DYNAMIC SYSTEMS

Dragos Vasile VÎGA - PhD ,DEPARTAMENT OF MACHINES AND INSTALLATIONS, UNIVERSITY OF PETROSANI, ROMANIA
(e-mail: dragosviga71@gmail.com)
Olimpia PECINGINA - Assistant Professor, "University of Constantin Brancusi", Tg-jiu ,, ROMANIA, pecinginaolimpia@yahoo.com
Constantin BOGDAN - .PhD., Machines and Installation Departement, University of Petroşani , ROMANIA, tica1234ticabogdt@yahoo.com

Abstract: This article introduces concepts of dynamical systems theory such as phase space, phase space reconstruction and analysis of phase space, and their application to theanalysis of behaving agents.

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1. Dynamical Systems Theory

An ideal pendulum, for instance, has one degree of freedom — the arc φ along which it is swinging, and the knowledge of the pendulum's position φ(t) and its velocity $\dot{\varphi}$ (t) describes the motion of the pendulum fully, for all times t. The phase space of the ideal pendulum, therefore, is the two-dimensional space defined by φ(t) and $\dot{\varphi}$ (t), and the physical motion of the pendulum can be fully described by the motion through that phase space.

It turns out that the phase space of the ideal pendulum is an ellipse (see Figure 1). As the pendulum swings backward and forward in physical space, its (φ, $\dot{\varphi}$) coordinates in phase space move from (φmax , 0) through (0, $-\dot{\varphi}$ max), (−φmax , 0) and (0, $\dot{\varphi}$ max) back to (φmax , 0).

The trajectory ("orbit") through phase space — in the pendulum's case the elipse shown in Figure 1 is referred to as the "attractor", because the dynam- ical system will follow that particular orbit, irrespective of initial conditions it is "attracted" to that orbit through phase space.

The phase space of a dynamical system following Hamiltonian mechanics is defined as position z(t) and impulse p(t) = mv(t) (v is the velocity) along each degree of freedom the system has. This is a strict definition of "phase space" and "degree of freedom"; however, the term "degree of freedom" is sometimes also simply used to mean "a single coordinate of

phase space". Unless otherwise stated, this is the notion used in this book.

It follows that if a Hamiltonian system has n degrees of freedom, its phase space has 2n dimensions, position and impulse along each of the n degrees of freedom of the system.

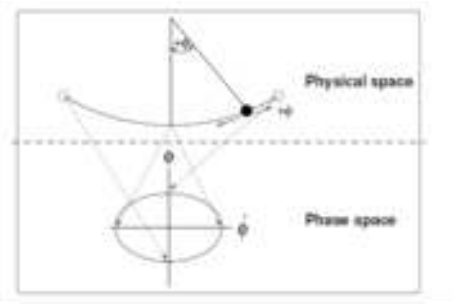

Figure 1. Physical movement and phase space of an ideal pendulum.

It is not always straightforward to state how many degrees of freedom a system has, and therefore what the size of its phase space is. Fortunately, this knowledge is not needed to reconstruct the attractor .

In the case of mobile robots, moving in a two-dimensional plane, however, one can say something about the degrees of freedom available to the robot. In the following, we will examine four different fundamental types of mobile robot: a fully holonomic robot (which we will refer to as "ball", because its motions are equivalent to those of a ball), a robot with differential drive, a robot with "Ackermann steering" (which we will refer to as "car", because its motions are equivalent to those of a conventional car), and a tracked robot (referred to as "train"). These four types of robot are shown in Figure 2.

The first three of these robots are all capable of assuming any position and orientation < x, y, φ > in space, but the means by which they can do so are different in the three cases.

We define a full degree of freedom as an axis (x, y or φ) along which any

position can be assumed without altering the positions along the remaining two axes. An equivalent definition would be that a full degree of freedom is an axis along which an external force can be applied without meeting resistance (assum- ing an "ideal" robot).

Following that definition, one can see that in the case of the ball, forces can be applied along all three axes independently, and the position along each of the three axes can be altered without altering the position in the other two axes. The ball therefore has three degrees of

freedom.

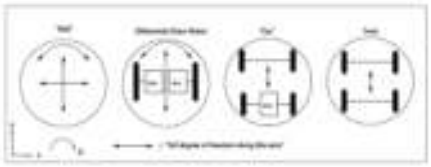

Figure 2. Different types of robot drive systems and their full degrees of freedom

The differential drive robot can change its position freely along the y and φ axes, but to change position along the x axis, the robot also has to change position along the φ axis at the same time. There are therefore two full degrees of freedom (y and φ), but because the robot is able to change position along the

x axis as well, it has more than two (but less than three) degrees of freedom.

