SOME PROPERTIES OF (α,β)- CUTS FORINTUITIONISTIC FUZZY SETS

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Abstract.*In this article, we give some basic definitionsfrom Intuitionistic fuzzy setstheory;we introducedthe notion of a complete system of eventsin intuitionistic fuzzy sets theory with examples and characterizations.*

Keywords: Intuitionistic fuzzy sets, *(α,β)*-cut, decomposition theorems, and complete system of eventsof a intuitionistic fuzzy sets.

1. INTRODUCTION

Zadeh introduced the notion of the *α cut*of fuzzy set (FS) and decomposition theorems [7],[8],[11]. The concept of intuitionistic fuzzy sets (IFS) was introduced by K.T. Atanassov [1] as a generalization of the notion of a fuzzy set. Starting from the decomposition theoremsfrom Intuitionistic fuzzy sets described in [7],[8],[11], I will try to develop this theoryby showing that IFS not forming acomplete system of events. similar to the classical theory of probabilities.

2.PRELIMINARIES

Definition 1.[11]Let *X* the non- empty set. A fuzzy subset*A* of *X* is

$$A=\left\{\left(x,\mu_A\left(x\right)\right)\middle|x\in X\right\} \quad (1)$$

Definition 2.[1]An intuitionistic fuzzy set (IFS) *A* in *X* is given by

$$A=\left\{\left(x,\mu_A\left(x\right),\nu_A\left(x\right)\right)\middle|x\in X\right\} \quad (2)$$

where $\mu_A : X \to [0,1]$ is called the membership function, $\mu_A\left(x\right)$ is calleddegree of membership, $\nu_A : X \to [0,1]$ is called non-membership, $\nu_A\left(x\right)$ is called degree of non-membership, with the condition

$$0\le\mu_A\left(x\right)+\nu_A\left(x\right)\le 1,\quad \forall x\in X \quad (3)$$

Definition 3.[11]In the case of fuzzy set theory a α-*cut*or a setof levelα, α $\in[0.1]$,of fuzzy set *A* is:

$$[A]^{\alpha}=\begin{cases}\left\{x\in X\,\middle|\,\mu_A\left(x\right)\ge\alpha\right. & \text{if}\quad 0<\alpha\le 1\}\\ \hline \left\{x\in X\,\middle|\,\mu_A\left(x\right)>0\right\} & \text{if}\quad \alpha=0\end{cases} \quad (4)$$

and

$$[A]^{+\alpha}=\begin{cases}\left\{x\in X\,\middle|\,\mu_A\left(x\right)>\alpha\right. & \text{if}\quad 0<\alpha\le 1\}\\ \hline \left\{x\in X\,\middle|\,\mu_A\left(x\right)>0\right\} & \text{if}\quad \alpha=0\end{cases} \quad (5)$$

is called strong *α-cut* as the IFS*A*.

Theorem 1. [7] **First Decomposition Theorem of FS**. For any fuzzy sets A,

$$A = \bigcup_{\alpha \in [0,1]} [A]_{\alpha}$$

Theorem 2. [7] **Second Decomposition Theorem of FS**. For any fuzzy sets A,

$$A = \bigcup_{\alpha \in [0,1]} [A]_{\alpha +}$$

3.(α,β) -CUT AS THE INTUITIONISTIC FUZZY SETS

In the case of IFS we have:

Definition 4.[3]We consider $\alpha,\beta \in [0.1]$ then, for any IFS set A

$$[A]^{(\alpha,\beta)} = \begin{cases} \{x \in X \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta, 0 \leq \mu_A(x) + \nu_A(x) \leq 1\} & \text{if } 0 < \alpha, \beta \leq 1 \\ \overline{\{x \in X \mid \mu_A(x) > 0, \nu_A(x) < 0, 0 \leq \mu_A(x) + \nu_A(x) \leq 1\}} & \text{if } \alpha = 0, \beta = 0 \end{cases} \quad (6)$$

is called (α,β) -cut as the IFSA.

$$[A]^{+(\alpha,\beta)} = \begin{cases} \{x \in X \mid \mu_A(x) > \alpha, \nu_A(x) < \beta, 0 \leq \mu_A(x) + \nu_A(x) \leq 1\} & \text{if } 0 < \alpha, \beta \leq 1 \\ \overline{\{x \in X \mid \mu_A(x) > 0, \nu_A(x) < 0, 0 \leq \mu_A(x) + \nu_A(x) \leq 1\}} & \text{if } \alpha = 0, \beta = 0 \end{cases} \quad (7)$$

is called strong (α,β) -cut as the IFSA.

