SOME CALCULUS RELATIONS FOR DETERMINING SOME OF THE MECHANICAL PROPERTIES OF COMPOSITE SANDWICH BARS REINFORCED WITH METAL FABRIC

[1]**Assistant Phd. Eng. Cosmin-Mihai MIRIȚOIU,** University of Craiova, Faculty of Mechanics, Department of Vehicles, Transports and Industrial Engineering, Calea Bucuresti Street, no. 107, Craiova, Code 200512, Romania, miritoiucosmin@yahoo.com

Abstract. I*n this paper, starting from some composite samples being in free vibration, I establish a way to determine the damping factor and the eigenfrequency for the first eigenmode. Using the experimental obtained data and the regression analysis, I will determine some calculus relations for the stiffness, eigenfrequency and the damping factor depending on the bars thickness.*

Keywords: sandwich bars, eigenfrequency, vibration, stiffness

Contents:

1. INTRODUCTION

Usually, the innovation in structural engineering is connected with the development of new materials. The most used materials in mechanical or civil engineering are made from metals and alloys. But nowadays, because of the higher mechanical properties (like less weight, higher breaking strength limit or vibration damping) the plastic materials (especially the ceramics) are replacing the ones made from metals. Also, the engineer can combine more ceramics to obtain materials with higher mechanical properties known as *composite materials*. The composite materials are widely used in aerospace engineering (specially for aircraft or satellites building), automotive engineering (for cars, trucks, and so on) wind energy systems and different civil constructions building (according to Ashby (2002)[1] and Vasiliev (2001)[2] and Vinson (2005)[3]).

Plastic materials are used in the domain of smart materials (the shape memory alloys). So, in Li (2008)[4], there is analyzed the dynamic damping of a magnetorheological elastomer (abbreviated in the paper as MRE). There was studied the influences of test strain amplitude, test frequency and test magnetic field and there was shown that the main source of dynamic damping is the friction between the iron particles and rubber matrix.

In Lopez (2009)[5] there was applied an original approach to produce high-damping composites with high rigidity. There was used a Cu-Al-Ni powder with metallic matrices of pure In, with the obtaining methodology described. The studied composites exhibit very high damping capacities in relatively wide temperature ranges.

In Asare (2012)[6] was investigated the vibration damping behaviour of $BaTiO_3$ ceramics, used as a high damping reinforcement in metal matrix composites. There have been examined the factors that affect the vibration damping behaviour by measuring its low frequency (in the domain between 0,1 and 10 Hz) damping loss factor using dynamic mechanical analysis. At temperatures bellow the Curies temperature, the damping factor increases with the frequency decrease of the imposed vibration. Also the damping factor tends to decrease with the number of vibration cycles imposed.

Ekel'chik (2010)[7] has presented a theory that takes into account the normal stresses and the inertial forces that act in the axial direction. This theory was used to determine the torsional vibrations natural complex frequencies for orthotropic composite cantilever roads. The obtained results are then compared to the ones obtained from the classical theory of rods torsional vibrations, the orthotropic plates vibrations theory and the finite element analysis. There was found that the difference between the classical and refined theories depends on the relations between geometrical sizes of a road and its axial elastic modulus and shear moduli and on the number of torsional vibrations modes.

2. EXPERIMENTAL SETUP

Sandwich composite bar specimens were made with polypropylene honeycomb core reinforced with metallic fabric and epoxy resin (fig. 1). The specimens have the next characteristics: thickness of 10, 15 and 20 mm, the length 400 mm and the width 40 and 50 mm. These were marked as follows: set 1- thickness= 10 mm, width= 40 mm; set 2- thickness= 10 mm, width= 50 mm; set 3- thickness= 15 mm, width= 40 mm; set 4- thickness= 15 mm, width= 50 mm; set 5- thickness= 20 mm, width= 40 mm; set 6- thickness= 20 mm, width= 50 mm. I have embedded the specimen bars in two variants: variant 1 – the embedding length= 100 mm, variant 2 – the embedding length= 50 mm.