In the case of the car, it can change its position freely along the y axis (one degree of freedom). But to change position along the φ axis, movement along the y axis is needed, so this isn't a full of degree of freedom. And in order to change position along the x axis, movement along the φ axis (and therefore also along the y axis) is needed, so this isn't a full degree of freedom either.

Finally, the train has only one full degree of freedom, y, and is incapable of assuming arbitrary positions in x and φ — positions along these two axes are pre-determined by the track.

It therefore has one degree of freedom, and its phase space is two-dimensional (y and $y^{\cdot}$).

The following hypothetical example is intended to illustrate the methods employed later in this book, to serve as a illustration of what we are trying to achieve by applying dynamical systems theory to mobile robotics.

Assume that a mobile robot is moving in some environment, perhaps along

a trajectory similar to that shown in Figure 4.8. The three variables that describe the robot's trajectory fully are position x(t) and y(t) and heading φ(t). As in all dynamical systems, these three variables can be described through differential equations. Furthermore, in the robot's case the three variables are coupled, be- cause the robot's control program, the physics of motion and the influence of the environment will mean that x, y and φ cannot change

completely independently from each other.

In a real robot, we do not know which differential equations describe the robot's motion, but let us assume, for argument's sake, a particular robot's motion was defined by the set of Equations :

$$\begin{aligned} \frac{dx}{dt} &= \dot{x} = -(y + \phi) \\ \frac{dy}{dt} &= \dot{y} = x + 0.15y \\ \frac{d\phi}{dt} &= \dot{\phi} = 0.2 + xz - 10z \end{aligned} \tag{1}$$

Suppose we had a dynamical system defined by the two differential equations given in Equation :

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -bx \end{aligned} \tag{2}$$

This is system is fully described by the variables x(t) and y(t), they form the phase space of the system.

In keeping with our "observation of robot behaviour" scenario, let us assume that we observe the behaviour of this system by logging a time series S (t) = x(t) at discrete points t in time. To reconstruct the phase space of this system from S (t), we obviously need to reconstruct only y(t), because x(t) is already given through S (t).

Equation indicates that

$$y = \frac{dx}{dt} = \frac{dS}{dt} \tag{3}$$

Therefore, the phase space can be reconstructed by plotting $S^{\cdot}$ (t) vs S (t).

The derivative of a variable x(t) is given by Equation .

$$\frac{dx}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h} \tag{4}$$

Following the definition given in Equation , we can approximate S˙ (t) by Equation :

$$\frac{dS(t)}{dt} = \dot{S}(t) = \frac{S(t+h) - S(t)}{h} \tag{5}$$

Because we have logged S at discrete times t, estimating S˙ means apply- ing Equation , using a suitable h, and consequently reconstructing the phase space means plotting S (t) vs

$$\frac{S(t+h) - S(t)}{h} \tag{6}$$

This method can be extended to higher dimensions than two, and therefore be used to reconstruct higher dimensional attractors. This will be discussed next. Again, suppose we measure some descriptive element of the agent's behavior over time, for example the movement of the agent in < x, y > space, obtain- ing two time series x(t) and y(t). The attractor D(tn) — the trajectory taken through phase space — can then be reconstructed through time-lag embedding as given in Equation :

$$D(tn) = (x(tn - (p-1)\tau), x(tn - (p-2)\tau), \ldots x(tn - \tau), x(tn)) \tag{7}$$

with x(t) being a sequential set of measurements (the time series), p being the embedding dimension and τ being the embedding lag. In order to reconstruct the system's phase space through time lag embedding from an observed time series, therefore, two parameters need to be chosen: the embedding dimension p and the embedding lag τ .

Choosing the embedding dimension. There are three possible scenarios: (i) the embedding dimension chosen is too small to reconstruct the attractor, (ii) it is "just right", or (iii) it is too large. Only the first case will result in errors, because an attractor whose dimension is larger than the chosen embedding dimension cannot be fully unfolded, which means that points that are distant in time end up as close neighbours in phase space (because these neighbours in space are distant in time they are referred to as "false nearest neighbours"). If the embedding dimension is the same or just slightly larger than the dimension of the attractor, reconstruction is obviously no problem. If the embedding dimension chosen is much larger than the attractor's dimension, there is theoretically no problem the attractor can be reconstructed perfectly but there are practical (computational and accuracy) reasons why this case is undesirable. It is therefore preferable to select the minimum embedding dimension.