Definition 5.[6] For $\alpha,\beta \in [0.1]$ with $\alpha + \beta \leq 1$, through$[A]_{(\alpha,\beta)}$ and $[A]_{+(\alpha,\beta)}$we understand

$$[A]_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x \in [A]^{(\alpha,\beta)} \\ (0,1) & \text{if } x \notin [A]^{(\alpha,\beta)} \end{cases} \quad (8)$$

Respectively

$$[A]_{+(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x \in [A]^{+(\alpha,\beta)} \\ (0,1) & \text{if } x \notin [A]^{+(\alpha,\beta)} \end{cases} \quad (9)$$

Definition 6.[3]Given two IFSs A and B over an universe of discourse X, one can define the following relations:

A ⊂ B iff∀x ∈ X, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$

A = B iffA ⊂ B andB ⊂ A

as well as the following operations [1]:

$\overline{A} = \{(x, \nu_A(x), \mu_A(x)) \mid x \in X\}$

A ∩ B $= \{(x, \mu_{A\cap B}(x), \nu_{A\cap B}(x)) \mid x \in X\}$, where

$\mu_{A\cap B} = \min\{\mu_A(x), \mu_B(x)\}$ and

$\nu_{A\cap B} = \max\{\nu_A(x), \nu_B(x)\}$

A ∪ B = $\{(x, \mu_{A\cup B}(x), \nu_{A\cup B}(x)) | x \in X\}$, where

$\mu_{A\cup B} = \max\{\mu_A(x), \mu_B(x)\}$ and

$\nu_{A\cup B} = \min\{\nu_A(x), \nu_B(x)\}$

Theorem 3.[3]Let A and B two IFS and one α,β, γ, δ ∈ [0.1]. Then following are true:

1) $[A]^{+(\alpha,\beta)} \subseteq [A]^{(\alpha,\beta)}$
2) If $\alpha \le \gamma$ and $\beta \le \delta$ then $[A]^{(\gamma,\delta)} \subseteq [A]^{(\alpha,\beta)}$
3) $[A \cup B]^{(\alpha,\beta)} = [A]^{(\alpha,\beta)} \cup [B]^{(\alpha,\beta)} \cup [D_{A,B}]^{(\alpha,\beta)} \cup [D_{B,A}]^{(\alpha,\beta)}$
4) $[A \cap B]^{(\alpha,\beta)} = [A]^{(\alpha,\beta)} \cap [B]^{(\alpha,\beta)}$
5) $[A \cup B]^{+(\alpha,\beta)} = [A]^{+(\alpha,\beta)} \cup [B]^{+(\alpha,\beta)}$
6) $[A \cap B]^{+(\alpha,\beta)} = [A]^{+(\alpha,\beta)} \cap [B]^{+(\alpha,\beta)}$
7) $[A^C]^{(\alpha,\beta)} \neq ([A]^{+(1-\alpha,1-\beta)})^C$

Where

$$[D_{A,B}]^{(\alpha,\beta)} = \begin{cases} \{x \in X \,|\, \mu_A(x) \ge \alpha, \nu_B(x) \le \beta, 0 \le \mu_A(x) + \nu_B(x) \le 1\} & \text{if } 0 < \alpha, \beta \le 1 \\ \hline \{x \in X \,|\, \mu_A(x) > 0, \nu_B(x) > 0, 0 \le \mu_A(x) + \nu_B(x) \le 1\} & \text{if } \alpha = 0, \beta = 0 \end{cases}$$

$$[D_{B,A}]^{(\alpha,\beta)} = \begin{cases} \{x \in X \,|\, \mu_B(x) \ge \alpha, \nu_A(x) \le \beta, 0 \le \mu_B(x) + \nu_A(x) \le 1\} & \text{if } 0 < \alpha, \beta \le 1 \\ \hline \{x \in X \,|\, \mu_B(x) > 0, \nu_A(x) > 0, 0 \le \mu_B(x) + \nu_A(x) \le 1\} & \text{if } \alpha = 0, \beta = 0 \end{cases}$$

Theorem 4.[3] Let *A* and *B* two IFS and one α, β, γ, δ ∈ [0.1]. Then:

1) A ⊆ B if and only if $[A]^{(\alpha,\beta)} \subseteq [B]^{(\alpha,\beta)}$
2) A ⊆ B if and only if $[A]^{+(\alpha,\beta)} \subseteq [B]^{+(\alpha,\beta)}$
3) A = B if and only if $[A]^{(\alpha,\beta)} = [B]^{(\alpha,\beta)}$

4.THE DECOMPOSITION THEOREMS OF INTUITIONISTIC FUZZY SETS

Theorem 5. [6] **First Decomposition Theorem of IFS**. Let *X* the non- empty set. For any intuitionistic fuzzy subset*A* in *X*,

$$A = \bigcup_{\alpha,\beta \in [0,1]} [A]_{(\alpha,\beta)}$$

Where ∪denotes union given in Definition 8 and$[A]_{(\alpha,\beta)}$given inDefinition.

Theorem 6. [6] **Second Decomposition Theorem of IFS**. Let *X* the non- empty set. For any intuitionistic fuzzy subset*A* in *X*,

$$A = \bigcup_{\propto,\beta \in [0,1]} [A]_{+(\alpha,\beta)}$$

Where ∪denotes union given in Definition 8 and$[A]_{(\alpha,\beta)}$given inDefinition .

