THE STUDY ON THE PRIMARY SEALING WITH COAXIAL ROUND-SHAPED SURFACES

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Abstract: The paperwork studies the way of calculation of the interstitial pressure with Reynolds equation for the particular case of the primary sealing with coaxial round-shaped surfaces, one fixed and the other one rotating around Oz axis.

Keywords: Primary sealing, equation, Reynolds, coaxial

1. INTRODUCTION

Through this paperwork the particular case of primary sealing analyzed in xxxx is studied. Let's presume that S_1 and S_2 are coaxial round-shaped surfaces, S_1 is fixed, S_2 has rotational movement with angular speed ω around Oz axis (fig. 1).

For determining $H_1 = H(r,\theta,t)$*,consider* $M_1(r,\theta,z) \in S_1$. It's noticed that M_1 and N_1 have the same z.

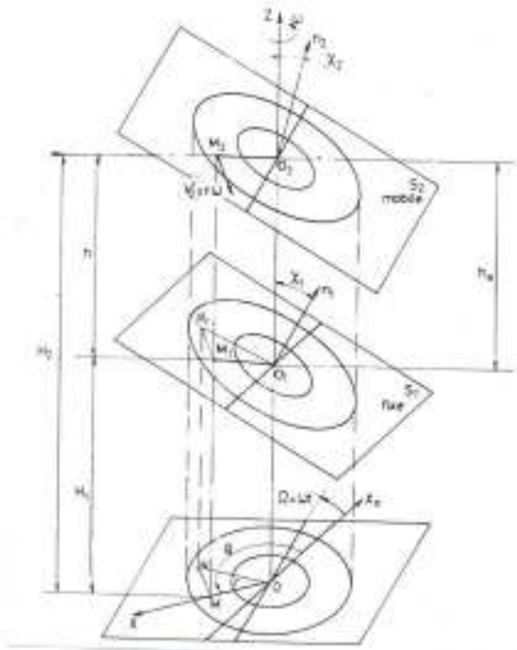

Figure 1.

$$z = NN_1 = OO_1 + O_1N_1 \cos NM_1O_1 = OO_1 + \frac{ON}{\sin NN_1O_1} \cos NM_1O_1 =$$

$$OO_1 + \frac{r\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \chi_1\right)} \cos\left(\frac{\pi}{2} - \chi_1\right) = OO_1 + R\sin\theta \ \mathrm{tg}\chi_1$$

Because χ_1 is low, it's approximated $\mathrm{tg}\chi_1 \approx \sin\chi_1 \approx \chi_1$, so:

$$H_1(r,\theta,t) = OO_1 + r\,\chi_1 \sin\theta \tag{1}$$

In a similar way for H_2 (replacing θ with θ-ωt due to S_2 movement), results:

$$H_2(r,\theta,t) = OO_1 + h_0 + r\,\chi_2 \sin(\theta - \omega t) \tag{2}$$

Because the fluid is thick (viscous), it sticks to the surfaces, so

$$v_1^\theta = 0,\ v_1^r = 0\ ,\ v_1^z = 0,\quad v_2^\theta = r\omega\ ,\ v_2^r = 0$$

We consider that h_0 is constant (uniform), so we could presume $v_2^z = 0$
The equation generalized Reynolds becomes:

$$\frac{\partial}{\partial r}\left[\frac{r}{\mu}(H_2 - H_1)^3 \frac{\partial p}{\partial r}\right] + \frac{\partial}{\partial\theta}\left[\frac{1}{\mu r}(H_2 - H_1)^3 \frac{\partial p}{\partial\theta}\right] = -6v_2^\theta \frac{\partial(H_1 + H_2)}{\partial\theta} \tag{3}$$

It's noted $h = H_2 - H_1$ and considered μ as constant. The equation (3) becomes:

$$\frac{\partial}{\partial r}\left(rh^3 \frac{\partial p}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{h^3}{r}\frac{\partial p}{\partial\theta}\right) = 6\mu\omega r^2\left[\chi_2\cos(\theta - \omega t) + \chi_1\cos\theta\right] \tag{4}$$

The equation (4) represents the Reynolds equation for the studied particular case. The admissible measures are introduced:

- the thickness $\bar{h} = \frac{h}{h_0}$
- the radius $\bar{r} = \frac{r}{R_e}$; $\bar{r}_i = \frac{R_i}{R_e}$

- the relative tilt $\chi = \frac{\chi_1}{\chi_2}$
- the tilt-thickness film parameter $e_2 = R_e \frac{\chi_2}{h_0}$
- the rotor angular position $\Omega = \omega r$
- the pressure $\overline{p} = \frac{h_0^2}{6\mu R_e^2 \omega} p$.

