OPTIMIZATION METHODS

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Abstract: Throughout the ages, man has continuously been involved with the process of optimization. In its *earliest form, optimization consisted of unscientific rituals and prejudices like pouring libations and sacrificing* animals to the gods, con- *sulting the oracles, observing the positions of the stars, and watching the flight of* birds. When the circumstances were appropriate, the timing was thought to be auspicious (or optimum) for planting the crops or embarking on a war.

Key words: transport, optimizations, mechanics

2. Introduction

As the ages advanced and the age of reason prevailed, unscientific rituals were replaced by rules of thumb and later, with the development of mathematics, mathematical calculations began to be applied.

Optimization problems occur in most disciplines like engineering, physics, mathematics, economics, administration, commerce, social sciences, and even politics. Optimization problems abound in the various fields of engineering like electrical, mechanical, civil, chemical, and building engineering. Typical areas of application are modeling, characterization, and design of devices, circuits, and systems; design of tools, instruments, and equipment; design of structures and buildings; process control; approximation theory, curve fitting, solution of systems of equations; forecasting, production scheduling, quality control; maintenance and repair; inventory control, accounting, budgeting, etc. Some recent innovations rely almost entirely on optimization theory, for example, neural networks and adaptive systems.

Several general approaches to optimization are available, as follows:

1. Analytical methods
2. Graphical methods
3. Experimental methods
4. Numerical methods

Analytical methods are based on the classical techniques of differential cal- culus. In these methods the maximum or minimum of a performance criterion is determined by finding the values of parameters $x1$, $x2$, ... , xn that cause the derivatives of $f(x1, x2, \ldots, xn)$ with respect to $x1$, $x2$, ... , xn to assume zero values. The problem to be solved must obviously be described in mathematical terms before the rules of calculus can be applied. The method need not entail the use of a digital computer. However, it cannot be applied to highly nonlinear problems or to problems where the number of independent parameters exceeds two or three. A graphical method can be used to plot the function to be maximized or min- imized if the number of variables does not exceed two. If the function depends on only one variable, say, x1 , a plot of f (x1) versus x1 will immediately reveal the maxima and/or minima of the function. Similarly, if the function depends on only two variables, say, x1 and x2 , a set of contours can be constructed. A contour is a set of points in the (x1 , x2) plane for which f (x1 , x2) is constant, and so a contour plot, like a topographical map of a specific region, will reveal readily the peaks and valleys of the function. For example, the contour plot of f (x1 , x2) depicted in Fig. 1 shows that the function has a minimum at point

A. Unfortunately, the graphical method is of limited usefulness since in most practical applications the function to be optimized depends on several variables, usually in excess of four.

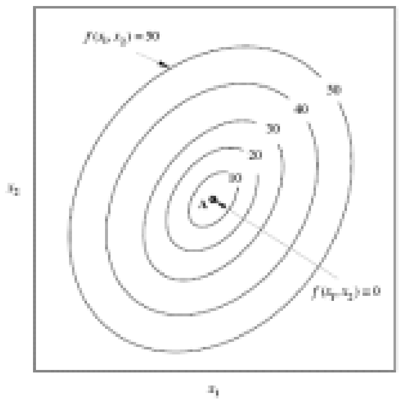

Figure 1. Contour plot of f (x1 , x2).

The optimum performance of a system can sometimes be achieved by direct experimentation. In this method, the system is set up and the process variables are adjusted one by one and the performance criterion is measured in each case. This method may lead to optimum or near optimum operating conditions. However, it can lead to unreliable results since in certain systems, two or more variables interact with each other, and must be adjusted simultaneously to yield the optimum performance criterion.

During the past 40 years, several branches of mathematical programming have evolved, as follows:

1. Linear programming
2. Integer programming
3. Quadratic programming
4. Nonlinear programming
5. Dynamic programming

Each one of these branches of mathematical programming is concerned with a specific class of optimization problems.

