MATHEMATICAL MODEL FOR DETERMINING KINEMATIC PARAMETERS OF THE CARDAN JOINT MECHANISM WITH TECHNICAL (GEOMETRICAL) DEVIATIONS

Ion BULAC
Doctor, University of Pitești, email: ionbulac57@yahoo.com

Abstract: The cardan joint mechanism is derived from the 4R spherical quadrilateral mechanism and a particular case of spatial quadrilateral mechanism RCCC where by C, R was noted the cylindrical kinematic pair respectively the rotation kinematic pair. In this paper are deducted the calculation relations, is established the mathematical model for determining the kinematic parameters of the cardan joint mechanism with technical (geometrical) deviations.

Keywords: cardan joint, kinematic pair, quadrilateral mechanism, cardan transmission.

1. INTRODUCTION

The technical (geometrical) deviations lead to the change of kinematic parameters of the cardan joint mechanism. This deviations is established in paper [3].

For determining the influence of both angular and axis deviations [1], over the kinematic parameters of the cardan joint mechanism, it is necessary to consider the cardan joint, not as a spherical quadrilateral but as a particular case of RCCC mechanism, where by C, R was noted the cylindrical kinematic pair respectively the rotation kinematic pair.

In this paper are deducted the calculation relations, is established the mathematical model for determining the kinematic parameters of the cardan joint mechanism with technical (geometrical) deviations.

2. THE POSITIONAL ANALYSIS OF THE RCCC MECHANISM

The RCCC mechanism (see Figure 1) is made of four elements noted with 1, 2, 3 and 4, the forth element (the base) being fixed and the elements being connected through the kinematic pairs \( O_1, O_2, O_3, \) and \( O_4 \), the \( O_1 \) being the rotation kinematic pair and \( O_2, O_3, \) and \( O_4 \) being the cylindrical kinematic pairs.

The axes of the kinematic pairs are noted with \( O_i'z_i, i=1,2,... \) and the following perpendiculars are noted with \( O_i'z_{i+1}, i=1,2,3,4 \), point \( O_5 \) being identical with point \( O_1 \).

One notates with \( \sigma_i, \alpha_i, i=1,2,3,4 \) the length of the axes and the angle between them.

So it is chosen a local reference system \( O_ix_iy_iz_i, i=1,2,3,4 \) so that the axes \( O_ix_i \) to be situated on the shared perpendicular of the axes \( O_i'z_i, O_{i+1}'z_{i+1} \). It is noted with \( s_i \) the distances \( O_iO_i' \) and with \( \theta_i \) the angle between the axes \( O_{i-1}x_{i-1}, O_i x_i, i=1,2,3,4 \).
In these conditions, the geometrical parameters \( s_i, \sigma_i, \alpha_i, i = 1,2,3,4 \) being known, the positional analysis for determining \( \theta_2, \theta_3, \theta_4, s_2, s_3, s_4 \) is based on the angle \( \theta_1 \).

From the equation of rotations closing, using the diagram „\( \theta \alpha \)“ [7] and the order 3,4,1 and 2 is obtained the following equation:

\[
A_3(\theta_1)s\theta_4 - B_3(\theta_1)c\theta_4 + C_3(\theta_1) = 0
\]

where:

\[
A_3(\theta_1) = s\alpha_3s\theta_1s\alpha_1
\]

\[
B_3(\theta_1) = s\alpha_3(c\alpha_4c\theta_1s\alpha_1 + s\alpha_4c\alpha)
\]

\[
C_3(\theta_1) = -c\alpha_3s\alpha_4c\theta_1s\alpha_1 + c\alpha_3c\alpha_4c\alpha_1 - c\alpha_2
\]

The trigonometrically functions \( \cos, \sin \) being noted with \( c, s \). Through the conventional derive \( D \) of the relations (1), (2) having as basis the relations [7].