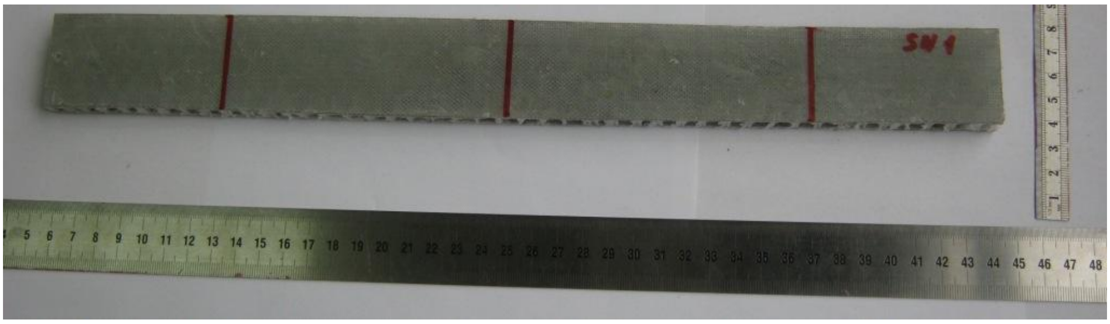

Fig. 1. General view with the composite bar with the thickness of 15 mm and width of 40 mm

A scheme with the experimental setup is shown in fig. 2. Using this experimental setup, I have determined the damping factor and the eigenfrequency of the first eigenmode. In fig. 3 there is shown a graphic with the damping factor and the eigenfrequency for the first eigenmode (set 6 bar, embedding length of 100 mm, the measurement was made in point 1).

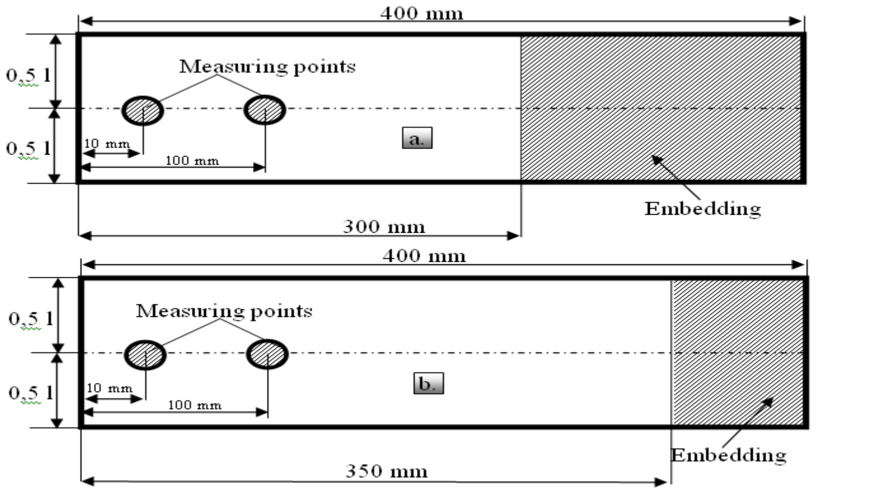

Fig. 2. Experimental scheme; a. variant 1 of embedding, b. variant 2 of embedding

