VIBRATIONS AND EQUILIBRIUM OF THE PLANAR KINEMATIC CHAINS WITH ROTATIONAL KINEMATICAL LINKS WITH CLEARANCES

Ph. D. Lecturer Jan Cristian GRIGORE, University of Piteşti, Faculty of Mechanics and Technology, Romania, jan_grigore@yahoo.com
Ph. D. Lecturer Monica BÂLDEA, University of Piteşti, Faculty of Mechanics and Technology, Romania, bldmonica@yahoo.com
Ph. D. Lecturer Mihaela ISTRATE, Faculty of Mechanics and Technology, University of Piteşti, Romania,mmihaela_1971@yahoo.com
Ph. D. Lecturer Ancuţa BĂLTEANU, University of Piteşti, Faculty of Mechanics and Technology, Romania, a_balteanu@yahoo.com

Abstract:*Based on our previous work in this paper we study the vibrations of a planar chain with rotational links with clearances.We also determined the matrix equation wich leads to the equilibrium positions*

Keywords: Lagrange's equations,nonlinear vibrations,multibody

1.INTRODUCTION

In our previous work we proved that general matrix equation of motion has the form.

$$\begin{bmatrix} [\mathbf{m}] & [\mathbf{B}]^T \\ [\mathbf{B}] & [\mathbf{0}] \end{bmatrix} \begin{bmatrix} \{\ddot{\mathbf{q}}\} \\ \{\mathbf{R}\} \end{bmatrix} \{\dot{\mathbf{q}}\} = \begin{bmatrix} \{\mathbf{F}\} \\ \{\dot{\mathbf{C}}\} - [\dot{\mathbf{B}}]\{\dot{\mathbf{q}}\} \end{bmatrix}. \tag{1.1}$$

The equilibrium equations are given by

$$\begin{cases} \{\mathbf{D}_k\} = \{\mathbf{0}\}, \text{if } O_k \text{ is rotationalkinematical joint without clearance} \\ \{\mathbf{D}_k\}^T \{\mathbf{D}_k\} - 1 = 0, \text{if } O_k \text{ is rotationalkinematical joint withclearance} \end{cases}, \tag{1.2}$$

$$[\mathbf{B}]^T \{\mathbf{R}\} - \{\mathbf{F}\} = \{\mathbf{0}\}. \tag{1.3}$$

2. VIBRATIONS OF THE PLANAR SYSTEMS WITH ROTATIONAL KINEMATIC LINKS WITH CLEARANCES

2.1. Nonlinear vibrations.

The motion of the system relative to an equilibrium position, position defined by the generalized coordinates having the values q_i^0, $i = \overline{1,n}$, values obtained from the system (1.2), (1.3), is given by the equations (1.1) in which, if we make the substitution $\{\mathbf{q}\} = \{\mathbf{q}^0\} + \{\mathbf{z}\}$, one obtains the matrix equations $[\mathbf{m}]\{\ddot{\mathbf{z}}\} + [\mathbf{B}]^T \{\mathbf{R}\} = \{\mathbf{F}\}$, $[\mathbf{B}]\{\ddot{\mathbf{z}}\} = \{\dot{\mathbf{C}}\} - [\dot{\mathbf{B}}]\{\dot{\mathbf{z}}\}$. By numerical solving of this system, we obtain the time histories both of the *displacements* $z_i = z_i(t)$, $i = \overline{1,n}$, and of the *reactions* $R_i = R_i(t)$, $i = \overline{1, 2n_1 + n_2}$.

2.2. Linear vibrations

In the case of the linear vibrations we make the development into the series of the functions $[\mathbf{B}]$, $\{\mathbf{F}\}$ and by retaining only the linear terms and using the notations $\left[\mathbf{B}_0\right] = \left[\mathbf{B}\right]\Big|_{\substack{q_i = q_i^0 \\ i=\overline{1,n}}}$, $\left[D\mathbf{B}_{i0}\right] = \left.\frac{\partial\left[\mathbf{B}\right]}{\partial q_i}\right|_{\substack{q_i = q_i^0 \\ i=\overline{1,n}}}$,

