EXTENSION OF AN ADDITIVE FUNCTIONS NUMARABLE

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ABSTRACT: All measures to start the construction of theories from user a fundamental theorem of a. Lebesgue whose contemporary formulations is due to c. Caratheodory.

NORMED ALGEBRAS
Lebesgue Theorem Caratheodory-Demonstration.

Either \( \xi \circ AB \sigma \) complete. We consider subalgebra \( \xi_0 \) with a quasimasura \( \phi \). We will now undertake the following tasks: construction of a subalgebra \( \sigma - \) regular \( \tilde{\xi} \) that includes \( \xi_0 \) and a quasimasura additive numarabila \( \phi \) on \( \tilde{\xi} \) so \( \phi(x) \leq \phi(x) \) , where it is possible, \( \phi(x) = \phi(x) \) for any \( x \in \xi_0 \). In order to accomplish the task, we will define an outer quasimasura \( \phi^* \) by associating with each \( x \in \xi \) the crowd \( S_x \) that is all most families numarabile \( \sigma \subset \xi_0 \) satisfying the condition \( \sup \sigma \geq x \) , and by:

\[
\phi^* = \inf_{\sigma \in \xi_0} \sum_{\alpha \in \sigma} \phi(3).
\]

The Demonstration. Either \( \xi \) a AB \( \sigma \) complete. We consider subalgebra \( \xi_0 \) with a quasimasura \( \phi \). We will now undertake the following tasks: construction of a subalgebra \( \sigma - \) regular \( \tilde{\xi} \) that includes \( \xi_0 \) and a quasimasura additive numarabila \( \phi \) on \( \tilde{\xi} \) so \( \phi(x) \leq \phi(x) \) , where possible, \( \phi(x) = \phi(x) \) for any \( x \in \xi_0 \). In order to accomplish the task, we will define an outer quasimasura \( \phi^* \) by associating with each \( x \in \xi \) the crowd \( S_x \) that is all most families numarabile \( \sigma \subset \xi_0 \) satisfying the condition \( \sup \sigma \geq x \) , and by:

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\phi^* = \inf_{\sigma \in \xi_0} \sum_{\alpha \in \sigma} \phi(3).
\]

3. If \( x = \bigvee_{k} k \) then \( \phi^*(x) \leq \sum_k \phi^*(x_k) \).
Considering that the measure is totally additive, $x \downarrow \Phi$. Indeed, involve establishing a chance $\varepsilon > 0$, associated with each $x_k$ family $\sigma_k \in S_k$ so that the inequality

$$\sum_{3=\sigma_k} \phi(3) \leq \phi^*(x) + \frac{\varepsilon}{2}$$

remains. Clearly, gender equality $\sigma = \bigcup_k \sigma_k$ belongs to $S_k$; and, that's why

$$\phi^*(x) \leq \sum_{3=\sigma_k} \phi(3) = \sum_{k} \sum_{3=\sigma_k} \phi(3) \leq \sum_{k} \phi^*(x_k) + \varepsilon.$$ 

Because $\varepsilon$ is random, we obtain the inequality. Now we arrange the $x \in \xi$ so equality

$$\phi^*(u \wedge x) + \phi^*(u \wedge C_x) = \phi^*(u)$$

keep for any $u \in \xi$. It is clear that always $\phi^*(u \wedge x) + \phi^*(u \wedge C_x) \geq \phi^*(u)$, so we no longer stays than to validate the seeming inequality backside.

Lemma 2. Either $Z_n \in \xi(n = 1, 2, \ldots)$ si be the sequence $\{Z_n\}$ What tends to repeat $z$. Then

$$\phi^*(u \wedge z) \leq \lim_{n \to \infty} \phi^*(u \wedge z_n) \text{ for any } u \in \xi.$$ 

Demonstration. Everything is obvious $\{z_n\}$ decreases. Suppose that $z_n \uparrow z$ and $z_1 = \emptyset$. Then $\xi$ complete all the steps, admits metric spaces $\{\xi, \mu\}$ are homeomorfe to each other.

$$\phi^*(u \wedge z) \leq \sum_{n=1}^{\infty} \phi^*[u \wedge (z_{n+1} - z_n)] = \sum_{n=1}^{\infty} \phi^*[u \wedge C_{z_n}]$$

$$= \sum_{n=1}^{\infty} \phi^*[u \wedge z_{n+1}] - \phi^*[u \wedge z_n]$$

$$= \lim_{n \to \infty} \phi^*(u \wedge z_n).$$ 

Lemma is proved.

