INFLUENCE OF THE SUSPENSION PARAMETERS UPON THE HUNTING MOVEMENT STABILITY OF THE RAILWAY VEHICLES

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Abstract: The hunting movement of the railway vehicles is essential for the safety, the comfort of the passengers and the integrity of the transported goods. The dynamic performance of a railway vehicle can be seriously damaged, due to the phenomenon of instability of the hunting motion that occurs when a certain speed is exceeded – the critical one. An important role is played by the suspension parameters in the extension of the stability interval during hunting motion at high velocities. And this is what the paper draws attention to, based on the analysis of how the elasticity and damping suspension characteristics influence the critical speed.

Keywords: railway vehicle, hunting, critical speed, suspension

1. INTRODUCTION

The hunting motion is specific for railway vehicles, as a result of the fact that the rolling surfaces of the wheels have a reversed conicity, provided that they are rigidly fixed on the axle. The hunting of the wheelsets is conveyed via the suspension elements to the entire vehicle, thus generating vibrations that can affect the safety, comfort of the passengers or the integrity of the transported goods [1, 2].

What is particular about the hunting movement is the dependence of its nature on the velocity, whether it is stable or unstable. For relatively low velocities, the vehicle has a stable hunting motion. Its movement occurs in the vicinity of the balance position and is maintained by the geometrical irregularities of the track. Starting from a certain speed, called critical speed, the hunting movement of the vehicle becomes unstable. The loss of stability is manifest in the increase of the amplitude in the movements of wheelsets, which will reach to hit the interior flanges of the rails. From a practical perspective, the phenomenon of the instability in the hunting motion is essential, since it restricts the maximum velocity of the vehicles.

The extension of the stability range of the hunting depends on a series of parameters of the vehicle– the equivalent conicity of the wheel rolling surface, the wheelset mass and inertia momentum, the bogie axle base, the bogie inertia momentum [3, 4, 5]. An important role is also played by the vehicle suspension by its stiffness and damping characteristics [5, 6, 7].

This paper focuses on the influence of stiffness and damping on the transversal and longitudinal direction of the suspension upon the stability of the hunting motion during the vehicle running on the alignment.
To this purpose, the vehicle is modelled via a 21-degree-of-freedom system, which includes all its basic components – the carbody, the suspended masses of the bogies and the wheelsets, as well as the elastic and damping elements of the two levels of suspension.

Solving the issue of stability is based on the analysis of the system’s matrix eigenvalues, comprising the movement equations describing the free vibrations of the vehicle and resulting into the establishment of the critical speed. The possibilities of an increase in the vehicle’s critical speed is pointed out at, via a better choice of the suspension parameters.

2. THE VEHICLE MECHANICAL MODEL AND MOVEMENT EQUATIONS

The mechanical model to study the horizontal-transversal vibrations of a 4-axle vehicle, two levels of suspension, travelling at a constant speed $V$ on a tangent track is shown in figures 1 and 2. This contains 7 rigid bodies representing the carbody, the suspended masses of the bogies and the 4 wheelsets, connected among them through Kelvin-Voigt systems, by which the two levels of suspension are modelled.

The vehicle carbody is assimilated to a three-degree-of-freedom rigid, with the following movements: lateral displacement $y_c$, rolling $\phi_c$ and yaw $\alpha_c$. This is supported in four points by the elements of the secondary suspension that have the transversal basis $2b_c$. It is about the secondary suspension’s springs of the vehicles, which can deform themselves in three directions and whose rigidities are noted with $k_{xc}$, $k_{yc}$ and $k_{zc}$. Every bogie has a system with an anti-rolling torsion bar, whose stiffness is $k_{\phi c}$. In the vertical direction, the secondary suspension of a bogie has two dampers with the damping constant $c_{yc}$, and in the horizontal direction, there is one damper whose damping constant is $c_{yc}$. Similarly, there will be considered the antiyawing dampers that are mounted on the lateral sides of the bogies, with the damping constant $c_{xc}$. The following carbody parameters are to be mentioned: mass $m_c$ and the inertia moments around the longitudinal axis $J_{xc}$ and the vertical axis $J_{zc}$, respectively. The height of the carbody centre of mass compared to the secondary suspension plan is $h_c$, and the vehicle axlebase is $2a_c$.