Then:
- for $r \in [R_i, R_e]$ it gets $\overline{r} \in [\overline{r}_i, 1]$, and
- $\frac{\partial}{\partial r} = \frac{\partial}{\partial \overline{r}} \frac{\partial \overline{r}}{\partial r}$ so $\frac{\partial}{\partial r} = \frac{1}{R_e} \frac{\partial}{\partial \overline{r}}$;

The fluid thickness equation is:

$$h = H_2 - H_1 = h_0 + r\chi_2 \left[\sin(\theta - \omega t) - \frac{\chi_1}{\chi_2} \sin\theta \right]$$

Divide with h_0 and results:

$$\frac{h}{h_0} = 1 + \frac{r}{R_e} \frac{R_e \chi_2}{h_0} \left[\sin(\theta - \omega t) - \frac{\chi_1}{\chi_2} \sin\theta \right],$$

so:

$$\overline{h} = 1 + \overline{r} e_2 [\sin(\theta - \Omega) - \chi \sin\theta] \tag{5}$$

The Equation (4) is re-written:

$$\frac{1}{R_e} \frac{\partial}{\partial \overline{r}} \left[\overline{r} R_e \left(\overline{h} h_0 \right)^3 \frac{1}{R_e} \frac{\partial}{\partial \overline{r}} \left(\frac{6\mu R_e^2 \omega}{h_0^2} \overline{p} \right) \right] + \frac{\partial}{\partial \theta} \left[\frac{\left(h_0 \overline{h} \right)^3}{R_e \overline{r}} \frac{\partial}{\partial \theta} \left(\frac{6\mu R_e^2 \omega}{h_0^2} \overline{p} \right) \right] =$$

$$= -6\mu\omega (\overline{r} R_e)^2 [\chi_2 \cos(\theta - \Omega) + \chi_1 \cos\theta],$$

so :

$$6\mu R_e h_0 \omega \frac{\partial}{\partial \overline{r}} \left[\overline{r} \overline{h}^3 \frac{\partial \overline{p}}{\partial \overline{r}} \right] + 6\mu R_e h_0 \omega \frac{\partial}{\partial \theta} \left[\frac{\overline{h}^3}{\overline{r}} \frac{\partial \overline{p}}{\partial \theta} \right] = -6\mu R_e h_0 \omega \frac{R_e \chi_2}{h_0} \overline{r}^2 \left[\cos(\theta - \Omega) + \frac{\chi_1}{\chi_2} \cos\theta \right]$$

Divide with $6\mu R_e h_0 \omega$ and results the Reynolds equation in dimensionless measures.

$$\frac{\partial}{\partial \overline{r}} \left[\overline{r} \overline{h}^3 \frac{\partial \overline{p}}{\partial \overline{r}} \right] + \frac{\partial}{\partial \theta} \left[\frac{\overline{h}^3}{\overline{r}} \frac{\partial \overline{p}}{\partial \theta} \right] = -e_2 \overline{r}^2 [\cos(\theta - \Omega) + \chi \cos\theta] \tag{6}$$

2. THE INTEGRATION OF THE REYNOLDS EQUATION

For easing the notations, instead of $\overline{h}, \overline{r}, \overline{p}$ we will use r, h, p. Because $e_2<1$ (from $R_e(\chi_1 + \chi_2) < h_0$, so $e(1+\chi)<1$) a serial of powers are developed after e_2 the *h şi p* functions.