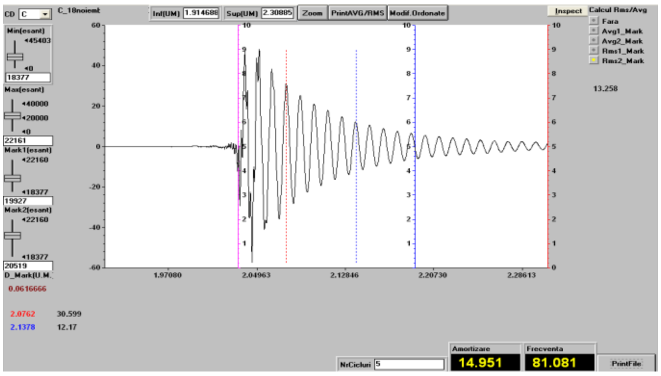

Fig. 3. The damping factor and the eigenfrequency

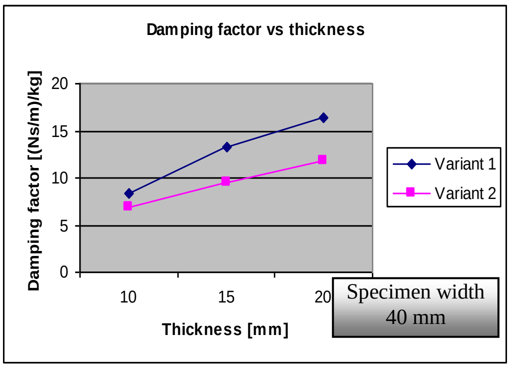

Fig. 4. Damping factor vs. thickness

Because I have not observed the significant differences given by the point of measurement, I have made for each set of bars, the average of values of the damping factor μ (half of the damping factor per unit mass of the bar) for all measurements.

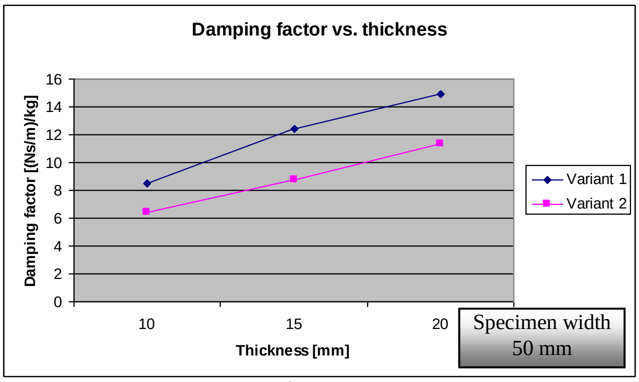

Fig. 5. Damping factor vs. thickness

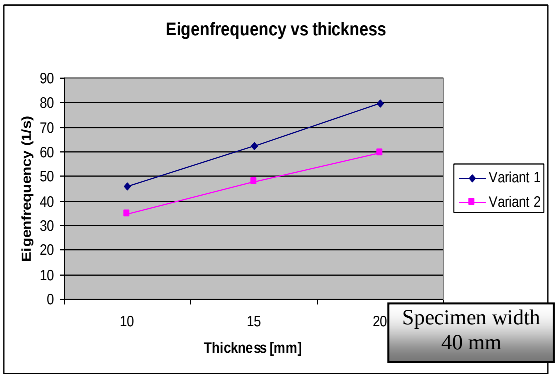

Fig. 6. Eigenfrequency vs. thickness

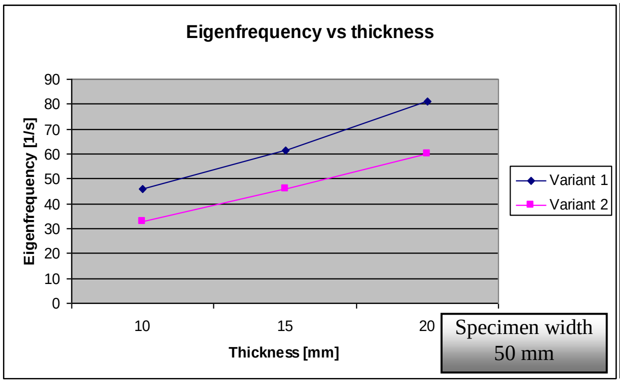

Fig. 7. Eigenfrequency vs. thickness

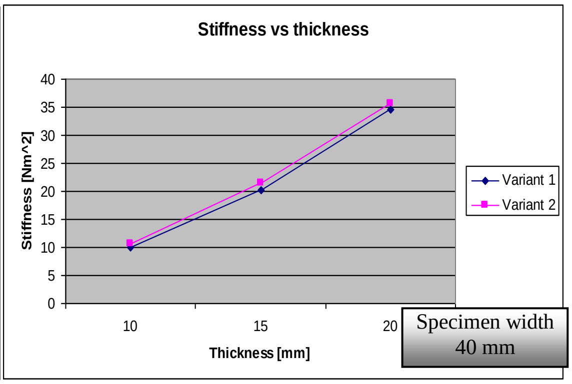

Fig. 8. Stiffness vs. thickness

The correlation between the damping factor and the thickness, for both specimen types, is presented in fig. 4 and 5. The correlation between eigenfrequency and thickness, for both specimens, is presented in fig. 6 and 7. The correlation between the stiffness and thickness is presented in fig. 8. The damping factor and the eigenfrequency were experimentally obtained. The stiffness was obtained using the relation (1).

$$\nu_n = \left(\xi_n / 2\pi l^2\right) / \left(EI^{0,5} / m^{0,5}\right) \quad (1)$$

In (1) I marked with: l – the bar length, EI – the bar stiffness, m – the mass per unit length of the bar, ξ – a factor determined from the supporting conditions of the bar and n is the number of the eigenfrequency. All the results are listed in table 1.