The bogie carriage (the bogie suspended mass) is also considered a three-degree-of-freedom rigid, namely lateral displacement $y_{bi}$, rolling $\phi_{bi}$ and yaw $\alpha_{bi}$, with $i = 1$ or 2, as it regards the front or the rear bogie. This carriage supports the elements of the secondary suspension and is resting in four points on the wheelsets, via the elements of the primary suspension. The main parameters of the bogie carriage are its mass $m_b$, the inertia moment in respect to the longitudinal axis $J_{xb}$, and the inertia moment in respect to the vertical axis $J_{zb}$. The bogie mass center is located at the distance of $h_{bi}$ from the primary suspension plan and at $h_{b2}$ from the secondary suspension plan. It should be mentioned that the bogie has the axlebase of $2a_b$. 
Fig. 1. The mechanical model of the vehicle – frontal view.

Fig. 2. The mechanical model of the vehicle – top view.
The elements of the primary suspension also operate after three directions and they have the rigidities \( k_{xb}, k_{yb}, k_{jy} \) and the damping constants \( c_{xb}, c_{yb} \) and \( c_{jy} \). The Kelvin-Voigt systems placed in the plan of the axles, at the distance \( h_{b1} \) from the mass center of the bogie suspended side, model their elastic driving. The transversal basis of the primary suspension is \( 2b_{y} \).

In terms of the wheelsets, they are considered to be able to perform the following independent movements: one movement of lateral translation, \( y_{oj,(j+1)} \), and a rotation movement – yaw, \( \alpha_{oj,(j+1)} \), where \( j = 2i - 1 \), with \( i = 1 \) or \( 2 \), mentioning that the bogie \( i \) has the axles \( j \) and \( j + 1 \). Similarly, the wheelset makes a rotation movement around its own axis, at the angle speed \( \Omega_{oj,(j+1)} = V/r_{o} + \omega_{oj,(j+1)} \), where \( \omega_{oj,(j+1)} \) is the angle sliding velocity of the axle compared to \( V/r_{o} \), and \( r_{o} \) is the radius of the rolling circle when the axle takes the median position on the track. The following parameters will be considered: mass \( m_{o} \) and the inertia moments \( J_{yo} \) and \( J_{zo} \).

The movement equations of the vehicle are as below:

- the carbody movement equations (lateral displacement, rolling, yaw)

\[
m_{c}\ddot{y}_{c} + c_{yc}[2(\dot{y}_{c} + h_{c}\dot{\phi}_{c}) - (\dot{y}_{b1} + \dot{y}_{b2}) + h_{b2}(\dot{\phi}_{b1} + \dot{\phi}_{b2})] + \\
+ 2k_{yc}[2(y_{c} + h_{c}\phi_{c}) - (y_{b1} + y_{b2}) + h_{b2}(\phi_{b1} + \phi_{b2})] = 0. \tag{1}
\]

\[
J_{xc}\ddot{\phi}_{c} + 2c_{xc}h_{c}^{2}[2\dot{\phi}_{c} - (\dot{\phi}_{b1} + \dot{\phi}_{b2})] + c_{yc}[2(\dot{y}_{c} + h_{c}\dot{\phi}_{c}) - (\dot{y}_{b1} + \dot{y}_{b2}) + h_{b2}(\dot{\phi}_{b1} + \dot{\phi}_{b2})] + \\
+ (k_{qc} + 2k_{xc}h_{c}^{2})[2\phi_{c} - (\phi_{b1} + \phi_{b2})] + 2k_{yc}[2(y_{c} + h_{c}\phi_{c}) - (y_{b1} + y_{b2}) + h_{b2}(\phi_{b1} + \phi_{b2})] - \\
- m_{c}g_{c}\phi_{c} = 0 \tag{2}
\]

\[
J_{zc}\ddot{\alpha}_{c} + 2c_{xc}h_{c}^{2}[2\dot{\alpha}_{c} - (\dot{\alpha}_{b1} + \dot{\alpha}_{b2})] + c_{yc}a_{c}[2a_{c}\dot{\alpha}_{c} - (\dot{y}_{b1} - \dot{y}_{b2}) + h_{b2}(\dot{\phi}_{b1} - \dot{\phi}_{b2})] + \\
+ 2k_{xc}h_{c}^{2}[2\alpha_{c} - (\alpha_{b1} + \alpha_{b2})] + 2k_{yc}a_{c}[2a_{c}\alpha_{c} - (y_{b1} - y_{b2}) + h_{b2}(\phi_{b1} - \phi_{b2})] = 0. \tag{3}
\]