Lemma 3. The Crowd $\tilde{\xi}$ is a subalgebra $\sigma$ on regular basis.

Demonstration. In condition of the main (4) items $x$ si $C_x$ have equal status, so $\tilde{\xi}$ contains each element together with the complementarul or. Is Yes $x, y \in \tilde{\xi}$ and we have $z = x \wedge y$ cu $u \in \xi$. We will have

$$\phi^*(u) = \phi^*(u \wedge x) + \phi^*(u \wedge C_x)$$

$$= \phi^*(u \wedge x \wedge y) + \phi^*(u \wedge x \wedge C_y)$$

$$+ \phi^*(u \wedge C_x \wedge y) + \phi^*(u \wedge C_x \wedge C_y)$$

$$\geq \phi^*(u \wedge x \wedge y)$$

$$+ \phi^*[u \wedge C_x \wedge y] \vee (u \wedge C_x \wedge C_y) \vee (u \wedge C_x \wedge C_y)]$$

$$= \phi^*(u \wedge z) + \phi^*(u \wedge C_z),$$
Since \((u \land x \land C_z) \lor (u \land C_x \land z) \lor (u \land C_x \land C_z)\)  
\(= u \land [(x \land C_z) \lor (C_x \land z) \lor (C_x \land C_z)]\)  
\(= u \land C (x \land z) = u \land C_z.\)

Thus \(z \in \xi\). Note that \(\xi\) is the subalgebra. We remain to show that \(\xi\) is \(\sigma\) on a regular basis. Either \(\{x_1, x_2, \ldots, x_n, \ldots\}\) a

numarabila and submultime \(x = \left(\bigvee_{i=m}^{m+k} y_i\right)\)

\(\mu \left(\bigvee_{i=m}^{m+k} y_i\right) \leq \sum_{i=m}^{m+k} \mu y_i \leq \sum_{i=m}^{n} \mu y_i.\)

Considering that so far as is (a) - continue:

In the previous Lemma,

\(\phi^*(u \land x) \leq \lim_{n} \phi^*(u \land y_n),\)

\(\phi^*(u \land C_x) \lim_{n} \phi^*(u \land C_{y_n})\)

Hence, \(\phi^*(u \land x) + \phi^*(u \land C_x) \leq \phi^*(u).\)

Thus, \(x \in \xi\). It has been demonstrated that \(\xi\) is a subalgebra \(\sigma\) on regular basis.

**Lemma 4** \(\xi_0 \subset \xi\)

Demonstration. Either \(x \in \xi_0\). Choose an item \(u\) and a correspondent by chance \(\sigma \in S_u\).

Families of elements of the form \(x \land z\ (z \in \sigma)\) and \(C_x \land z\) the corresponding elements are \(x \land u\) and that \(C_x \land u.\)

So \(\sum_{z \in \sigma} \phi(z) = \sum_{z \in \sigma} \left(\phi(z \land x) + \phi(z \land C_x)\right)\)

\(= \sum_{z \in \sigma} \phi(z \land x) + \sum_{z \in \sigma} \phi(z \land C_x)\)

\(\geq \phi^*(x \land u) + \phi^*(C_x \land u).\)

Taking into account that \(\sigma\) by chance, we conclude that \(\phi(u) \geq \phi^*(x \land u) + \phi^*(C_x \land u)\)

Thus, \(x \in \xi\). Axiona is demonstrated.