Table 1. Obtained results for the bars

Set	*m* [kg/m]	Variant 1			Variant 2		
		μ [(Ns/m)/kg]	*ν* [1/s]	*EI* [Nm^2]	*μ* [(Ns/m)/kg]	*ν* [1/s]	*EI* [Nm^2]
1	0.185	8.40	45.84	10.06	6.87	34.55	10.58
2	0.236	8.47	45.93	12.88	6.43	32.79	12.16
3	0.201	13.36	62.22	20.13	9.47	47.95	21.41
4	0.251	12.42	61.54	24.59	8.73	46.01	25.36
5	0.210	16.43	79.73	34.54	11.87	59.46	35.58
6	0.272	14.95	80.95	46.11	11.34	59.84	46.68

3. CALCULUS RELATIONS

In the next part of the paper, I aim to find a formula between the damping factor, eigenfrequency, stiffness and the vibration frequency. I have searched for the next formulas:

- linear: $(\nu, \mu, EI)(g) = \alpha_1 \cdot g + \alpha_2$
- logarithmic: $(\nu, \mu, EI)(g) = \beta_1 \cdot \ln(g) + \beta_2$
- power: $(\nu, \mu, EI)(g) = \eta_1 \cdot g^{\eta_2}$
- exponential: $(\nu, \mu, EI)(g) = \lambda_1 \cdot e^{\lambda_2 \cdot g}$

*Important remark: in the relations above, I have marked with **g** the specimen thickness.*

By using the regression analysis and the experimental data, I have obtained the values for α, β, η, λ and the correlation factor R^2. For every parameter, I have written the results in tables 2, 3, 4, 5, 6 and 7.

Table 2 The parameters α, β, η, λ, values for the damping factor μ (specimen width 40 mm)

Function type	Linear	Logarithmic	Power	Exponential	Variant
Parameters	α_1= 4,015 α_2= 4,7	β_1= 7,2927 β_2= 8,3744	η_1= 8,4828 η_2= 0,617	λ_1= 6,2693 λ_2= 0,3354	1
Correlation factor R^2 [%]	98,19	99,98	99,54	95,33	
Parameters	α_1= 2,5 α_2= 4,4033	β_1= 4,465 β_2= 6,7366	η_1= 6,8304 η_2= 0,494	λ_1= 5,31 λ_2= 0,2734	2
Correlation factor R^2 [%]	99,95	98,4	99,75	99	

Table 3 The parameters α, β, η, λ, values for the damping factor μ (specimen width 50 mm)

Function type	Linear	Logarithmic	Power	Exponential	Variant
Parameters	α_1= 3,24 α_2= 5,4667	β_1= 5,8768 β_2= 8,4367	η_1= 8,5196 η_2= 0,521	λ_1= 6,5886 λ_2= 0,2841	1
Correlation factor R^2 [%]	98,42	99,94	99,77	96,13	
Parameters	α_1= 2,455 α_2= 3,9233	β_1= 4,3453 β_2= 6,2381	η_1= 6,3498 η_2= 0,5083	λ_1= 4,8777 λ_2= 0,2837	2
Correlation factor R^2 [%]	99,87	96,56	98,9	99,8	

Table 4 The parameters α, β, η, λ, values for the eigenfrequency ν (specimen width 40 mm)

