# STUDY ON THE PRIMARY SEALING WITH TILT SURFACES AND EQUABLE CENTER DISTANCE 

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> Abstract: The pressure distribution for the fluid film presumes a separation area model. Through the paperwork the mentioned model is the one where the primary sealing has tilt surfaces and equable center distance; the Reynolds equation is implied.

Keywords: primary sealing, Reynolds, model, fluid.

## 1. INTRODUCTION

The simplest model is the one where the surfaces are perfect leveled, even and parallel, with uniform film and a linear, straight distribution of the pressures. The nature of the film pressure was the subject of many researches and most of them mention the following bearing ways:

- hydrodynamic;
- hydrostatic;
- hydrodynamic and hydrostatic;
- elastic-hydrodynamic;
- mixed.

It hasn't yet reached any unanimous conclusion referring to a single bearing mechanism, although more hypotheses were studied. Most of the theoretical studies consider that the thickness on the rotation axis of the two surfaces remains constant (uniform).

When analyzing the performances of a frontal sealing the pressure from the casing (sealing space) should be known. .

Through this paperwork the hydrodynamic model is studied which is preponderantly considered in the front sealing

For the hydrodynamic tilt levelled surfaces the typical configuration of oil cone (the tilt angle $\alpha$ ) studied at the known hydrodynamic and under the name of skate with fixed geometry is a typical one for axial bearings (figure 1). The A detail represents an equivalent simplified geometry at a medium diameter.


Figure 1.

## 2. INFERRING THE REYNOLDS EQUATION FOR THE PRMARY SEALING WITH TILT SURFACES AND UNIFORM CENTER DISTANCE.

We study the movement of a thick fluid between two surfaces (fig.2) for the following hypotheses:

- The fluid is Newtonian and the flow is laminar;
- The exterior and inertial forces are negligible;
- The film thickness is so small relating the other dimensions of sealing space;
- The fluid speed is very low relating with the components in the other two directions; the speeds ratio is the same with the corresponding dimensions ratio;
- The irregularities level is low comparing the film thickness.


Figure 2
For the general case described in the 2 and 3 figures, the movement equations in cylindrical coordinates are:

$$
\begin{gather*}
\frac{1}{r} \frac{\partial}{\partial r}\left(\rho r v^{f}+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho v^{\theta}\right)\right)+\frac{\partial\left(\rho v^{z}\right)}{\partial z}+\frac{\partial \rho}{\partial t}=0  \tag{1}\\
\frac{\partial p}{\partial r}=\frac{\partial}{\partial z}\left[\mu \frac{\partial v^{r}}{\partial z}\right] \\
\frac{1}{r} \frac{\partial p}{\partial \theta}=\frac{\partial}{\partial z}\left[\mu \frac{\partial v^{\theta}}{\partial z}\right]  \tag{2}\\
\frac{\partial p}{\partial z}=0
\end{gather*}
$$

where:
$\mathrm{r}, \theta, \mathrm{z}$ are cylindrical coordinates;
$v^{r}, v^{\theta}, v^{z}$ are the speed components in cylindrical coordinates;
p is the pressure, $\rho$ is the specific density;
$\mu$ is the viscous factor.
On the $\mathrm{S}_{1}$ şi $\mathrm{S}_{2}$ surfaces (equations $z=H_{1}$ siz $z=H_{2}$ ) we consider the following conditions on limit:

$$
\begin{align*}
& v^{r}=v_{1}^{r} ; v^{\theta}=v_{1}^{\theta} ; v^{z}=v_{1}^{z}, \text { for } z=H_{l}(r, \theta, t) \\
& v^{r}=v_{2}^{r} ; v^{\theta}=v_{2}^{\theta} ; v^{z}=v_{2}^{z}, \text { for } z=H_{2}(r, \theta, t) \tag{3}
\end{align*}
$$

We reduce to a problem for the $p$ unknown. From (2) results $p=p(r, \theta, t)$. We consider $\mu=$ $\mu(r, \theta, t)$ and then (2) becomes:

$$
\begin{aligned}
\frac{\partial^{2} v^{r}}{\partial z^{2}} & =\frac{1}{\mu} \frac{\partial p}{\partial r} \\
\frac{\partial^{2} v^{\theta}}{\partial z^{2}} & =\frac{1}{\mu r} \frac{\partial p}{\partial \theta}
\end{aligned}
$$