2. The Basic Optimization Problem

Before optimization is attempted, the problem at hand must be properly formulated. A performance criterion F must be derived in terms of n parameters x1 , x2 , ... , x_n as

F = f (x1 , x2 , ... , x_n)

F is a scalar quantity which can assume numerous forms. It can be the cost of a product in a manufacturing environment or the difference between the desired performance and the actual performance in a system. Variables x1 , x2 , ..., x_n are the parameters that influence the product cost in the first case or the actual performance in the second case. They can be independent variables, like time, or control parameters that can be adjusted. The most basic optimization problem is to adjust variables x1 , x2 , ..., x_n in such a way as to minimize quantity F . This problem can be stated mathe matically as

minimize F = f (x1 , x2 , ... , x_n) (1)

Quantity F is usually referred to as the objective or cost function.

The objective function may depend on a large number of variables, sometimes as many as 100 or more. To simplify the notation, matrix notation is usually employed. If x is a column vector with elements x1 , x2 , ... , xn , the transpose of x, namely, xT , can be expressed as the row vector

xT = [x1 x2 ••• x_n]

In this notation, the basic optimization problem of Eq. (1) can be expressed as minimize F = f (x) for x ∈ E_n where E_n represents the n-dimensional Euclidean space. On many occasions, the optimization problem consists of finding the maxi- mum of the objective function.

Since max[f (x)] = −min[−f (x)] the maximum of F can be readily obtained by finding the minimum of the negative of F and then changing the sign of the minimum. Consequently, in this and subsequent chapters we focus our attention on minimization without loss of generality. In many applications, a number of distinct functions of x need to be optimized simultaneously. For example, if the system of nonlinear simultaneous equations fi (x) = 0 for i = 1, 2, ... , m needs to be solved, a vector x is sought which will reduce all fi (x) to zero simultaneously. In such a problem, the functions to be optimized can be used to construct a vector F(x) = [f1 (x) f2 (x) ••• fm (x)]T

The problem can be solved by finding a point x = x∗ such that F(x∗) = 0. Very frequently, a point x∗ that reduces all the fi (x) to zero simultaneously may not exist but an approximate solution, i.e., F(x∗) ≈ 0, may be available which could be entirely satisfactory in practice.

A similar problem arises in scientific or engineering applications when the function of x that needs to be optimized is also a function of a continuous independent parameter (e.g., time, position, speed, frequency) that can assume an infinite set of values in a specified range. The optimization might entail adjusting variables x1 , x2 , ... , x_n so as to optimize the function of interest over a given range of the independent parameter. In such an application, the function of interest can be sampled with respect to the independent parameter, and a vector of the form

F(x) = [f (x, t_1) f (x, t_2) ••• f (x, t_m)]T can be constructed, where t is the independent parameter. Now if we let f_i (x) ≡ f (x, t_i) we can write F (x) = [f_1 (x) f_2 (x) ••• f_m (x)]T

A solution of such a problem can be obtained by optimizing functions fi (x) for i = 1, 2, ..., m simultaneously. Such a solution would, of course, be approximate because any variations in f (x, t) between sample points are ig- nored. Nevertheless, reasonable solutions can be obtained in practice by using a sufficiently large number of sample points. This approach is illustrated by the following example.

Example 1 The step response y(x, t) of an nth-order control system is re- quired to satisfy the specification

$$y_0(\mathbf{x}, t) = \begin{cases} t & \text{for } 0 \le t < 2 \\ 2 & \text{for } 2 \le t < 3 \\ -t + 5 & \text{for } 3 \le t < 4 \\ 1 & \text{for } 4 \le t \end{cases} \qquad (2)$$

as closely as possible. Construct a vector F(x) that can be used to obtain a function f (x, t) such that $y(x, t) \approx y_0(x, t)$ for $0 \le t \le 5$