Function type	Linear	Logarithmic	Power	Exponential	Variant
Parameters	α_1= 16,945 α_2= 28,707	β_1= 30,071 β_2= 44,637	η_1= 45,361 η_2= 0,497	λ_1= 35,093 λ_2= 0,2767	1
Correlation factor R^2 [%]	99,96	97,16	99,19	99,64	
Parameters	α_1= 12,455 α_2= 22,41	β_1= 22,314 β_2= 33,993	η_1= 34,427 η_2= 0,4919	λ_1= 26,836 λ_2= 0,9859	2
Correlation factor R^2 [%]	99,81	98,87	99,9	98,59	

Table 5 The parameters α, β, η, λ, values for the eigenfrequency ν (specimen width 50 mm)

Function type	Linear	Logarithmic	Power	Exponential	Variant
Parameters	α_1= 17,51 α_2= 27,707	β_1= 30,869 β_2= 44,37	η_1= 45,218 η_2= 0,5057	λ_1= 34,703 λ_2= 0,2834	1
Correlation factor R^2 [%]	99,61	95,55	98,29	99,96	
Parameters	α_1= 13,525 α_2= 19,163	β_1= 24,024 β_2= 31,865	η_1= 32,47 η_2= 0,5412	λ_1= 24,582 λ_2= 0,3008	2
Correlation factor R^2 [%]	99,98	97,36	99,4	99,47	

Table 6 The parameters α, β, η, λ, values for the stiffness EI (specimen width 40 mm)

Function type	Linear	Logarithmic	Power	Exponential	Variant
Parameters	α_1= 12,24 α_2= -2,9033	β_1= 21,448 β_2= 8,767	η_1= 9,8572 η_2= 1,1097	λ_1= 5,5701 λ_2= 0,6168	1
Correlation factor R^2 [%]	98,96	93,78	99,39	99,48	
Parameters	α_1= 12,5 α_2= -2,4767	β_1= 21,988 β_2= 9,3909	η_1= 10,428 η_2= 0,9968	λ_1= 5,9619 λ_2= 0,6064	2
Correlation factor R^2 [%]	99,41	94,93	99,68	99,13	

Table 7 The parameters α, β, η, λ, values for the stiffness EI (specimen width 50 mm)

Function type	Linear	Logarithmic	Power	Exponential	Variant
Parameters	α_1= 16,615 α_2= -5,37	β_1= 28,809 β_2= 10,654	η_1= 12,4 η_2= 1,1363	λ_1= 6,8277 λ_2= 0,6377	1
Correlation factor R^2 [%]	97,18	90,17	98	99,99	
Parameters	α_1= 17,26 α_2= -6,4533	β_1= 30,089 β_2= 10,096	η_1= 11,832 η_2= 1,2068	λ_1= 6,3369 λ_2= 0,6726	2
Correlation factor R^2 [%]	98,19	94,93	99,07	99,71	

4. CONCLUSIONS

In this paper I have built some new original composite sandwich bars with the core made of polypropylene honeycomb and the exterior layers reinforced with metallic fabric. Then, for each built specimen, I have determined the damping factor and the eigenfrequency of the first eigenmode. Then, by using relation (1) I have also determined the specimens stiffness. In the third part of the paper, I have tried to find correlations between the eigenfrequency, damping factor, stiffness and the bars thickness using the regression analysis. All these correlations have been listed in tables 2, 3, 4, 5, 6 and 7. From these tables I can extract the following conclusions:

- for the damping factor calculus at specimens embedded at 100 mm, the logarithmic function can be used, but for the specimens embedded at 50 mm the linear function can be used (for these, the correlation factor is above 99,87%);

- for the eigenfrequency calculus, at the samples with 40 mm width, the linear function can be used (for these, the correlation factor is above 99,81%);

- for the eigenfrequency calculus, at the samples with 50 mm width, the exponential function can be used for the specimens embedded in variant 1 and the linear function for the specimens embedded in variant 2;

- for the stiffness calculus, at the samples with 50 mm width, the exponential function can be used (for these, the correlation factor is above 99,71%);

- for the stiffness calculus, at the samples with 40 mm width, the exponential function can be used for the specimens embedded in variant 1 and the power function for the specimens embedded in variant 2.

I have not used the polynomial functions at the regression analysis, because I have taken into account the recommandations from Miriţoiu (2013)[8].

4. ACKNOWLEDGEMENT

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