# PLANAR MECHANISMS USED FOR GENERATING CURVE LINE TRANSLATION MOTION 

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#### Abstract

The curve line translation motion can be generated in the particular form of the circular translation, through mono-mobile mechanisms with articulated links of simple parallelogram type (with a fixed side) or through transmission with toothed belt with a fixed wheel. Also, the circular translation can be generated through planar mechanisms with two cylindrical gears with a fixed central wheel. It is mentioned that the two cylindrical gearings of the Fergusson mechanisms are both exterior and interior.


Keywords: planar mechanism, circular translation motion, cylindrical gear, kinematic scheme

## 1. Introduction

Curve line translation motion, in particular circular translation, can be generated by means of planar mechanisms with articulated links [1, 3, 4] such as the articulated parallelogram (fig. 1), or by means of planar mechanisms with cylindrical gears [1, 2, 3] such as the Fergusson mechanism (fig. 2).

With the planar mechanism of the articulated parallelogram type (fig. 1), the reciprocating rod 2 does a circular translation motion as the velocities of points A and B are equal, which can be noticed by maintaining the segment AB in a parallel position to itself.


Fig. 1. The parallelogram mechanism


Fig. 2. Fergusson planar mechanism

With the multi-level planar mechanism (fig. 2), where the gear wheels 1 and 3 are equal, if the central gear wheel 1 is fixed $\left(\omega_{1} \equiv 0\right)$, the satellite wheel 3 does a circular translation motion ( $\omega_{3}=0$ ).

Indeed, by actuating the planet wheel carrier $p\left(\omega_{p} \neq 0\right)$, by means of the distribution of linear velocities (fig. 2), it results that the velocities of points B and D are equal:

$$
(C c) \equiv 0 ;(B b)=(D d)=2(A a)
$$

## 2. Using the circular translation motion in auxiliary mechanisms of motor vehicles

### 2.1. Doors closing / opening mechanism without hinges

The doors of some modern buses no longer contain hinges, and they are actuated by means of the articulated parallelogram mechanism (fig. 3) in which the door side MN is one with the reciprocating rod AB of this planar mechanism.


Fig. 3. Kinematic scheme of the mechanism used for actuating the bus door


Fig. 4. Kinematic scheme of the screen wiper

The kinematic scheme of the parallelogram mechanism (fig. 3) is presented by means of a continuous line in an intermediate position (when the door side MN is outside the bus), and by means of a dotted line in the extreme closing (right) and opening (left) positions.

The mechanism is pneumatically driven by means of an actuator situated below, under the bus platform, at the same level with the stairs used by passengers to go on and off the bus.

### 2.2. The screen wiper mechanism

The parallelogram mechanism is used as screen wiper (fig. 4) for urban buses, where the wiping shim is fixed in the $M N$ position perpendicular to the reciprocating rod $A B$ in a point situated to the right of the $A B$ segment.

The kinematic scheme of the parallelogram mechanism (fig. 4) is shown in three distinct positions, of which the extreme position to the right is represented by a continuous line, and the other two positions (middle and left) are represented by means of a dotted line.

This mechanism is usually electrically driven by means of an equalizing rod - crank mechanism.

This type of parallelogram mechanism with a fixed small edge is used for the doors of motor vehicles in terms of a crane to lift / lower the glass.

The reciprocating rod of such a parallelogram does a circular translation motion, and by means of some rollers, gets into contact with the glass frame, which is vertically guided.

This kind of mechanism is driven manually or electrically by means of a cylindrical gear having a large multiplication ration, placed inside or outside, which ensures a certain self-locking degree with regard to the intention of actuating from the led element to the leading one.

## 3. Geometry and kinematics of the Fergusson planar mechanism

The Fergusson planar mechanism (fig. 2, 5) consists of three geared wheels to the outside $(1,2,3)$ placed in a series, articulated in points $\mathrm{O}, \mathrm{A}$ and B on the planet wheel carrier $p$, which does a translation motion around the fixed point O .

Providing that the distances between the axes of the wheels 1 and 2 , respectively 2 and 3 are equal $(\mathrm{OA}=\mathrm{AB})$, the gear wheels 1 and 3 have equal diameters, and the same number of teeth $\left(\mathrm{z}_{1}=\mathrm{z}_{3}\right)$.