\[
D(c\theta_i) = -s_i s\theta_i; D(s\theta_i) = s_i c\theta_i
\]

\[
D(c\alpha_i) = -\sigma_i s\alpha_i; D(s\alpha_i) = \sigma_i c\alpha
\]

is obtained the equation:

\[
D_3s_4 + F_3s_1 + F_3\sigma_1 + G_3\sigma_2 + H_3\sigma_3 + K_3\sigma_4 = 0
\]

where:

The angle \( \theta_4 \) is determined by solving the equation (1) and through the equation (5) is known the parameter \( s_4 \).
With circular permutations the relations follows:

\[
A_2(\theta_4)s_3\theta_3 - B_2(\theta_4)c_3\theta_3 + C_2(\theta_4) = 0 \quad (6)
\]

\[
A_1(\theta_3)s_2\theta_2 - B_1(\theta_3)c_2\theta_2 + C_1(\theta_3) = 0 \quad (7)
\]

from where are determined, in order, the angles \( \theta_3 \) and \( \theta_2 \) and also the equations:

\[
D_3s_3 + E_3s_4 + F_3\sigma_4 + G_3\sigma_1 + H_3\sigma_2 + K_3\sigma_3 = 0 \quad (8)
\]

\[
D_2s_2 + E_2s_3 + F_2\sigma_3 + G_2\sigma_4 + H_2\sigma_1 + K_2\sigma_2 = 0 \quad (9)
\]

from which are determined the parameters \( s_3, s_2 \).

The expression of the coefficients \( A_i, B_i, C_i, D_i, E_i, F_i, G_i, H_i, K_i \), \( i=3,2,1 \), are given in TABLE 1 from the paper \([2]\).

In the initial position \( \theta_i^0 = 0 \) the expressions are obtained

\[
A_3 = 0, B_3 = s\alpha_3s(\alpha_1 + \alpha_4), C_3 = c\alpha_3c(\alpha_1 + \alpha_4) - c\alpha_2
\]

and it results that:

\[
c\theta_i^0 = \frac{c\alpha_3c(\alpha_1 + \alpha_4) - c\alpha_2}{s\alpha_3s(\alpha_1 + \alpha_4)}
\]

3. THE TECHNICAL (GEOMETRICAL) DEVIATIONS OF CARDAN JOINT

The cardanic joint enables the transmission of the rotation movement from the shaft 3 through the cardanic cross 2.

The cardanic cross is tied to the brackets of the shafts 1 and 2 through the cinematic rotation couples \( A, A' \) and also \( B, B' \).

In the papers \([1], [3]\) is established the technical (geometrical) deviations of cardan joint mechanism.

A kinematic diagram that represents a mechanism with one cardan joint, with all geometrical deviations possible \([1], [3]\), is presented in Figure. 3.
These deviations are small and fulfill the condition:

\[ \alpha_i = \frac{\pi}{2} + \Delta \alpha_i, \quad i = 1, 2, 3, \alpha_4 = \pi - \alpha \]  
\[ \sigma_i = O_iO'_{i+1}, \quad i = 1, 2, \sigma_4 = O_4O'_4 \]  

- The angularly deviation of the main shaft bracket is defined by the parameter \( \Delta \alpha_1 \) and the smoothness deviations for the same bracket is given by the parameter \( \sigma_1 \).
- The angularly deviation of the cardanic cross 2 is given by the parameter \( \Delta \alpha_2 \) and also the deviation from smoothness is given by the parameter \( \sigma_2 \).
- The angularly deviation of the driven shaft bracket 3 is given by the parameter \( \Delta \alpha_3 \) and the smoothness deviation is given by the parameter \( \sigma_3 \).
- The angularly deviation of the driven shaft 3 depending on the driving shaft 1 is given by the parameter \( \sigma_4 \).

As shown in the papers [1], [3] in default of shafts 1 and 3 are known the points \( O_4, O'_1, O_1, O'_2, O_2, O'_3, O_3, O'_4 \) are overlaid with point O (see Figure 3.) and the kinematic cylindrical couples A, B and D (see Figure 1.) become rotation kinematic couples (there are no displacements \( s_2, s_3, s_4 \) along the axes \( Oz_2, Oz_3, Oz_4 \).

