

# BERNOULLI QUARTIC GENERATION BY AN ORIGINAL MECHANISM

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***Abstract:** The paper is based on a geometrical method for generating Bernoulli Quartic and design a mechanism that can generate it. The found mechanism is R-RTT-RTR - Triad of order 3. The analytical equations are obtained through the outlines method and the desired curve is obtained using a computer program. The curve is generated and the functioning of the generating mechanism is analyzed on the basis of some diagrams.*

**Keywords:** Bernoulli quartic, 2D curves, mechanisms generating curves

## 1. INTRODUCTION

2D curves can be generated by graphical methods, with computer, either using their equations or with mechanisms. Over time these curves were drawn with traditional drawing tools, but many mathematicians have built mechanisms in the form of models, to check if one can get the desired curves. Are known in this type of work Cebâșev [2], Reuleaux [5] and others. Artobolevskii [1] has created several mechanisms generating curves from geometric considerations. In [4] are given some original mechanisms that generate 2D curves. In [3] are given many 2D and 3D curves with their equations and geometric considerations. From here we started this work, building a mechanism that generates the Bernoulli quartic.

## 2. INITIAL DATA

Bernoulli quartic (fig. 1) was studied in 1687 by Jacques Bernoulli and Leibniz in 1696.

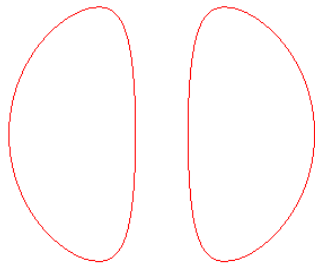


Fig.1 Bernoulli quartic [3]

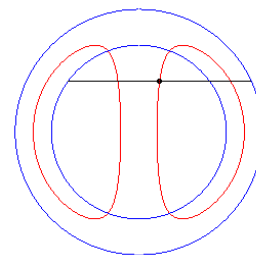


Fig.2 Geometric data [3]

In [3] it is given this curve and it's Cartesian equation:

$$x^2y^2 = \left[ \left( \frac{a+b}{2} \right)^2 - y^2 \right] \left[ y^2 - \left( \frac{a-b}{2} \right)^2 \right] \quad (1)$$

indicating also the geometric considerations in fig.2.

### 3. SYNTHESIS OF THE GENERATOR MECHANISM

Geometrically, a line moving with the ends on two concentric circles (fig. 2), is always parallel to itself so that the middle point of this line describes a branch of Bernoulli quartic.

For the synthesis of the mechanism was started from the following considerations:

- the straight line has variable length, so it takes two slides on the C and B (fig. 3);
- the straight line has a circular translational motion, so it is necessary a slide in D, providing vertical movement of the straight line, parallel to the previous position;
- the slides of C and B are required rotational joints to ensure rotation AB and AC elements, with C and B on the concentric circles;
- point G should be set at the middle of the variable segment BC, so adding BECG kinematic chain.

### 4. THE MECHANISM ANALYSIS

The mechanism is quite complex, with the structural scheme of fig. 4. It consists (fig. 5) of driving element 1, dyad DFBB, type RTT, dyad ACC, type RTR and triad of order 3, CEBG. So the mechanism is type: R-RTT-RTR-Triad of order 3.

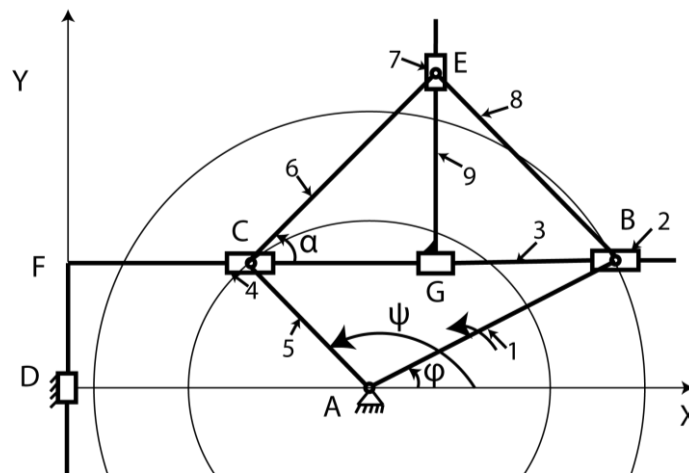


Fig. 3. The achieved mechanism

The kinematic groups of the mechanism are shown in fig.5.

