A COUNTER-EXAMPLE OF COMPLETE SYSTEMS FORINTUITIONISTIC FUZZY SETS EVENTS

Iuliana Carmen BĂRBĂCIORU , Lecturer Ph. D.
”Constantin Brâncuşi” University, Tg. Jiu

Abstract: *In this article, starting from the decomposition theoremof a intuitionistic fuzzy setsfrom [6],[2], we give some basic definitions from The level set of Intuitionistic fuzzy sets andwe I will show that thelevel cuts do not form a partition.*

Keywords: Intuitionistic fuzzy sets, *(α,β)*-cut, decomposition theorems, and the level setof a intuitionistic fuzzy sets.

1. INTRODUCTION

Zadeh introduced the notion of the *α cut*of fuzzy set (FS) and decomposition theorems [7],[8],[11]. The concept of intuitionistic fuzzy sets (IFS) was introduced by K.T. Atanassov [1] as a generalization of the notion of a fuzzy set. The*(α,β)*-cut of IFS and their properties were studied P.K. Sharma [4] and me an [3]. Starting from the decomposition theoremsfrom Intuitionistic fuzzy sets described in [7],[8],[11], I will try to develop this theoryby showing that IFS not forming acomplete system of events, similarly to the classical theory of probabilities.

2.PRELIMINARIES

Basic definitions and properties of these setsare those used in [2].

Also in [6], [2] are given decomposition theorems of an intuitionistic fuzzy sets for $A = \cup_{\propto\in[0,1]}[A]_{\propto}$and $A = \cup_{\propto\in[0,1]}[A]_{\propto+}$.

Definition 1.[7]Denote by $[A]_{\propto}(x) = \propto * [A]^{\propto}(x)$ and $[A]_{+\propto}(x) = \propto * [A]^{+\propto}(x)$

The level set of *A* [7] denoted by $\Lambda(A)$, is defined as$\Lambda(A) = \left\{\alpha \middle| A(x) = \alpha, \forall x \in X\right\}$.

Theorem 1. [7] **Third Decomposition Theorem of FS**. For any fuzzy sets A,

$$A = \bigcup_{\propto\in\Lambda A} [A]_{\propto}$$

3.(α,β) -CUT AS THE INTUITIONISTIC FUZZY SETS

Definition 2.[3]We consider α,β∈[0.1] then, for any IFS set *A*

$$[A]^{(\alpha,\beta)} = \begin{cases} \left\{x \in X \middle| \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta, 0 \leq \mu_A(x) + \nu_A(x) \leq 1\right\} & \text{if } 0 < \alpha, \beta \leq 1 \\ \hline \left\{x \in X \middle| \mu_A(x) > 0, \nu_A(x) < 0, 0 \leq \mu_A(x) + \nu_A(x) \leq 1\right\} & \text{if } \alpha = 0, \beta = 0 \end{cases} \quad (1)$$

is called (α,β) -cut as the IFS*A*.

$$[A]^{+(\alpha,\beta)} = \begin{cases} \{x \in X \mid \mu_A(x) > \alpha, \nu_A(x) < \beta, 0 \le \mu_A(x) + \nu_A(x) \le 1\} & \text{if } 0 < \alpha, \beta \le 1 \\ \overline{\{x \in X \mid \mu_A(x) > 0, \nu_A(x) < 0, 0 \le \mu_A(x) + \nu_A(x) \le 1\}} & \text{if } \alpha = 0, \beta = 0 \end{cases} \quad (2)$$

is called strong (α,β) -cut as the IFS*A*.

Definition 3.[6] For α,β∈[0.1] with $\alpha + \beta \le 1$, through$[A]_{(\alpha,\beta)}$ and $[A]_{+(\alpha,\beta)}$we understand

$$[A]_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x \in [A]^{(\alpha,\beta)} \\ (0,1) & \text{if } x \notin [A]^{(\alpha,\beta)} \end{cases} \quad (3)$$

Respectively

$$[A]_{+(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x \in [A]^{+(\alpha,\beta)} \\ (0,1) & \text{if } x \notin [A]^{+(\alpha,\beta)} \end{cases} \quad (4)$$

Definition 4.[6]The level set of an IFS *A*denoted by $\Lambda(A)$, is defined as

$$\Lambda(A) = \{(\alpha,\beta) \mid A(x) = (\alpha,\beta), \forall x \in X\} \quad (5)$$

4.THE DECOMPOSITION THEOREMS OF INTUITIONISTIC FUZZY SETS

Theorem 2. [6] **Third Decomposition Theorem ofIFS**. Let *X* the non- empty set. For any intuitionistic fuzzy subset*A* in *X*,

$$A = \bigcup_{\alpha,\beta \in \Lambda A} [A]_{(\alpha,\beta)}$$

Where $\wedge A$ denotes the level set of *A*, given in Definition 4.

Theorem 3.Let *X* the non- empty set, $\alpha, \beta \in \Lambda A$with $\alpha + \beta \le 1$. For any intuitionistic fuzzy subset*A* in *X*, $[A]_{(\alpha,\beta)}$not form a complete system of events of A.

Proof: The fact that$A = \bigcup_{\alpha,\beta \in \Lambda A} [A]_{(\alpha,\beta)}$is demonstrated by [6] Theorem 2. Still need to show that

$[A]_{(\alpha,\beta)} \cap [A]_{(\gamma,\delta)} \ne \emptyset, \forall\, (\alpha,\beta) \ne (\gamma,\delta), \alpha,\beta,\gamma,\delta \in \Lambda A, \alpha + \beta \le 1, \gamma + \delta \le 1$ (6)

If the absurd relationship were true,
then$\exists\ (\alpha,\beta)\neq(\gamma,\delta), \alpha,\beta,\gamma,\delta\ \in \wedge A, \alpha+\beta\leq 1, \gamma+\delta\leq 1]$ so that$[A]_{(\alpha,\beta)}\ \cap [A]_{(\gamma,\delta)} = \emptyset$.