$\left\{\mathbf{F}_0\right\} = \left\{\mathbf{F}\right\}\Big|_{\substack{q_i = q_i^0 \\ i=\overline{1,n}}}$, $\left[D\mathbf{F}_{i0}\right] = \left.\frac{\partial\left\{\mathbf{F}\right\}}{\partial q_i}\right|_{\substack{q_i = q_i^0 \\ i=\overline{1,n}}}$, $\left\{\mathbf{R}\right\} = \left\{\mathbf{R}_0\right\} + \left\{\Delta\mathbf{R}\right\}$, $\left[\widetilde{\mathbf{B}}_{i0}\right] = \left[D\mathbf{B}_{i0}\right]\left\{\mathbf{R}_0\right\}$,

$\left[\widetilde{\mathbf{B}}_0\right] = \left[\left\{\mathbf{B}_{10}\right\}\ \left\{\mathbf{B}_{20}\right\}\ \ldots\ \left\{\mathbf{B}_{n0}\right\}\right]$, $\left[D\mathbf{F}_0\right] = \left[\left\{D\mathbf{F}_{10}\right\}\ \left\{D\mathbf{F}_{20}\right\}\ \ldots\ \left\{D\mathbf{F}_{n0}\right\}\right]$ in the conditions of the equality deduced from the equation (1.2) $\left[\mathbf{B}_0\right]^T\left\{\mathbf{R}_0\right\} + \left\{\mathbf{F}_0\right\}$ one obtains the matrix equations $\left[\mathbf{m}\right]\left\{\ddot{\mathbf{z}}\right\} + \left[\widetilde{\mathbf{B}}_0\right]^T\left\{\mathbf{z}\right\} + \left[\mathbf{B}_0\right]\left\{\Delta\mathbf{R}\right\} = \left[D\mathbf{F}_0\right]\left\{\mathbf{z}\right\}$, $\left[\mathbf{B}_0\right]\left\{\ddot{\mathbf{z}}\right\} = \left\{\dot{\mathbf{C}}\right\}$, wherefrom, with the notation $\left[\mathbf{K}\right] = \left[\widetilde{\mathbf{B}}_0\right] - \left[D\mathbf{F}_0\right] + \left[\mathbf{B}_0\right]^T\left[\left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[\mathbf{B}_0\right]^T\right]^{-1}\left[\left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[D\mathbf{F}_0\right] - \left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[\widetilde{\mathbf{B}}_0\right]\right]$, we get the equalities $\left\{\Delta\mathbf{R}\right\} = \left[\left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[\mathbf{B}_0\right]^T\right]^{-1}\left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[D\mathbf{F}_0\right]\left\{\mathbf{z}\right\} - \left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[\widetilde{\mathbf{B}}_0\right]^T\left\{\mathbf{z}\right\} - \left\{\dot{\mathbf{C}}\right\}$,
$\left[\mathbf{m}\right]\left\{\ddot{\mathbf{z}}\right\} + \left[\mathbf{K}\right]\left\{\mathbf{z}\right\} = \left[\left[\mathbf{B}_0\right]\left[\mathbf{m}\right]^{-1}\left[\mathbf{B}_0\right]^T\right]^{-1}\left\{\dot{\mathbf{C}}\right\}$.

The eigenpulsations for such a system are obtained from the nth degree equation in p^2

$$\det\left(\left[\mathbf{K}\right] - p^2\left[\mathbf{m}\right]\right) = 0, \qquad (2.1)$$

equation that has n_1 roots equal to zero, where n_1 is the number of the constraint equations, number which is equal to the number of lines of the matrix $\left[\mathbf{B}\right]$.

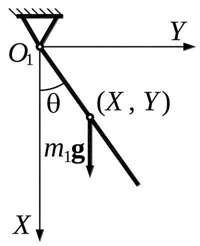

Fig.1. Vibrations of the bar articulated at O acted only by its own weight

As example, for the vibrations of the homogenous bar articulated at O, of length $2l$, Fig.1, at which the equilibrium position corresponds to $X = l$, $Y = 0$, $\theta = 0$, one successively deduces the expressions $\left\{\mathbf{F}\right\} = \left\{\mathbf{F}_0\right\} = \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$, $\left\{\mathbf{R}_0\right\} = \begin{bmatrix} mg \\ 0 \end{bmatrix}$, $\left[\mathbf{B}\right] = \begin{bmatrix} 1 & 0 & l\sin\theta \\ 0 & 1 & -l\cos\theta \end{bmatrix}$, $\left[\mathbf{B}_0\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -l \end{bmatrix}$,