- the bogie movement equations (lateral displacement, rolling, yaw), for \( i = 1, 2 \) and \( j = 2i - 1 \)

\[
m_{b}\ddot{y}_{bi} + c_{yc}(\dot{y}_{bi} - h_{b2}\dot{\phi}_{b1} - \dot{y}_{c} - h_{c}\dot{\phi}_{c} - (-1)^{j+1}a_{c}\alpha_{c}) + 2c_{yb}[2(\dot{y}_{bi} + h_{b1}\dot{\phi}_{bi}) - (\dot{\phi}_{oj} + \dot{\phi}_{o(j+1)})] + \\
+ 2k_{yc}(\dot{y}_{bi} - h_{b2}\phi_{b1} - y_{c} - h_{c}\phi_{c} - (-1)^{j+1}a_{c}\alpha_{c}) + 2k_{yb}[2(y_{bi} + h_{b1}\phi_{bi}) - (y_{oj} + y_{o(j+1)})] = 0. \tag{4}
\]
\[ J_{xb} \ddot{\phi}_{bi} + 2c_{zc} b_c^2 (\dot{\phi}_{bi} - \dot{\phi}_c) + c_{xc} b_2 (h_{b2} \dot{\phi}_{bi} - \dot{y}_c + h_c \dot{\phi}_c + (-1)^{i+1} a_c \alpha_c) + \\
2c_{yb} h_{b1} \left[ 2(h_{b1} \dot{\phi}_{bi} + \dot{y}_c) - (\ddot{y}_{oj} + \dot{y}_{o(j+1)}) \right] + 4c_{zc} b_c^2 \dot{\phi}_{bi} + (k_{qc} + 2k_{zc} b_c^2) (\dot{\phi}_{bi} - \dot{\phi}_c) + \\
2k_{yc} h_{b2} (h_{b2} \dot{\phi}_{bi} - y_c + h_c \dot{\phi}_c + (-1)^{i+1} a_c \alpha_c) + 2k_{yb} h_{b1} \left[ 2(h_{b1} \dot{\phi}_{bi} + y_c) - (\ddot{y}_{oj} + \dot{y}_{o(j+1)}) \right] + \\
+ \left[ 4k_{zb} b_c^2 - g \left( h_{12} \frac{m_c}{2} + h_{b1} m_b \right) \right] \dot{\phi}_{bi} = 0 \]  

(5)

\[ J_{zb} \ddot{\alpha}_{bi} + 2c_{xc} b_c^2 (\dot{\alpha}_{bi} - \dot{\alpha}_c) + 2c_{xb} b_c^2 [2 \ddot{\alpha}_{bi} - (\dot{\alpha}_{oj} + \dot{\alpha}_{o(j+1)})] + \\
+ 2c_{yb} a_b [2a_b \dot{\alpha}_{bi} - (\ddot{y}_{oj} - \ddot{y}_{o(j+1)})] + 2k_{xc} b_c^2 (\alpha_{bi} - \alpha_c) + \\
+ 2k_{xb} b_c^2 [2 \alpha_{bi} - (\alpha_{oj} + \alpha_{o(j+1)})] + 2k_{yb} a_b [2a_b \alpha_{bi} - (\ddot{y}_{oj} - \ddot{y}_{o(j+1)})] = 0. \]  

(6)

- the axle movement equations (lateral displacement, yaw, rotation around its own axis), for 
\( i = 1, 2 \) and \( j = 2i - 1 \)

\[ m_o \ddot{y}_{oj,(j+1)} + 2c_{yb} [\dddot{y}_{oj,(j+1)} - \dot{y}_c + h_{b1} \dot{\phi}_{bi} \mp a_b \dot{\alpha}_{bi} ] + \\
+ 2k_{yb} [\dddot{y}_{o(j+1)} - y_c + h_{b1} \dot{\phi}_{bi} \mp a_b \dot{\alpha}_{bi}] = Y_1 + Y_2 \]  

(7)

\[ J_{zo} \ddot{\alpha}_{oj,(j+1)} + 2c_{xb} b_c^2 (\dot{\alpha}_{oj,(j+1)} - \dot{\alpha}_{bi}) + 2k_{xb} b_c^2 (\alpha_{oj,(j+1)} - \alpha_{bi}) + \\
+ J_{yo} \frac{V}{r_o} \dot{\phi}_{oj} = -e_o (X_{j,(j+1)1} - X_{j,(j+1)2}). \]  

(8)

\[ J_{yo} \ddot{\alpha}_{oj,(j+1)} = -r_o (X_{j,(j+1)1} + X_{j,(j+1)2}). \]  

(9)

where \( X_{j,(j+1)1,2} \) represent the longitudinal forces, and \( Y_{j,(j+1)1,2} \) – the guiding forces acting upon the axle \( j \) and \( (j+1) \) respectively, at the contact points of the wheels 1 or 2 with the rails.