The resulted system is twice integrated related to z , it results:

$$
\begin{align*}
& v^{f}=\frac{1}{\mu} \frac{\partial p}{\partial r} \frac{z^{2}}{2}+C_{1} z+C_{3} \\
& v^{\theta}=\frac{1}{\mu r} \frac{\partial p}{\partial \theta} \frac{z^{2}}{2}+C_{2} z+C_{4} \tag{4}
\end{align*}
$$



Figure 3.
For determining the $C_{1}, C_{2}, C_{3}, C_{4}$ constants we use the (3) conditions (we consider that $\mu, \frac{\partial p}{\partial r}, \frac{\partial p}{\partial \theta}$ don't depend on z ), it results:

$$
\begin{align*}
& v_{1}^{f}=\frac{1}{2 \mu} \frac{\partial p}{\partial r} H_{1}^{2}+\mathrm{C}_{1} \mathrm{H}_{1}+\mathrm{C}_{3} \\
& v_{1}^{\theta}=\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta} H_{1}^{2}+\mathrm{C}_{2} \mathrm{H}_{1}+\mathrm{C}_{4} \tag{5}
\end{align*}
$$

$$
\begin{align*}
& v_{2}^{f}=\frac{1}{2 \mu} \frac{\partial p}{\partial r} H_{2}^{2}+\mathrm{C}_{1} \mathrm{H}_{2}+\mathrm{C}_{3} \\
& v_{2}^{\theta}=\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta} H_{2}^{2}+\mathrm{C}_{2} \mathrm{H}_{2}+\mathrm{C}_{4} \tag{5'}
\end{align*}
$$

SO:

$$
\begin{align*}
& C_{1}=\frac{v_{1}^{r}-v_{2}^{r}}{H_{1}-H_{2}}-\frac{1}{2 \mu} \frac{\partial p}{\partial r}\left(H_{1}+H_{2}\right) ; \\
& C_{2}=\frac{v_{1}^{\theta}-v_{2}^{\theta}}{H_{1}-H_{2}}-\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta}\left(H_{1}+H_{2}\right) \tag{6}
\end{align*}
$$

so:

$$
\begin{align*}
& \mathrm{C}_{3}=v_{1}^{r}-\frac{v_{1}^{r}-v_{2}^{r}}{H_{1}-H_{2}} H_{1}+\frac{1}{2 \mu} \frac{\partial p}{\partial r} H_{1} H_{2} ; \\
& \mathrm{C}_{4}=v_{1}^{\theta}-\frac{v_{1}^{\theta}-v_{2}^{\theta}}{H_{1}-H_{2}} H_{1}+\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta} H_{1} H_{2} \tag{6'}
\end{align*}
$$

We replace in (4) the given constants from (6) and (6'), and results:

$$
\begin{gathered}
\mathrm{v}^{\mathrm{r}}=\frac{1}{2 \mu} \frac{\partial p}{\partial r} z^{2}+\left[\frac{v_{1}^{r}-v_{2}^{r}}{H_{1}-H_{2}}-\frac{1}{2 \mu} \frac{\partial p}{\partial r}\left(H_{1}+H_{2}\right)\right] z+v_{1}^{r}-\frac{v_{1}^{r}-v_{2}^{r}}{H_{1}-H_{2}} H_{1}+\frac{1}{2 \mu} \frac{\partial p}{\partial r} H_{1} H_{2} \\
\mathrm{v}^{\theta}=\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta} z^{2}+\left[\frac{v_{1}^{\theta}-v_{2}^{\theta}}{H_{1}-H_{2}}-\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta}\left(H_{1}+H_{2}\right)\right] z+v_{1}^{\theta}-\frac{v_{1}^{\theta}-v_{2}^{\theta}}{H_{1}-H_{2}} H_{1}+\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta} H_{1} H_{2},
\end{gathered}
$$

deci:

$$
\begin{align*}
& v^{f}=\frac{1}{2 \mu} \frac{\partial p}{\partial r}\left[z^{2}-z\left(H_{1}+H_{2}\right)+H_{1} H_{2}\right]+\frac{v_{1}^{r}-v_{2}^{r}}{H_{1}-H_{2}}\left(z-H_{1}\right)+v_{1}^{r} \\
& \mathrm{v}^{\theta}=\frac{1}{2 \mu r} \frac{\partial p}{\partial \theta}\left[z^{2}-z\left(H_{1}+H_{2}\right)+H_{1} H_{2}\right]+\frac{v_{1}^{\theta}-v_{2}^{\theta}}{H_{1}-H_{2}}\left(z-H_{1}\right)+v_{1}^{\theta} \tag{7}
\end{align*}
$$