$\left[\widetilde{\mathbf{B}}_0\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & mgl \end{bmatrix}$, $\left[D\mathbf{F}_0\right] = \left[\mathbf{0}\right]$, $\left[\mathbf{K}\right] = \frac{mgl}{J_0}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & ml \\ 0 & 0 & J \end{bmatrix}$, where $J_0 = J + ml^2$ and the equation (2.1) becomes $\begin{vmatrix} -mp^2 & 0 & 0 \\ 0 & -mp^2 & \dfrac{m^2l^2g}{J_0} \\ 0 & 0 & J\left(-p^2+\dfrac{m\,\mathrm{lg}}{J_0}\right) \end{vmatrix} = 0$ and, as easily can be seen, it has two roots equal to zero and the third given by

$$p^2 = \frac{mlg}{J_0}. \tag{2.2}$$

In the general case, if we consider that the independent variables define the column matrix $\{\mathbf{q}_1\}$, and the dependent variables define the column matrix $\mathbf{q}_2$, then in the linear calculus when $[\dot{\mathbf{B}}]\{\dot{\mathbf{q}}\} = \{\mathbf{0}\}$, keeping into account the diagonal form of the matrix $[\mathbf{m}]$, the system (1.1), can be brought to the form $[\mathbf{m}_{11}]\{\ddot{\mathbf{q}}_1\} + [\mathbf{B}_1]^T\{\mathbf{R}\} = \{\mathbf{F}_1\}$, $[\mathbf{m}_{22}]\{\ddot{\mathbf{q}}_2\} + [\mathbf{B}_2]^T\{\mathbf{R}\} = \{\mathbf{F}_2\}$ and from here, eliminating the matrices $\{\ddot{\mathbf{q}}_1\}$, $\{\mathbf{R}\}$ and using the matrices $[\mathbf{m}_2^*] = [\mathbf{m}_{22}] + [\mathbf{B}_2]^T\left[[\mathbf{B}_1]^{-1}\right]^T[\mathbf{m}_{11}][\mathbf{B}_1]^{-1}[x_2]$, $[\mathbf{F}^*] = [\mathbf{B}_2]^T\left[[\mathbf{B}_1]^{-1}\right]^T[\mathbf{m}_{11}][\mathbf{B}_1]^{-1}[\dot{\mathbf{C}}]$ we obtain the matrix equation

$$[\mathbf{m}_2^*]\{\ddot{\mathbf{q}}_2\} + [\mathbf{B}_2]^T\left[[\mathbf{B}_1]^{-1}\right]^T\{\mathbf{F}_1\} - \{\mathbf{F}_2\} = \{\mathbf{F}^*\}. \tag{2.3}$$

For the system drawn in Fig. 6.1 we successively obtain the expressions $[\mathbf{m}_{11}] = \begin{bmatrix} m_1 & 0 \\ 0 & m_1 \end{bmatrix}$, $[\mathbf{m}_{22}] = J$, $[\mathbf{B}_1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $[\mathbf{B}_2] = \begin{bmatrix} l\sin\theta \\ -l\cos\theta \end{bmatrix}$, $\{\mathbf{F}_1\} = \begin{bmatrix} mg \\ 0 \end{bmatrix}$, $\{\mathbf{F}_2\} = 0$ and the equation (2.3) becomes $J_0\ddot{\theta} + mgl\theta = 0$ and from here we obtain the eigenpulsation given by the relation (2.2).