We have:

$$h= 1+e_2h_1(r,\theta)\ ,\ cu\ h_1(r,\theta)=\ r\left[\sin(\theta-\Omega)-\chi\ \sin\theta\right]$$

$$p=p_0(r,\theta)+e_2p_1(r,\theta)+e_2^2p_2(r,\theta)+..... \tag{7}$$

With (6) the equation (7) becomes

$$\frac{\partial}{\partial r}\left[r(1+e_2h_1)^3\frac{\partial}{\partial r}\left(\sum_{k\ge 0}e_2^k p_k\right)\right]+\frac{\partial}{\partial\theta}\left[\frac{1}{r}(1+e_2h_1)^3\frac{\partial}{\partial\theta}\left(\sum_{k\ge 0}e_2^k p_k\right)\right]=-e_2r^2\left[\cos(\theta-\Omega)+\chi\cos\theta\right]$$

.

$$\frac{\partial\left[r(1+e_2h_1)^3\right]}{\partial r}\sum_{k\ge 0}e_2^k\frac{\partial p_k}{\partial r}+r(1+e_2h_1)^3\sum_{k\ge 0}e_2^k\frac{\partial^2 p_k}{\partial r^2}+$$
$$+\frac{1}{r}\frac{\partial\left[(1+e_2h_1)^3\right]}{\partial\theta}\sum_{k\ge 0}e_2^k\frac{\partial p_k}{\partial\theta}+\frac{1}{r}(1+e_2h_1)^3\sum_{k\ge 0}e_2^k\frac{\partial^2 p_k}{\partial\theta^2}=-e_2r^2\left[\cos(\theta-\Omega)+\chi\cos\theta\right]$$

or:

$$\left[(1+e_2h_1)^3+3e_2r(1+e_2h_1)^2\frac{\partial h_1}{\partial r}\right]\sum_{k\ge 0}e_2\frac{\partial p_k}{\partial r}+r(1+e_2h_1)^3\times$$
$$\times\sum_{k\ge 0}e_2\frac{\partial^2 p_k}{\partial r^2}+\frac{3e_2}{r}(1+e_2h_1)^2\frac{\partial h_1}{\partial\theta}\sum_{k\ge 0}e_2^k\frac{\partial p_k}{\partial\theta}\ \frac{1}{r}(1+e_2h_1)^3\times \tag{8}$$
$$\times\sum_{k\ge 0}e_2{}^k\frac{\partial^2 p_k}{\partial\theta^2}=-e_2r^2\left[\cos(\theta-\Omega)+\chi\cos\theta\right]$$

For identification of the e_2^k factors we get:

For e_2^0: $\dfrac{\partial p_0}{\partial r}+r\dfrac{\partial^2 p_0}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial^2 p_0}{\partial\theta^2}=0$.

The conditions $p_0(r,\theta)=p_0(1,\theta)=0$ şi $p_0(r,\theta)=p_0(r,\theta+2\pi)=0$

will get $p_0=0$.

For e^m, $m\ge 4$:

$$\frac{\partial p_m}{\partial r}+\left[3h_1+3r\frac{\partial h_m}{\partial r}\right]\frac{\partial p_{m-1}}{\partial r}+\left[3h_1^2+6rh_1\frac{\partial h_1}{\partial r}\right]\frac{\partial p_{m-2}}{\partial r}+\left[h_1^3+3rh_1^2\frac{\partial h_1}{\partial r}\right]\frac{\partial p_{m-3}}{\partial r}+r\frac{\partial^2 p_m}{\partial r^2}+$$
$$3rh_1\frac{\partial^2 p_{m-1}}{\partial r^2}+3rh_1^2\frac{\partial^2 p_{m-2}}{\partial r^2}+rh_1^3\frac{\partial^2 p_{m-3}}{\partial r^2}+\frac{3}{r}\frac{\partial h_1}{\partial\theta}\frac{\partial p_{m-1}}{\partial\theta}+\frac{6}{r}h_1\frac{\partial h_1}{\partial\theta}\frac{\partial p_{m-2}}{\partial\theta}+\frac{3}{r}h_1^2\frac{\partial h_1}{\partial\theta}\frac{\partial p_{m-3}}{\partial\theta}+$$
$$\frac{1}{r}\frac{\partial^2 p_m}{\partial\theta^2}+\frac{3h_1}{r}\frac{\partial^2 p_{m-1}}{\partial\theta^2}+\frac{3}{r}h_1^2\frac{\partial^2 p_{m-2}}{\partial\theta^2}+\frac{1}{r}h_1^3\frac{\partial^2 p_{m-3}}{\partial\theta^2}=0$$

