AX}^{0} + M_{AY}^{0} \\ F_{AZ}^{0} & F_{AZ}^{0} & M_{AZ}^{0} \end{bmatrix}$$

3. CONCLUSIONS

By using the pluckerian coordinates and the method of relative displacements [11], it is possible to obtain final analytical results for calculating the influence of technical deviations over the efforts from the 4R spherical mechanism triple statically undetermined.

The second relation (29) is about the components expressed in the fixed reference system $O_{A}X_{A}Y_{A}Z_{A}$, that are null.

If the expression of the components it's made in a local reference system with the origin in point A, then the moments components (real components) are different then zero.

The reaction force given by the relation (30), depends only on the linear displacement of the technological deviation expressed in the $O_{A}X_{A}Y_{A}Z_{A}$ reference system.

The way that different deviations of the sections or bracket displacement influence the efforts from the parts, can be shown by numerical simulations, thing that will be discus in the next paper.

REFERENCES

1. BULAC, I., Vibrations of the bicardanic transmissions with elastic supports, 5th „Durability and Reliability of Mechanical Systems”, Târgu-Jiu, Romania, 18-19 may 2012.
2. BULAC, I., PANDREA, N., Vibrations of the tricardanic transmissions with elastic supports, SISOM 2012, Bucharest, 30-31 may.

5. BULAC, I., The numerical study of efforts from a RRRR spherical mechanism (în curs de publicare), Annals of the Oradea University, 2013.

6. BULAC, I., The influence of elasticity of some sections over the efforts from the 4R spherical spatial mechanism, (în curs de publicare), Annals of the Oradea University, 2013.