**Lemma 5.** If \(\phi^*(x) = 0\) then \(x \in \xi\).

Demonstration. Se Yes \(u \in \xi\), as a result \(\phi^*(u) \geq \phi^*(x \land u) + \phi^*(C_x \land u)\)

\(x_{n_u} \rightarrow \emptyset.\)
In fact, the coincidence of topology (o) and (os) It is sufficient to refer to Theorem 4

Suppose that $x, y \in \hat{\xi}$ and $x \sim d \sim y$. Atunci

$$\hat{\phi}(x \vee y) = \phi^*(x \vee y)$$

$$= \phi^*[\{(x \vee y) \wedge x\}] + \phi^*[\{(x \vee y) \wedge C_x\}]$$

$$= \phi^*(x) + \phi^*(y) = \hat{\phi}(x) + \hat{\phi}(y).$$

So, see, for example, the equations. So, for any sequence disjuncta $\{x_n\}$ in $\hat{\xi}$, we have

$$\hat{\phi}\left(\bigvee_{n=1}^{\infty} x_n\right) \geq \phi \left(\bigvee_{n=1}^{\infty} x_n\right)$$

$$\left(\bigvee_{n=1}^{m} x_n\right) = \sum_{n=1}^{m} \phi(x_n),$$

and thus:

$$\hat{\phi}\left(\bigvee_{n=1}^{\infty} x_n\right) \geq \sum_{n=1}^{m} \phi(x_n).$$

Considering the inequality:

$$\hat{\phi}\left(\bigvee_{n=1}^{\infty} x_n\right) = \phi^*$$

$$\left(\bigvee_{n=1}^{\infty} x_n\right) \leq \sum_{n=1}^{m} \phi^*(x_n) = \sum_{n=1}^{m} \phi(x_n),$$

which is maintained, we get to equality:

$$0 \leq \mu\left(\lim_{n} y_i\right) = \mu\left(\bigcap_{m=1}^{\infty} \bigvee_{i=m}^{\infty} y_i\right) \leq \inf_{m} \sum_{i=m}^{\infty} \mu y_i = 0 \quad (4)$$

The right side of this inequalities disappears:

They are studying the structure of subalgebrei $\hat{\xi}$ that is defined quasimasura $\hat{\phi}$. Se da un $x \in \hat{\xi}$ Incidentally, we choose the corresponding $\sigma_{n} \in S_{x} \quad (n = 1, 2, ...)$ so
\[ \bar{\phi}(x) = \phi^*(x) \leq \sum_{y \in \sigma_n} \phi(y) < \bar{\phi}(x) + \frac{1}{n} \]

\((n = 1, 2, \ldots)\) to be valid.

Now we will put \( \bar{x} \equiv \bigwedge_n \sup \sigma_n \). It is clear that \( x \leq \bar{x} \) and \( \bar{x} \) belongs to subalgebra \( \xi \langle \xi_0 \rangle \) and also
\[ \bar{\phi}(x) \leq \bar{\phi}(\bar{x}) \leq \bar{\phi}(\sup \sigma_n) \]
\[ \leq \sum_{y \in \sigma_n} \phi(y) < \bar{\phi}(x) + \frac{1}{n}. \]

So, \( \bar{\phi}(x) = \bar{\phi}(\bar{x}), \bar{\phi}(\bar{x} - x) = \phi^*(\bar{x} - x) = 0. \)

Thus, each element \( x \in \xi \) is represented as \( x = \bar{x} - u \),
where \( \bar{x} \in \xi \langle \xi_0 \rangle, \phi^*(u) = 0, \) si \( \bar{\phi}(x) = \bar{\phi}(\bar{x}). \)

Lemma 1, \( [x_n - x_{n+1}] \overset{\to}{\longrightarrow} x \). So, we have

Even more so, since the \( \sigma \) is there a submultim time numarabil \( \sigma_0 \subset \sigma \) numarabila such as:
\[ \bar{\phi}(\bar{x} - \frac{\epsilon}{\bar{x}}) < \bar{\phi}(\sup \sigma_0) < \bar{\phi}(\bar{x}). \]

where:
\[ \bar{\phi}(\bar{x} - \sup \sigma_0) \leq \bar{\phi}(\bar{x}) - \bar{\phi}(\sup \sigma_0) < \frac{\epsilon}{2}. \]

Now we can consider \( \sup \sigma_0 \) a substitute for \( x_0 \). Therefore,
\[ \bar{\phi}(\bar{x} - x_0) \leq \bar{\phi}(\bar{x} - \bar{x}) + \bar{\phi}(\bar{x} - x_0) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \]

Lemma is proved.