We take the equations:

$$r\frac{\partial^2 p_1}{\partial r^2}+\frac{1}{r}\frac{\partial^2 p_1}{\partial \theta^2}+\frac{\partial\ p_1}{\partial r}=-r^2\left[\cos(\theta-\Omega)+\chi\cos\theta\right] \tag{9}$$

And respectively:

$$r\frac{\partial^2 p_2}{\partial r^2}+\frac{1}{r}\frac{\partial^2 p_2}{\partial \theta^2}+\frac{\partial\ p_2}{\partial r}=$$
$$-\left[3h_1+3r\frac{\partial h_1}{\partial r}\right]\frac{\partial p_1}{\partial r}-3rh_1\frac{\partial^2 p_1}{\partial r^2}-\frac{3}{r}\frac{\partial h_1}{\partial \theta}\frac{\partial p_1}{\partial \theta}-\frac{3}{r}h_1\frac{\partial^2 p_1}{\partial \theta^2} \tag{10}$$

Because $p(r,\theta) = p(r,\theta+2\pi)$ it's expected that also $p_1(r,\theta) = p_1(r,\theta+2\pi)$ so we can solve p_1 related to θ in a Fourier series, meaning:

$$p_1(r,\theta)=p_1^0(r)+\sum_{k\geq1}\left[p_1^{ck}(r)\cos k\theta+p_1^{sk}(r)\sin k\theta\right] \tag{11}$$

We imply the equation solving (9)

$$p_1(r,\theta)=-\frac{1}{8}\left[r^3-\left(r_i^2+1\right)r+r_i^2\frac{1}{r}\right]\left[(\cos\Omega+\chi)\cos\theta+\sin\Omega\sin\theta\right] \tag{12}$$

For solving the (10) equation, because *p(r,θ)= p(r,θ+2π) şi* $p_2(r,\theta)= p_2(r,\theta+2\pi)$ we can develop p_2 related to θ in a Fourier series, meaning:

$$p_2(r,\theta)=p_2^0(r)+\sum_{k\geq1}\left[p_2^{ck}(r)\cos k\theta+p_2^{sk}(r)\sin k\theta\right] \tag{13}$$

We introduce (13), $(8)_1$ and (9) în (10) and get:

$$r\left[\frac{d^2p_2^0}{dr^2}+\sum_{k\geq1}\left(\frac{d^2p_2^{ck}}{dr^2}\cos k\theta+\frac{d^2p_2^{sk}}{dr^2}\sin k\theta\right)\right]-\frac{1}{r}\left[\sum_{k\geq1}k^2\left(p_2^{ck}\cos k\theta+p_2^{sk}\sin k\theta\right)\right]+\frac{dp_2^0}{dr}+$$
$$\sum_{k\geq1}\left[\frac{dp_2^{ck}}{dr}\cos k\theta+\frac{dp_2^{sk}}{dr}\sin k\theta\right]=\frac{6}{8}\left[r(\cos\Omega-\chi)\cos\theta-\sin\Omega\sin\theta\right]\left[3r^3-\left(r_i^2+1\right)-r_i^2\frac{1}{r^2}\right]$$
$$\left[(\cos\Omega+\chi)\cos\theta+\sin\Omega\sin\theta\right]+\frac{3}{8}r\left[r(\cos\Omega-\chi)\sin\theta-\sin\Omega\cos\theta\right]\left[6r+r_i^2\frac{1}{r^2}\right]$$
$$\left[(\cos\Omega+\chi)\cos\theta+\sin\Omega\sin\theta\right]+$$
$$\frac{3}{8}\frac{1}{r}\left[r(\cos\Omega-\chi)\sin\theta-\sin\Omega\cos\theta\right]\left[r^3-\left(r_i^2+1\right)r+r_i^2\frac{1}{r}\right]\left[(-\cos\Omega+\chi)\sin\theta+\sin\Omega\sin\theta\right]+$$

$$\frac{3}{8}r\left[r(\cos\Omega-\chi)\sin\theta-\sin\Omega\cos\theta\right]\left[r^3-\left(r_i^2+1\right)r+r_i^2\frac{1}{r}\right]\left[(-\cos\Omega+\chi)\cos\theta+\sin\Omega\sin\theta\right]$$