Solution The difference between the actual and specified step responses, which constitutes the approximation error, can be expressed as f (x, t) = y(x, t) − y0 (x, t) and if f (x, t) is sampled at t = 0, 1, 2, ..., 5, we obtain $F(x) = [f_1(x)\ f_2(x) \cdots f_6(x)]T$

where

$f_1(x) = f(x, 0) = y(x, 0)$

$f_2(x) = f(x, 1) = y(x, 1) - 1$

$f_3(x) = f(x, 2) = y(x, 2) - 2$

$f_4(x) = f(x, 3) = y(x, 3) - 2$

$f_5(x) = f(x, 4) = y(x, 4) - 1$

$f_6(x) = f(x, 5) = y(x, 5) - 1$

The problem is illustrated in Fig. 2. It can be solved by finding a point x = x∗ such that F(x∗) ≈ 0. Evidently, the quality of the approximation obtained for the step response of the system will depend on the density of the sampling points and the higher the density of points, the better the approximation. Problems of the type just described can be solved by defining a suitable objec-
tive function in terms of the element functions of F(x). The objective function

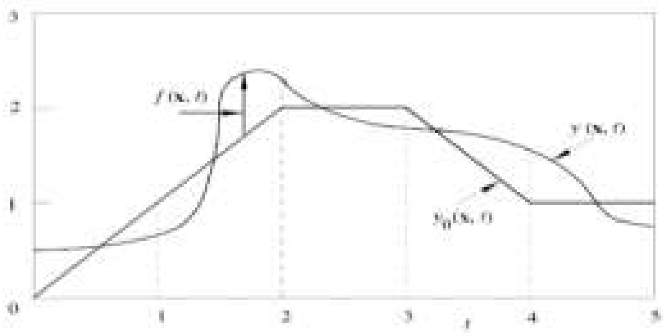

Figure 2. Graphical construction for Example 1.

must be a scalar quantity and its optimization must lead to the simultaneous optimization of the element functions of F(x) in some sense. Consequently, a norm of some type must be used. An objective function can be defined in terms of the Lp norm as

$$F \equiv L_p = \left\{ \sum_{i=1}^{m} |f_i(\mathbf{x})|^p \right\}^{1/p} \tag{3}$$

where p is an integer.

Several special cases of the Lp norm are of particular interest.

If p = 1

$$F \equiv L_1 = \sum_{i=1}^{m} |f_i(\mathbf{x})| \tag{4}$$

and, therefore, in a minimization problem like that in Example 1.1, the sum of the magnitudes of the individual element functions is minimized. This is called an L1 problem. If p = 2, the Euclidean norm

$$F \equiv L_2 = \left\{ \sum_{i=1}^{m} |f_i(\mathbf{x})|^2 \right\}^{1/2} \tag{5}$$

is minimized, and if the square root is omitted, the sum of the squares is minimized. Such a problem is commonly referred as a least-squares problem.

1 For more details on vector and matrix norms. In the case where p = ∞, if we assume that there is a unique maximum of $|f_i(x)|$ designated F^ such that

$$\hat{F} = \max_{1 \le i \le m} |f_i(\mathbf{x})| \tag{6}$$

then we can write

$$F \equiv L_\infty = \lim_{p\to\infty}\left\{\sum_{i=1}^{m}|f_i(\mathbf{x})|^p\right\}^{1/p}$$
$$= \hat{F}\lim_{p\to\infty}\left\{\sum_{i=1}^{m}\left[\frac{|f_i(\mathbf{x})|}{\hat{F}}\right]^p\right\}^{1/p} \quad (7)$$

Since all the terms in the summation except one are less than unity, they tend to zero when raised to a large positive power. Therefore, we obtain

$$F = \hat{F} = \max_{1\le i\le m}|f_i(\mathbf{x})| \quad (8)$$

Evidently, if the $L\infty$ norm is used in Example 1.1, the maximum approximation error is minimized and the problem is said to be a minimax problem.
Often the individual element functions of F(x) are modified by using con- stants w1 , w2 , ... , wm as weights. For example, the least-squares objective function can be expressed as

$$F = \sum_{i=1}^{m}[w_i f_i(\mathbf{x})]^2 \quad (9)$$

so as to emphasize important or critical element functions and de-emphasize unimportant or uncritical ones. If F is minimized, the residual errors in wi fi (x) at the end of the minimization would tend to be of the same order of magnitude, i.e., and so error in |wi fi (x)| ≈ ε error in |f (x)| ≈ ε $|w_i|$ Consequently, if a large positive weight wi is used with fi (x), a small residual error is achieved in |fi (x)|.

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