Definition: a quasimasa \( v \) a subalgebra \( \mathfrak{A} \subset \xi \). It is called complete as long as the conditions \( x \in \mathfrak{A}, \mathfrak{A} \in \xi, x \in \xi \) and \( v\mathfrak{A} = 0 \) involve \( x \in \mathfrak{A} \) (and obviously \( v\mathfrak{A} = 0 \)).

As well as in Lemma 6, quasimasa \( \bar{\phi} \) I built it is complete.

Lemma 9. Either \( v \) an additive numarabila full quasimasa \( \mathfrak{A} \subset \xi \) - that includes regular \( \xi_0 \). It is assumed that \( \psi(x) \leq \phi(x) \) for any \( x \in \xi_0 \). Then:

1) \( \mathfrak{A} \supset \xi \)

considered as a metric space. What else can we say about the properties in this space? For example, will be connected? The answer is negative, in general, since some lie algebras are normed
listed all finite AB, which are clearly disconnected.
Teorema 4
An ABN is a metric space continue Astfel, \( \psi(x) \leq \phi^*(x) \). Applying this to an element \( x \in \tilde{\xi} \), result \( \psi(x) \leq \tilde{\phi}(x) \). It remains to prove the Lemma enunt first. First we will show that if \( \phi^*(v) = 0 \) then \( v \in \mathcal{S} \) and \( \psi(v) = 0 \). Indeed, as in Lemma 8, There is an element \( \sigma \delta \tilde{v} \geq v \) satisfying \( \phi^*(\tilde{v}) = \tilde{\phi}(\tilde{v}) = 0 \). All items \( \sigma \delta \) belong to him \( \xi \langle \xi_0 \rangle \) and thus his \( \mathcal{S} \). So \( \tilde{v} \in \mathcal{S} \) and \( 0 \leq \psi(\tilde{v}) \leq \phi^*(\tilde{v}) = 0 \).

Quasimasura \( \psi \) It is thus complete; \( v \in \mathcal{S} \) and \( \psi(x) = 0 \). Now it is clear that each \( x \in \tilde{\xi} \) belong to him \( \mathcal{S} \), taking into account that \( x \) can be written as \( x = \tilde{x} - u \), where \( \tilde{x} \) It is an element \( \sigma \delta \) and \( \phi^*(u) = 0 \). So inclusiveness \( \mathcal{S} \supset \tilde{\xi} \) and Juliet are demonstrated. We need. \( \psi \) complete the demonstration only point 1).

Lemma 10. If
\[ \phi(\sup_{\sigma} \sigma) \leq \sum_{y=\sigma} \phi(y) \]
In general: the regularity of the algebra. Thus, each can be considered a ABN space metric. What else can we say about the properties in this space? For example, will be connected? The answer is negative, in general, since some lie algebras are normed listed all finite AB, which are clearly disconnected. Indeed, in this case
\[ \tilde{\phi}(x) = \phi^*(x) \geq \phi(x) \] for any \( x \in \xi_0 \), so the 1 of an external steps leads to the desired equality.

Teorema 6. Either \( \xi \) a AB \( \sigma \) complete, and \( \xi_0 \) a subalgebra of \( \xi \), and \( \phi \) a quasimasura in \( \xi_0 \). Then there is a subalgebra) \( \sigma \) regular \( \tilde{\xi} \) including \( \xi_0 \) and (b)) a quasimasura additive numarabila \( \phi \) in \( \tilde{\xi} \) with the following properties:
1) \( \tilde{\phi}(x) \leq \phi(x) \) for any \( x \in \xi_0 \);
2) for each additive numarabila quasimasura \( \psi \) in \( \tilde{\xi} \) and for any \( x \in \xi_0 \) Image check \( \psi(x) \leq \phi(x), \) the inequality \( \psi(x) \leq \phi(x) \) is true for any \( x \in \tilde{\xi} \).
How \( \phi \) is numarabila then the additive equality
\[ \phi(x) = \tilde{\phi}(x) \]
It is true for any \( x \in \xi_0 \), in other words \( \tilde{\phi} \) It is an extension of \( \phi \).

Call quasimasura \( \phi \) It is built in the proof theorem 6
CONCLUSIONS
The main issue is topical because it approaches the fuzzy systems underlying the artificial intelligence that is implemented in the economic and industrial machines.

REFERENCES
[1] Balbes si Dwinger, The curatours of the University of Missouri, 1974