This method determines the number of false nearest neighbours (close in the reconstructed phase space, but far apart in time) in the reconstructed phase space when this number is near zero, the attractor is properly unfolded and contains no self-intersections.

Choosing the embedding lag. The second variable to be chosen for the time lag embedding method is the embedding lag τ. The right choice of τ means determining that point at which the sample $x(t + \tau)$ of the observed time series contains new information, compared with x(t). For example, if a slow moving system is sampled at a high sampling frequency, τ is going to be large, because it will take many samples before $x(t + \tau)$ actually contains new information. On the other hand, if the sampling rate is low with respect to the motion of the system, τ is going to be small. First of all, there is a qualitative method to see the influence of increaseing τ. For a small τ, x(t) and $x(t + \tau)$ are essentially identical. If they are plotted against each other, therefore, all points would lie on the diagonal iden- tity line. As τ increases, the reconstructed attractor will expand away from the identity line. There are two further ways to determine the point in time at which x(t) and $x(t + \tau)$ contain different information. First suggest a suitable τ is found when the autocorrelation between x(t) and $x(t + \tau)$ has fallen below $e^{-1} = 0.37$ which can be considered a generalisation of the autocorrelation function (Equation (6)), has its first minimum:

$$MI = H(x) + H(x + \tau) - H(x, x + \tau) \quad (8)$$

with

$$H(x),\ H(x+\tau) \text{ and } H(x, x+\tau) \qquad (9)\ .$$

2. Testing for Determinism and Stationarity

All considerations presented in this chapter refer to deterministic systems, i.e. systems that are not mainly governed by stochastic (random) behaviour. We therefore need to establish first whether the time series x(t) is deterministic, i.e. casually dependent on past events, or not. To do this, we use the following method, described by Kaplan and Glass . The underlying assumption in determining whether the signal is deterministic or not is that in a deterministic signal D of length 2T , the first half of the signal should be usable as a "good" predictor for the second half in a purely stochastic (random) system this assumption would not hold. In other words: if a model-based prediction of the system is perfect (zero prediction error), the sys- tem is purely deterministic. If there is some small prediction error, the system has a deterministic component, and if the model-based prediction is only as good as a random guess, the system is not deterministic at all.

To find out whether the first half of D is a good predictor of the second half, we split the time series D into two halves of length T each, and construct an embedding D :

$$D(T+i) = [D(T+i), D(T+i-1), D(T+i-2)],\ \forall i = 3 \ldots T \qquad (10)$$

In other words, we construct an embedding for the second half of the time series, using an time lag τ of 1 and an embedding dimension p of 3 (of course, one could use other values for τ and p). To make a prediction of D(tk + 1) ($T < tk \leq 2T$), we determine the closest point Dc (tc) ($0 < tc \leq T$) to D(tk) in Euclidean distance, and select D(tc + 1) as the prediction of D(tk + 1). In this fashion all points of the second half are predicted (we always only predict one-step ahead). In order to decide whether this error is "large" or "small", we set it in relation to the error b of a baseline prediction of simply using the average of the first half of the signal as a prediction of the second. In a purely stochastic signal the ratio / b is 1 or larger than 1, indicating that the mean would have been the best prediction possible, and therefore that the system is non-deterministic. If, on the other hand, the ratio / b is smaller than 1, this indicates that the first half of the time series indeed is a good predictor of the

second, and that therefore the time series has a deterministic component.

There is a second way of establishing whether the time series is deterministic (i.e. signal values are dependent of signal values in the past) or stochastic (i.e. Non-stationary signals can often be made stationary by simple transformations.The simplest of these transformations is to compute the first difference between successive values of the time series. If the first difference is still not stationary, the process can be repeated (second difference). Another obvious step to take is to remove linear trends. This is achieved by simply subtracting that linear function $y = ax + b$ that best fits the time series (linear regression).

If the signal x(t) shows exponential growth over time, it sometimes can be made stationary by using $x(t)/x(t-1)$. Other transformations that may render non-stationary signals stationary are logarithmic or square root transformations. An exponential signal, for instance, can be linearised by computing the logarithm, and then made stationary by computing the first difference. Similarly, time series that follow a power law can be linearised by computing square roots or higher order roots.

Having established that the descriptor of the agent's behaviour (the logged time series) is indeed mainly deterministic and stationary, we are now ready to analyse the system's phase space quantitatively.

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