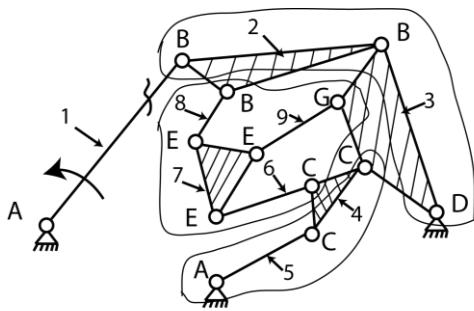


Fig. 4. The structural scheme

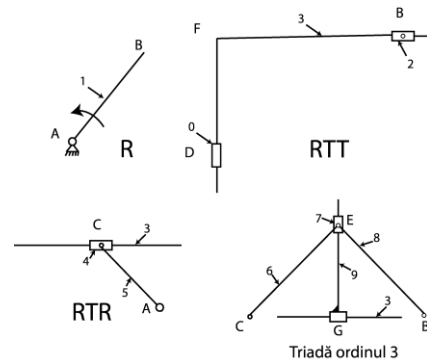


Fig. 5. The kinematic groups

The following relations are written:

$$x_B = x_A + AB \cos \varphi \quad (2)$$

$$y_B = y_A + AB \sin \varphi \quad (3)$$

$$x_D + FB = x_B \Rightarrow FB \quad (4)$$

$$y_F = y_B \quad (5)$$

$$y_C = y_B = DF = y_A + AC \sin \psi \Rightarrow \psi \quad (6)$$

$$x_C = x_A + AC \cos \psi \quad (7)$$

$$CB = x_B - x_C \quad (8)$$

$$EB^2 = EC^2 + CB^2 - 2EC \cdot CB \cos \alpha \quad (9)$$

$$x_E = x_C + EC \cos \alpha \quad (10)$$

$$EG = EC \sin \alpha \quad (11)$$

$$CG = EC \cos \alpha \quad (12)$$

$$x_G = x_C + CG \quad (13)$$

$$y_G = y_B \quad (14)$$

$$y_E = y_B + EG \quad (15)$$

$$EB = EC \quad (16)$$

## 5. THE OBTAINED RESULTS

In fig. 6 it is shown the outcome mechanism (in one position) for the following initial data :  $XA=75$ ;  $AB=68$ ;  $AC=42$ ;  $FG=92$ ;  $BE=63$ ;  $EC=65$ ;  $S=-1$ ;  $S1=1$ .  $S$  and  $S1$  are the signs of the square roots that appear to solve the above system.

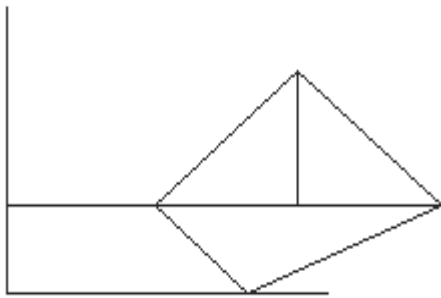


Fig. 6. The mechanism in one position

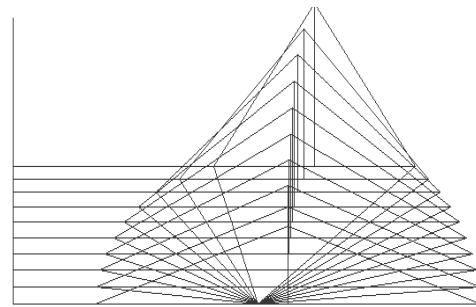


Fig. 7. Successive positions

Because the mechanism moves only in a narrow field, two lengths have changed the values:  $AB=50$ ,  $AC=130$ , resulting the successive positions of the mechanism in fig. 7.

It appears that the mechanism is well built since G is always in the middle of variable length BC. However, it also noted the following:

- The obtained curve (Fig. 8) is not the required one;
- $AC > AB$  unlike the indication in fig.2.

By attempts, resulted fig. 9 at  $AB = 50$ ,  $AC = 69$ , achieving a branch of the curve similar to the desired curve.

For  $AB = 55$ ,  $AC = 69$  was obtained the curve in fig. 10, which is precisely the required curve.