Based on the analysis of the system matrix eigenvalues derived from the equations (1 - 9) turned into first-order differential equations, a quality-based evaluation can be done for the stability in the vehicle movement, with the calculation of the critical speed during hunting as a result.
3. NUMERICAL APPLICATION

This section deals with the influence of the elasticity and damping characteristics in the longitudinal and transversal direction of the suspension upon the critical speed while hunting. During the numerical simulations, the parameters have been appropriately adapted to a Y 32R bogie-fitted passenger railway vehicle, with a maximum velocity of 200 km/h. For the reference values of the vehicle, the critical speed of 261.56 km/h has been calculated.

![Fig. 3. Influence of the secondary suspension upon the critical speed: (a) influence of the elasticity characteristics; (b) influence of the damping characteristics.](image)

![Fig. 4. Influence of the primary suspension upon the critical speed: (a) influence of the elasticity characteristics; (b) influence of the damping characteristics.](image)

Figure 3 shows the critical speeds calculated for values of the secondary suspension rigidities within the range of $10^4$ N/m and $10^6$ N/m and dampings between $10^3$ Ns/m and $10^5$ Ns/m.
In diagram (a), the high value of the critical speed is visible ($374.89 \text{ km/h}$) for high rigidities of the secondary suspension ($k_{xc} = k_{yc} = 10^6 \text{ N/m}$). The increase of the critical speed compared to the reference value is still possible should only one of the two suspension parameters be modified. While the reference value ($k_{xc} = 1.7 \cdot 10^5 \text{ N/m}$) is maintained in the transversal direction and the value of $10^6 \text{ N/m}$ is adopted for $k_{xc}$, the result is the critical speed of 302.34 km/h. Upon considering the reference value $k_{xc} = 1.7 \cdot 10^5 \text{ N/m}$ and increasing the transversal stiffness of the suspension up to $10^6 \text{ N/m}$, the critical speed of 314.43 km/h is obtained. The diagram (b) shows, on the one hand, the increase of the critical speed during the raising of the secondary suspension damping in the transversal direction. Should the situation corresponding to the damping reference value in the longitudinal direction $c_{xc} = 6.46 \cdot 10^4 \text{ Ns/m}$ be looked at and the transversal damping is increased to $10^5 \text{ Ns/m}$, the critical speed of 319.22 km/h will be the result. On the other hand, the rise in the critical speed can be also obtained by the increase in the secondary suspension damping in the longitudinal direction. Thus, if the reference value of $c_{yc} = 1.52 \cdot 10^4 \text{ Ns/m}$ is maintained and only the suspension longitudinal damping is increased ($c_{xc} = 10^5 \text{ Ns/m}$), the critical speed can be raised up to 269 km/h. Finally, should $c_{xc} = c_{yc} = 10^5 \text{ Ns/m}$ be adopted, the critical speed can be 320 km/h.

The diagram (a) in figure 4 shows the results concerning the critical speed for various values of the primary suspension rigidities within the interval of ($10^5$ ... $10^8$) N/m, and in diagram (b) the critical speed is calculated for dampings that can have values from $10^3$ Ns/m to $10^5$ Ns/m. The diagram (a) points out at a series of interesting aspects related to the way in which the stiffness in the longitudinal and transversal directions of the primary suspension affects the value of the critical speed. It is about the fact that an increase in the longitudinal stiffness leads to a higher critical speed, but this only occurs for certain values of the stiffness in the transversal direction. Should this range be exceeded, an opposite effect will result, i.e. a decrease in the critical speed. Further, the diagram (b) of the figure 4 shows that a slightly less increase of the critical speed is possible by lowering the damping in the primary suspension. As an example, for $c_{xb} = c_{yb} = 10^3 \text{ Ns/m}$, the critical speed rises from the reference value of 261 km/h to 269 km/h.

4. CONCLUSIONS

The dynamic performance of the railway vehicles, defined in terms of safety, comfort of the passenger and the integrity of the transported merchandise, can be seriously damaged, due to the phenomenon of instability of the hunting motion that occurs at higher speed than the critical one. The stability interval for the hunting can be extended at high velocities by adopting certain parameters of the vehicle, among which the stiffness and the suspension damping are essential.
This paper examines the possibilities to increase the critical speed by adopting the best values of the vehicle suspension parameters. It has been proved that an increase of the stiffness and damping in the longitudinal and transversal directions of the secondary suspension, the critical speed of the vehicle can be higher. Likewise, it is demonstrated that higher critical speeds can result from the correlation of the values of longitudinal and transversal stiffness of the primary suspension. A less increase in the critical speed turns into a lower damping in the primary suspension of the vehicle.

REFERENCES