If gear wheel 1 is fixed by means of blocking (fig. 2, 5), then point C becomes the instantaneous rotation centre of gear wheel 2 , with a null velocity $(\mathrm{Cc}=0)$.


b)

Fig.
5. Kinematic scheme (a) and general kinematic scheme (b)
of Fergusson planar mechanism in a double orthogonal projection
For one rotation of the planet wheel carrier $p$ at an angular velocity $\omega$, points A and B have velocities in the ratio $\mathrm{OA} / \mathrm{OB}=1 / 2$. The distribution of velocities (fig. 2) on the gear wheel 2 is obtained by uniting the point $a$ (the peak of the A point velocity) with point C whose velocity is null.

Point $d$, peak of the velocity of point D , is located on the extension of the $a c$ segment, so that the ratio of the velocities of points A and D is $\mathrm{CA} / \mathrm{CD}=1 / 2$. For the velocities of points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D we can define the ratios (fig. 2, 5):

$$
\begin{equation*}
\frac{A a}{B b}=\frac{O A}{O B}=\frac{1}{2} ; \frac{A a}{D d}=\frac{C A}{C D}=\frac{1}{2} \tag{3.1,2}
\end{equation*}
$$

Of the ratios (2.1) and (2.2) it results $B b=D d$, that is the velocities of points $B$ and $D$ on the gear wheel 3 are equal, which determines $\omega_{3}=0$, so gear wheel 3 does a circular translation motion.

The same conclusion may be obtained by means of the analytic method, writing the relative transmittal ratios provided the planet wheel carrier $p$ does not move:

$$
\begin{equation*}
i_{21}^{p}=\frac{\omega_{2}-\omega_{P}}{\omega_{1}-\omega_{P}}=-\left(\frac{\omega_{2}}{\omega_{P}}-1\right) ; i_{32}^{p}=\frac{\omega_{3}-\omega_{P}}{\omega_{2}-\omega_{P}}=\frac{\frac{\omega_{3}}{\omega_{P}}-1}{\frac{\omega_{2}}{\omega_{P}}-1} \tag{3.3,4}
\end{equation*}
$$

Multiplying relations (3.3) and (3.4) we obtain

$$
\begin{equation*}
\frac{\omega_{3}}{\omega_{p}}=1-i_{21}^{p} \cdot i_{32}^{p}=1-\frac{z_{1}}{z_{3}} \tag{3.5}
\end{equation*}
$$

For $z_{1}=z_{3}$ and $\omega_{p} \neq 0$ it results from (3.5) that $\omega_{3}=0$, which corresponds to the circular translation motion.

## 4. New cylindrical planar mechanisms for generating circular translation motion

A new kinematic scheme (fig. 5b) is obtained starting from the Fergusson planar mechanism (fig. 5a), where the interim gear wheel 2 has been replaced by two joint and several gear wheels 2 and 2' with different radii.

Let us consider the formula (3.5) which is now written as

$$
\begin{equation*}
\frac{\omega_{3}}{\omega_{p}}=1-i_{21}^{p} \cdot i_{32^{\prime}}^{p}=1-\frac{z_{1} z_{2^{\prime}}}{z_{2} z_{3}} \tag{4.1}
\end{equation*}
$$

Provided that gear wheel 3 does a circular translation motion, that is $\omega_{3}=0$, we obtain from formulae (4.1):

$$
\begin{equation*}
\frac{z_{1} z_{2^{\prime}}}{z_{2} z_{3}}=1 \tag{4.2}
\end{equation*}
$$

Thus, if we impose the transmittal ratio $i_{0}$ of the exterior cylindrical gear (1,2), from (4.2) we obtain:

$$
\begin{equation*}
\frac{z_{2}}{z_{1}}=\frac{z_{3}}{z_{2^{\prime}}}=i_{0}>1 \tag{4.3}
\end{equation*}
$$

Relation (4.2) can be checked by means of the graphical method of the linear velocities distribution (fig. 6), where, if the velocities of two points on gear wheel 3 are equal, for example $V_{B}=V_{D}$, the instantaneous rotation centre is to infinity, that is the angular velocity $\omega_{3}$ is null. Considering the distribution of linear velocities (fig. 6) we notice that the equivalence $B b=D d$ implies the relations

$$
\begin{equation*}
\frac{A C}{A D}=\frac{O A}{A B} \text { or } \frac{r_{2}}{r_{2^{\prime}}}=\frac{r_{1}+r_{2}}{r_{2^{\prime}}+r_{3}} \tag{4.4}
\end{equation*}
$$

Considering the second relation of (4.4), we obtain $r_{1} r_{2}=r_{2} r_{3}$, equivalent to (4.2), as the radii of the dividing circles are proportional to the numbers of teeth.