Fig. 8. Initial curve

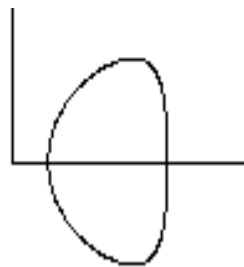


Fig. 9. An alternative of the curve

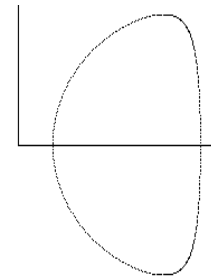


Fig. 10. The desired curve

Successive positions of the mechanism for the curve in fig. 10, are given in fig. 11. More in detail the successive positions are shown in fig. 12 ( $\varphi=0\dots90$ ), fig. 13 ( $\varphi=90\dots180$ ), fig. 14 ( $\varphi=180\dots270$ ), fig. 15 ( $\varphi=270\dots 360$ ).

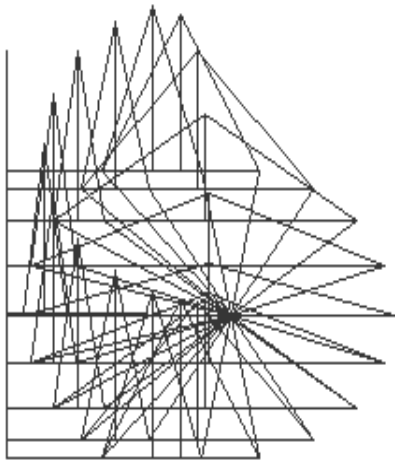


Fig. 11 Successive positions for the desired curve

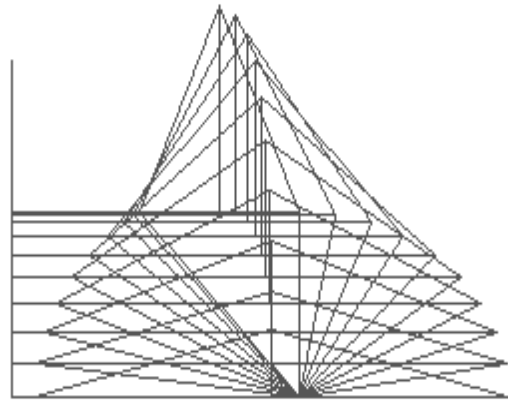


Fig. 12. Successive positions in quadrant I

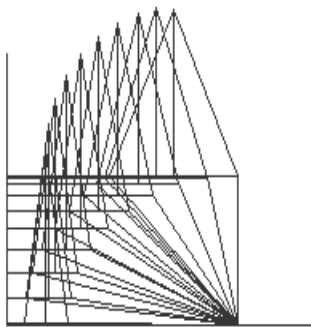


Fig. 13. Successive positions in quadrant II

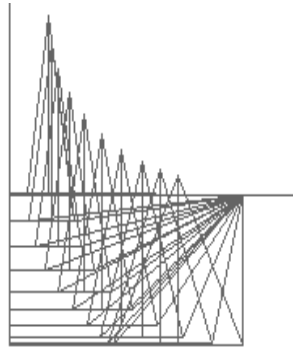


Fig. 14. Successive positions in quadrant III

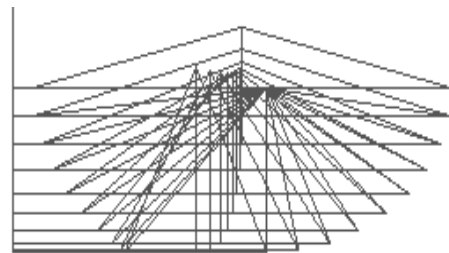


Fig. 15. Successive positions in quadrant IV

For the curve in fig. 10, the data sliders CG and BG variations are given in fig. 16, observing that they are equal, so they are on the same curve in the diagram, ie confirming that the mechanism ensures that point G is in the middle of the variable segment BC.