3. EQUILIBRIUM OF THE PLANAR SYSTEMS WITH ROTATIONAL KINEMATICAL JOINTS WITH CLEARANCES

The equilibrium equations are obtained from the equalities (1.1), and from the equations $\begin{bmatrix} [\mathbf{m}] & [\mathbf{B}]^T \\ [\mathbf{B}] & [\mathbf{0}] \end{bmatrix}\begin{bmatrix} \{\ddot{\mathbf{q}}\} \\ \{\mathbf{R}\} \end{bmatrix} = \begin{bmatrix} \{\mathbf{F}\} \\ \{\dot{\mathbf{C}}\} - [\dot{\mathbf{B}}]\{\dot{\mathbf{q}}\} \end{bmatrix}$ in which $\{\ddot{\mathbf{q}}\} = \{\mathbf{0}\}$, $\{\dot{\mathbf{q}}\} = \{\mathbf{0}\}$ and they write in the form (1.2) and (1.3).

Thus, for a system with n elements, n_1 rotational kinematical joints without clearance and n_2 rotational kinematical joints with joints, one obtains $2n_1 + n_2 + 3n$ equations ($2n_1 + n_2$ equations from (1.1) and $3n$ equations from (1.2)) with $2n_1 + n_2 + 3n$ unknowns, name them: $2n_1$ reactions for the kinematical joints without clearance, n_2 reactions for the kinematical joints with clearance and $3n$ kinematical parameters of the type X_i, Y_i, θ_i, $i = \overline{1,n}$, for the n elements. By solving the system of equations (1.1), (1.2), we determine the values of the generalized coordinates q_1, q_2, …, q_{3n}, and the reactions generically denoted by λ_1, λ_2, …,

$\lambda_{2n_1+n_2}$, values that correspond to the equilibrium positions.

In the case when the matrix of the forces $\{\mathbf{F}\}$ does not depend on the coordinates X_i, Y_i, $i=\overline{1,n}$, then the matrix equation (1.2), using the expressions $\left[\mathbf{E}_k^{(i)}\right]=\left[\cos\alpha_k \;\; \sin\alpha_k \;\; -x_k^{(i)}\sin(\theta_i-\alpha_k)-y_k^{(i)}\cos(\theta_i-\alpha_k)\right]$ for the matrices $\left[\mathbf{E}_k^{(i)}\right]$ separates in $3n$ equations with $n+2n_1+2n_2$ unknowns (n angular parameters θ_i, $2n_1$ reactions in the kinematical joints without clearance, n_2 reactions in the kinematical joints with clearance and n_2 angular parameters α_k).

For open kinematical chains there exists the relation $n=n_1+n_2$ and, as a consequence, for these, the equilibrium position can be determined from the matrix equation (1.2).

4. CONCLUSIONS

Based on the differential matrix equation of motion, we obtained the equations of the vibrations for a planar chain with rotational linkages with clearances. This equation is treated both in the nonlinear case as well as in the linear case. We also determined the equilibrium positions.

REFERENCES

[1] AMIROUCHE, F., *Fundamentals of multibody dynamics*,Birkhänser,Boston,Berlin, 2004.

[2] ERKAYA, S., UZMAY, I., *Investigation on effect of joint clearance on dynamics of four-bar mechanism*. Nonlinear Dyn., 58, 179-198, (2009).

[3] FLORES, P., AMBRÓSIO, J., *Revolute joints with clearance in multibody systems*. Comput. Struct. 82, 1359-1369(2004).

[4] FLORES, P., *Modeling and simulation of wear in revolute clearance joints in multibody systems*. Mechanism and Machine Theory, 44, 1211-1222(2009).

[5] GRIGORE,J.C.,*Contribuţii la studiul dinamic al mecanismelor cu jocuri*. Teză de doctorat, Universitatea din Piteşti, 2008.

[6] PANDREA, N., *Calculul dinamic al convertorului mecanic de cuplu*,, G. Constantinescu" IFToMM Int. Symp. SYROM`89 pag. 673-679, Bucharest, Romania, 1989.

[7] PANDREA, N., POPA, D., *Mecanisme*. Editura Tehnică, București 2000.

[8] PENESTRI, E., VALENTINI, P., P., VITO, L., *Multibody dynamics simulation of planar linkages with Dahl friction*, Multibody Syst. Dyn. 17, 321-347(2007).

[9]PFEIFFER,F.,GLOCKER,C.,*Multibody dynamics with unilateral contacts*.Wiley,New York(1996).

[10] RAVN, P., *A continuous analysis method for planar multibody systems with joint clearance*. Multibody Syst. Dyn. 2, 1-24,(1998).