The existence of technical deviations conducts to the displacements \( s_i, i = 1, 2, 3, 4 \) and.

In order to determine these displacements it is first necessary to calculate the angularly parameters \( \theta_2, \theta_3, \theta_4 \) variation depending on the angle \( \theta_1 \).

4. THE DETERMINING PARAMETERS \( \theta_2, \theta_3, \theta_4 \)

The angularly parameters \( \theta_2, \theta_3, \theta_4 \) is determined from the system of equations

\[ A_i s \theta_{i+1} - B_i c \theta_{i+1} + C_i = 0, \quad i = 1, 2, 3. \]  

(13)
To this purpose, one uses the Newton method \[10\], \[11\], and with the notations

\[
\begin{bmatrix}
\theta_2 \\
\theta_3 \\
\theta_4 
\end{bmatrix}
= \begin{bmatrix}
\Delta \theta_2 \\
\Delta \theta_3 \\
\Delta \theta_4 
\end{bmatrix}
\]

\(\Psi_i = A_i s \theta_{i+1} - B_i c \theta_{i+1} + C_i, i = 1,2,3.\)  

\[
\begin{bmatrix}
\Psi_1 \\
\Psi_2 \\
\Psi_3 
\end{bmatrix}
= \begin{bmatrix}
A_1^* s \alpha_3 c \theta_3 \\
B_1^* = -s \alpha_4 c a_2 s \theta_3 s \alpha_3 \\
C_1^* = c a_1 s a_2 s \theta_3 s \alpha_2 \\
A_2^* = s a_2 s a_4 c \theta_4 \\
B_2^* = -s a_2 c a_3 s \theta_4 s \alpha_4 \\
C_2^* = c a_2 s a_3 s \theta_4 s \alpha_4 
\end{bmatrix}
\]

\[
\begin{bmatrix}
A_1 c \theta_2 + B_1 s \theta_2 \\
A_2^* s \theta_2 - B_1^* c \theta_2 + C_1^* \\
0 \\
A_2 c \theta_3 + B_2 s \theta_3 \\
A_2^* s \theta_3 - B_2^* c \theta_3 + C_2^* \\
0 \\
A_2 c \theta_4 + B_3 s \theta_4
\end{bmatrix}
\]

is obtained the matrix equation

\[
\Delta \theta = [J]^{-1} \{\Psi\}
\]

from which results the variation \(\Delta \theta\) for the known values of angles \(\theta_1, \theta_2, \theta_3, \theta_4\).

After the determining the angularly parameters \(\theta_2, \theta_3, \theta_4\) variation depending on the angle \(\theta_1\), is determined the parameters \(s_2, s_3, s_4\) with the relations

\[
s_4 = -\frac{1}{D_3} (E_3 s_1 + F_3 \sigma_1 + G_3 \sigma_2 + H_3 \sigma_3 + K_3 \sigma_4)
\]

\[
s_3 = -\frac{1}{D_2} (E_2 s_4 + F_2 \sigma_4 + G_2 \sigma_1 + H_2 \sigma_2 + K_2 \sigma_3)
\]

\[
s_2 = -\frac{1}{D_1} (E_1 s_3 + F_1 \sigma_3 + G_1 \sigma_4 + H_1 \sigma_1 + K_1 \sigma_2)
\]

where the coefficients \(E_i, F_i, G_i, H_i, K_i, i = 1,2,3\). are calculated from the TABLE 1. from the paper [2].
5. CONCLUSIONS
The mathematical model presented in this paper, makes possible the study of technological deviations influence over the kinematic parameters of the cardan joint mechanism. Using this mathematical model, made possible the elaboration an calculation algorithm that by numerical simulation can highlight the influence of different technical deviations that appear in the mechanism elements.

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