The (14) connection is implied

$$\begin{aligned}
&p_2(r,\theta)=p_2^0(r)+p_2^{C2}(r)\cos 2\theta+p_2^{s2}(r)\sin 2\theta= \\
&+2\left(r_i^2+1\right)r^2-3r^4\ \left.\right]+\frac{3}{32}\chi\sin\Omega\left[1-2r_i^2+\left(r_i^4-1\right)\frac{\ln r}{\ln r_i}+\frac{1}{32}\sin 2\Omega\right. \\
&\left[\frac{5+8r_i^2+5r_i^4}{1+r_i^2}r^2--2\frac{r_i^4}{1+r_i^2}\frac{1}{r^2}-3r_i^2-5r_i^4\ \right]\cos 2\theta+ \\
&+\frac{1}{32}\left(\cos 2\Omega-\chi^2\right)\left[-\frac{5+8r_i^2+5r_i^4}{1+r_i^2}r^2++2\frac{r_i^4}{1+r_i^2}\frac{1}{r^2}+3r_i^2+5r^4\ \right]\sin 2\theta\,.
\end{aligned}\tag{14}$$

From (8), (12) and (14) it results (15):

$$\begin{aligned}
&p\ (r,\theta)=e_2p_1(r,\theta)+e_2^2p_2(r,\theta)=-\frac{e_2}{8}\left[r^3-\left(r_i^2+1\right)r+\frac{r_i^2}{r}\right] \\
&\left[(\ \cos\Omega+\chi\ )\cos\theta-\sin\Omega\sin\theta\ \right]+ \\
&+\frac{e_2^2}{32}\left\{\chi\sin\Omega\left[1-2r_i^2+\left(r_i^4-1\right)\frac{\ln r}{\ln r_i}+2\left(r_i^2+1\right)r^2-3r^4\right]+\right. \\
&\left[\sin 2\Omega\cos 2\Omega-\left(\cos 2\Omega-\chi^2\right)\sin 2\theta\right]\times \\
&\times\left[\ \frac{5+8r_i^2+5r_i^4}{1+r_i^2}r^2-2\frac{r_i^4}{1+r_i^2}\frac{1}{r^2}-3r_i^2-5r^4\ \right]\Big\}.
\end{aligned}\tag{15}$$

We come again to the notations $\bar{p},\bar{r},\bar{r}_i$ and get the film pressure as (16):

$$\begin{aligned}
&\bar{p}\ (\bar{r},\theta)=-\frac{e_2}{8}\left[\bar{r}^3-\left(\bar{r}_i^2+1\right)\bar{r}+\frac{\bar{r}_i^2}{\bar{r}}\right]\left[(\ \cos\Omega+\chi\ )\cos\theta-\sin\Omega\sin\theta\ \right]+ \\
&+\frac{e_2^2}{32}\left\{\chi\sin\Omega\left[1-2\bar{r}_i^2+\left(\bar{r}_i^4-1\right)\frac{\ln\bar{r}}{\ln\bar{r}_i}+2\left(\bar{r}_i^2+1\right)\bar{r}^2-3\bar{r}^4\right]\right. \\
&+\left[\sin 2\Omega\cos 2\Omega-\left(\cos 2\Omega-\chi^2\right)\sin 2\theta\right] \\
&\left[\ \frac{5+8\bar{r}_i^2+5\bar{r}_i^4}{1+\bar{r}_i^2}\bar{r}^2-2\frac{\bar{r}_i^4}{1+\bar{r}_i^2}\frac{1}{\bar{r}^2}-3\bar{r}_i^2-5\bar{r}^4\ \right]\Big\}.
\end{aligned}\tag{16}$$

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