Theorem 7.Let X the non- empty set, α,β∈[0.1] with $\alpha + \beta \leq 1$. For any intuitionistic fuzzy subset$A$ in X, $[A]_{(\alpha,\beta)}$not form a complete system of events of A.

Proof: The fact that$A = \cup_{\propto,\beta \in [0,1]} [A]_{(\alpha,\beta)}$is demonstrated by [6]..Still need to show that

$$[A]_{(\alpha,\beta)} \cap [\mathrm{A}]_{(\gamma,\delta)} \neq \emptyset, \forall (\alpha,\beta) \neq (\gamma,\delta), \propto,\beta,\gamma,\delta \in [0,1], \alpha+\beta \leq 1, \gamma+\delta \leq 1 \qquad (10)$$

If the absurd relationship were true, then$\exists\ (\alpha,\beta) \neq (\gamma,\delta), \propto,\beta,\gamma,\delta \in [0,1], \alpha+\beta \leq 1, \gamma+\delta \leq 1]$ so that$[A]_{(\alpha,\beta)} \cap [\mathrm{A}]_{(\gamma,\delta)} = \emptyset$. But$[A]_{(\alpha,\beta)} \cap [\mathrm{A}]_{(\gamma,\delta)}$

$$= \left\{ \left(x, \mu_{[A]_{(\alpha,\beta)} \cap [\mathrm{A}]_{(\gamma,\delta)}}(x), \nu_{[A]_{(\alpha,\beta)} \cap [\mathrm{A}]_{(\gamma,\delta)}}(x) \right) \middle| x \in X \right\} =$$

$$\left\{ \left(x, \min\left\{ \mu_{[A]_{(\alpha,\beta)}}(x), \mu_{[\mathrm{A}]_{(\gamma,\delta)}}(x) \right\}, \max\left\{ \nu_{[A]_{(\alpha,\beta)}}(x), \nu_{[\mathrm{A}]_{(\gamma,\delta)}}(x) \right\} \right) \middle| x \in X \right\} \neq \emptyset.$$

Example 1. Let$X = \{a,b,c,d,e,f\}$and

$A = \{(a,0.2,0.4),(b,0.8,0.2),(c,0.6,0.3),(d,0.4,0.5),(e,0,1),(f,1,0)\}$.Let us denote A for convenience as

$$A = \frac{(0.2,0.4)}{a} + \frac{(0.8,0.2)}{b} + \frac{(0.6,0.3)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(1,0)}{f}.$$ Then

$$[\mathrm{A}]^{(0.2,0.4)} = \frac{(0.2,0.4)}{a} + \frac{(0.8,0.2)}{b} + \frac{(0.6,0.3)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(1,0)}{f}$$. And, by Definition 6

$$[A]_{(0.2,0.4)} = \frac{(0.2,0.4)}{a} + \frac{(0.2,0.4)}{b} + \frac{(0.2,0.4)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(0.2,0.4)}{f}.$$

Similarly

$$[A]_{(0.8,0.2)} = \frac{(0,1)}{a} + \frac{(0.8,0.2)}{b} + \frac{(0,1)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(0.8,0.2)}{f}$$

$$[A]_{(0.6,0.3)} = \frac{(0,1)}{a} + \frac{(0.6,0.3)}{b} + \frac{(0.6,0.3)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(0.6,0.3)}{f}$$

$$[A]_{(0.4,0.5)} = \frac{(0,1)}{a} + \frac{(0.4,0.5)}{b} + \frac{(0.4,0.5)}{c} + \frac{(0.4,0.5)}{d} + \frac{(0,1)}{e} + \frac{(0.4,0.5)}{f}$$

$$[A]_{(0,1)} = \frac{(0,1)}{a} + \frac{(0,1)}{b} + \frac{(0,1)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(0,1)}{f}$$

$$[A]_{(1,0)} = \frac{(0,1)}{a} + \frac{(0,1)}{b} + \frac{(0,1)}{c} + \frac{(0,1)}{d} + \frac{(0,1)}{e} + \frac{(1,0)}{f}.$$

Observe that $[A]_{(\alpha,\beta)} \cap [A]_{(\gamma,\delta)} \neq \emptyset, \forall (\alpha,\beta) \neq (\gamma,\delta)$.

Theorem 8.Let *X* the non- empty set, $\alpha,\beta \in [0.1]$ with $\alpha + \beta \leq 1$. For any intuitionistic fuzzy subset*A* in *X*, $[A]_{+(\alpha,\beta)}$ not form a complete system of events of A.

Proof of Theorem 8areexactly similar to those of Theorem 7. Only difference is instead of inequality here we will have > inequality.

5. CONCLUSIONS

Starting from the definition of a complete system of events in probability theoryI tried to show thatset of parts of(α, β)- cuts and strong (α, β)- cutsdoes not form a complete system of events.

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