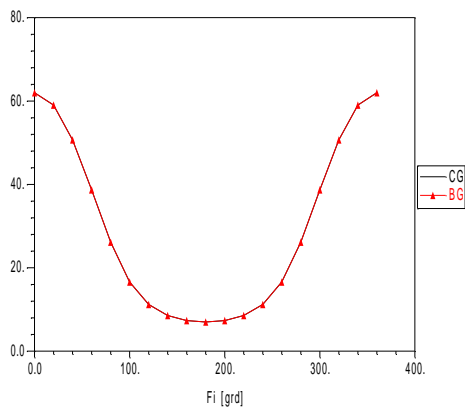


Fig. 16. Identical curves CG and BG

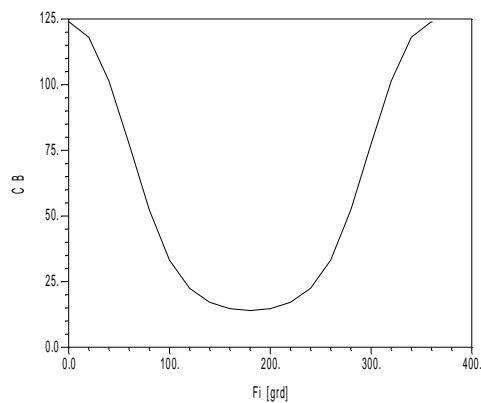


Fig. 17. Curve CB

The change of distance CB is given in fig. 17, similar to fig. 16, but for some other values. It's noticed that CB decreases, reaches a minimum and then increases, the curve's branches being symmetrical.

In fig. 18 are observed the curves for EG and YG quota. YG has sinusoidal variation and ED increases at first, remains approximately constant in a field, then decreases, its branches are symmetrical.

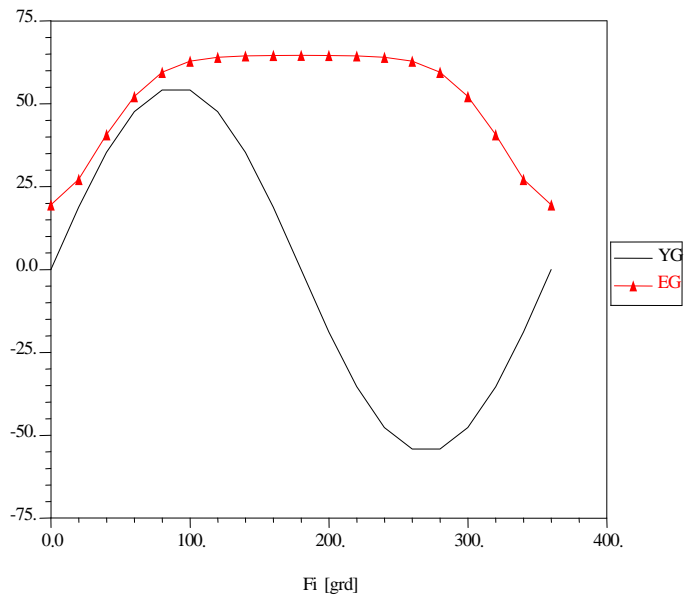


Fig. 18. Curves YG and EG

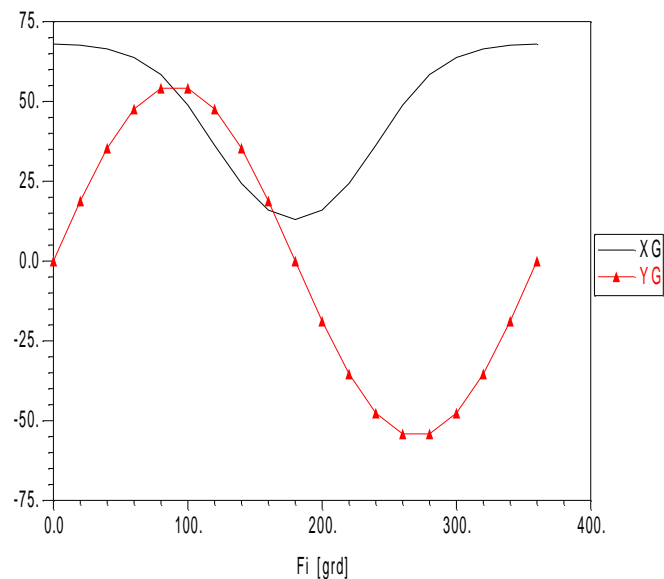


Fig. 19. Curves XG and YG

In fig. 19 are shown the coordinates variations of the tracer point, G. YG and XG sine and cosine varies. Curves are continuous, so the mechanism does not break the cycle.

## 6. CONCLUSIONS

- It's started from geometric considerations and it was obtained the generating mechanism;
- the lengths of some kinematic elements were adjusted by attempts, up to the required curve obtained;
- it was made a structural analysis of the mechanism, observing that it has two dyads and a triad;
- the required curve was obtained;
- the movement of the mechanism was studied, based on the curves of variable sliders and successive positions (in the quadrants).

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