A smaller gauge alternative of the kinematic scheme presented above (fig. 5b) is obtained if the gear wheel 3 gears with the gear wheel $2^{\prime}$ on the same side of the $\Delta_{2}$ axis with the exterior gear mechanism $(1,2)$ (fig. 6).


Fig. 6. Kinematic scheme of the Fergusson mechanism - compact alternative
The position of the $\Delta_{3}$ axis, correspondingly of point $B$, is obtained by means of the distribution of velocities $O a$ and $C a$, depending on the position of point $D$. Thus, knowing the vector $D d$ with $d \in C a$, the position of $B$ is given by the peak of its velocity where the parallel line from $d$ to $O A$ meets $O a$.

In this case (fig. 7), following the distribution of linear velocities in the hypothesis $B b=D d$, that is $\omega_{3}=0$, the geometrical construction implies

$$
\begin{equation*}
\frac{O B}{A B}=\frac{C D}{A D} \text { or } \frac{r_{1}+r_{2}-r_{2^{\prime}}-r_{3}}{r_{2^{\prime}}+r_{3}}=\frac{r_{2}-r_{2^{\prime}}}{r_{2^{\prime}}} \tag{4.5}
\end{equation*}
$$

which leads to the relation $r_{1} r_{2^{\prime}}=r_{2} r_{3}$, the same as with the previous alternative, namely (4.2), $z_{1} z_{2^{\prime}}=z_{2} z_{3}$. For the particular case when $O D=0$ it results (fig. 7a) $O C=A C$, and points $A$ and $B$ are antipodal $[1,3]$, which allows for better balancing.


Fig. 7a. Kinematic scheme of the planar mechanism, improved version


Fig. 7b. Kinematic scheme of the planar mechanism with interior gears


Fig. 8. Images foto of mechanism model with external cylindrical gearings
Coming back to formula (4.1), for $\omega_{3}=0$ we can write the general expression

$$
\begin{equation*}
i_{21}^{o} \cdot i_{32^{\prime}}^{p}=1 \tag{4.6}
\end{equation*}
$$

Analysing the relation (4.6) it results that the transmittal ratios $i_{21}^{p}$ and $i_{32}^{p}$ must have the same sign, that is the cylindrical gears $(1,2)$ and $\left(2^{\prime}, 3\right)$ are either both exterior (fig. 7a) or both interior (fig. 7b).

If both gearings are interior (fig. 7b), we shall start by conveniently choosing the characteristic points $O, A, C$ and $D$, respectively the numbers of teeth of the gear wheels 1 , 2 and $2^{\prime}$.

We build the velocity of point $A$ through the segment $A a$ oriented to the left (fig. 7b). Knowing that the velocity of point $C$ is zero, the velocity of point $D$ is obtained by means of the linear distribution of velocities, points $a, c$ and $d$ are collinear.

From point we draw a parallel line to the reference line $O A$ till it crosses the extension of the $O a$ segment in point $b$. Thus we obtain on the drawing (fig. 7b) the $O B$ segment, positioning the mobile axis of the satellite wheel 3 with interior dents, which does a circular translation motion.

## 5. Conclusions

The curve line translation motion can be generated in the particular form of circular translation, by means of mono-mobile mechanisms with articulated links of the simple parallelogram type (with a fixed side) or by means of transmissions with a toothed belt with a fixed wheel.

Also, circular translation can be generated through planar mechanisms with two cylindrical gears with a fixed central wheel. It is mentioned that the two cylindrical gearings of the Fergusson mechanisms are both exterior and interior.

The general form of the curve line translation is obtained by means of planar bimobile manipulating mechanisms with articulated links, auxiliary kinematic chains mounted in parallel with the main kinematic chain.

## Bibliography

1. Antonescu, P., Mechanisms -Structural and Kinematic Calculation, U.P.B. Printing Press, Bucharest, 1979;
2. Maros, D., Theory of Mechanisms and Machines - Gear Kinematics, Tech. Publ. House, Bucharest, 1958;
3. Antonescu, P., Antonescu, E., Synthesis of Cylindrical Planetary Mechanisms in view of the Circular Translation, SYROM'81 Bucharest, Vol. III, pp 9-14, 1981;
4. Antonescu, P., Antonescu, O., Mechanisms and the Dynamics of Machines, Printech Publishing House, Bucharest, 2005.