But$[A]_{(\alpha,\beta)}\ \cap [A]_{(\gamma,\delta)}$

$$=\left\{\left(x,\mu_{[A]_{(\alpha,\beta)}\cap[A]_{(\gamma,\delta)}}(x),\nu_{[A]_{(\alpha,\beta)}\cap[A]_{(\gamma,\delta)}}(x)\right)\middle| x\in X\right\}=$$

$$\left\{\left(x,\min\left\{\mu_{[A]_{(\alpha,\beta)}}(x),\mu_{[A]_{(\gamma,\delta)}}(x)\right\},\max\left\{\nu_{[A]_{(\alpha,\beta)}}(x),\nu_{[A]_{(\gamma,\delta)}}(x)\right\}\right)\middle| x\in X\right\}\neq\emptyset.$$

Example 1. Let$X=\{a,b,c,d,e,f\}$and

$A=\{(a,0.2,0.4),(b,0.8,0.2),(c,0.6,0.3),(d,0.4,0.5),(e,1,0)\}$.Let us denote A for convenience as

$$A=\frac{(0.2,0.4)}{a}+\frac{(0.8,0.2)}{b}+\frac{(0.6,0.3)}{c}+\frac{(1,0)}{e}.$$ Then

$$[A]^{(0.2,0.4)}=\frac{(0.2,0.4)}{a}+\frac{(0.8,0.2)}{b}+\frac{(0.6,0.3)}{c}+\frac{(1,0)}{e}$$. And, by Definition 6

$$[A]_{(0.2,0.4)}=\frac{(0.2,0.4)}{a}+\frac{(0.2,0.4)}{b}+\frac{(0.2,0.4)}{c}+\frac{(0.2,0.4)}{e}.$$

Similarly

$$[A]_{(0.8,0.2)}=\frac{(0.8,0.2)}{b}+\frac{(0.8,0.2)}{e}$$

$$[A]_{(0.6,0.3)}=\frac{(0.6,0.3)}{b}+\frac{(0.6,0.3)}{c}+\frac{(0.6,0.3)}{e}$$

$$[A]_{(0.4,0.5)}=\frac{(0.4,0.5)}{b}+\frac{(0.4,0.5)}{c}+\frac{(0.4,0.5)}{d}+\frac{(0.4,0.5)}{e}$$

$$[A]_{(1,0)}=\frac{(1,0)}{e}.$$

Observe that$[A]_{(\alpha,\beta)}\ \cap [A]_{(\gamma,\delta)}\neq\emptyset, \forall\ (\alpha,\beta)\neq(\gamma,\delta)$.

5. CONCLUSIONS

Starting from the definition of a complete system of events in probability theoryI tried to show thatset of parts of(α, β)- cuts, $\alpha,\beta\in\wedge A$, does not form a complete system of events.

REFERENCES

[1] Atanasov, K.T.,*Intuitionistic fuzzy sets: past, present and future*, in*EUSFLAT Conf.*, M. Wagenknecht and R. Hampel, Eds. University ofApplied Sciences at Zittau/G¨orlitz, Germany, 2003, pp. 12–19.

[2] Bărbăcioru, I.,C.,*Some Properties of (α,β)- cuts for intuitionistic fuzzy sets* , Fiability& Durability 1/2015, Editura " AcademicaBrâncuşi", Târgu Jiu.

[3] Bărbăcioru,I.,C., *A Note on (α,β)-cut in Intuitionistic Fuzzy Sets Theory*, Fiability& Durability 1/2014, Editura " AcademicaBrâncuşi", Târgu Jiu, ISSN 1844-640X, 200-206.

[4] Che, L.,Zadeh, A., *Fuzzy sets*, Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

[5] Despi, I., Opriş, D., Yalcin, E., *Generalized Atanasov Intuitionistic fuzzy sets*, The Fifth International Conference on Information, Process, and Knowledge Management,eKNOW 2013.

[6] Jose, S., Kuriakose, S., Decomposition theorems of an *Intuitionistic fuzzy set*, Notes on Intuitionistic fuzzy sets, Vol. 18, 2012, no. 31-36.

[7] Klir, G. J., B. Yuan. *Fuzzy Sets and Fuzzy Logic: Theory and Applications*,Prentice Hall ofIndia Private Limited, New Delhi, 2005.

[8] Sharma, P. K., *(α, β) cut for Intuitionistic Fuzzy Groups*, International Mathematical Forum,Vol. 6(53), 2011, 2605–2614.

[9] Veeramani,V.,Batulan R.,*Some Characterisations of α-cut in Intuitionistic Fuzzy Set Theory*.

[10] Zeng, W. and Li, H., *Correlation coefficient of intuitionistic fuzzy sets*, Journal of Industrial Engineering International, vol. 3, pp. 33–40, July 2007.

[11] Zadeh, L.A., *Fuzzy sets*, Information and Control, Vol. 8, 1965, No. 3, 338–353.

[12] Yusoff, B.,Taib, I., Abdullah, L.,and Wahab, A. F., *A new similarity measure on intuitionistic fuzzy sets*,World Academy of Science Engineeringand Technology, vol. 78, pp. 36–40, 2011.