The continuity equation (1) is integrated related with z on the interval $\left[H_{1}, H_{2}\right]$ and results:

$$
\begin{equation*}
\int_{H_{1}}^{H_{2}} \frac{\partial}{\partial r}\left(\rho r v^{r}\right) d z+\int_{H_{1}}^{H_{2}} \frac{\partial}{\partial \theta}\left(\rho v^{\theta}\right) d z+\int_{H_{1}}^{H_{2}} r \frac{\partial}{\partial z}\left(\rho v^{z}\right) d z+\int_{H_{1}}^{H_{2}} r \frac{\partial \rho}{\partial t} d z=0 \tag{8}
\end{equation*}
$$

Considering the deriving formulas of the integral with parameter

$$
\frac{d}{d \lambda} \int_{a(\lambda)}^{b(\lambda)} f(x, \lambda) d x=\int_{a(\lambda)}^{b(\lambda)} \frac{\partial f}{\partial \lambda}(x, \lambda) d x+b^{\cdot}(\lambda) f(b(\lambda), \lambda)-a^{\prime}(\lambda) f(a(\lambda), \lambda)
$$

and considering $\rho=\rho(r, \theta, t)$, then ( 8 ) becomes:

$$
\begin{array}{r}
\frac{\partial}{\partial r} \int_{H_{1}}^{H_{2}} \rho r v^{r} d z-\frac{\partial H_{2}}{\partial r} \rho r v_{2}^{r}+\frac{\partial H_{1}}{\partial r} \rho r v_{1}^{r}+ \\
+\frac{\partial}{\partial \theta} \int_{H_{1}}^{H_{2}} \rho v^{\theta} d z-\frac{\partial H_{2}}{\partial \theta} \rho v_{2}^{\theta}+\frac{\partial H_{1}}{\partial \theta} \rho v_{1}^{\theta}+r \rho\left(v_{2}^{z}-v_{1}^{z}\right)+r \frac{\partial \rho}{\partial t}\left(H_{2}-H_{1}\right)=0
\end{array}
$$

For the calculation of the above integrals we consider (7) and results:

$$
\begin{align*}
\frac{\partial}{\partial r}\left[\frac{\rho r}{\mu}\left(H_{2}-H_{1}\right)^{\frac{\partial}{\partial p}} \frac{\partial r}{\partial r}\right]+ & \frac{\partial}{\partial \theta}\left[\frac{\rho}{\mu r}\left(H_{2}-H_{1}\right)^{3} \frac{\partial p}{\partial \theta}\right]=6 \frac{\partial}{\partial r}\left[\rho r\left(v_{1}^{r}+v_{2}^{r}\right)\left(H_{2}-H_{1}\right)\right]-12 \rho r v_{2}^{r} \frac{\partial H_{2}}{\partial r}+12 \rho r v_{1}^{r} \frac{\partial H_{1}}{\partial r}+ \\
& +6 \frac{\partial}{\partial \theta}\left[\rho\left(v_{1}^{\theta}+v_{2}^{\theta}\right)\left(H_{2}-H_{1}\right)\right]-12 \rho v_{2}^{\theta} \frac{\partial H_{2}}{\partial \theta}+12 \rho v_{1}^{\theta} \frac{\partial H_{1}}{\partial \theta}+ \\
& +12 r \rho\left(v_{2}^{z}-v_{1}^{z}\right)+12 r\left(H_{2}-H_{1}\right) \frac{\partial p}{\partial t} \tag{9}
\end{align*}
$$

For an incompressible liquid, the (9) equation becomes:

$$
\begin{gathered}
\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left(H_{2}-H_{1}\right)^{3} \frac{\partial p}{\partial r}\right]+\frac{\partial}{\partial \theta}\left[\frac{1}{\mu r r}\left(H_{2}-H_{1}\right) \frac{\partial}{\partial \theta} \frac{\partial p}{\partial \theta}\right]= \\
6\left(H_{2}-H_{1}\right) \frac{\partial}{\partial r}\left[r\left(v_{1}^{r}+v_{2}^{r}\right)\right]+6 r\left(v_{1}^{r}+v_{2}^{r}\right) \frac{\partial\left(H_{2}-H_{1}\right)}{\partial r}-12 r v_{2}^{r} \frac{\partial H_{2}}{\partial r}+12 r v_{1}^{r} \frac{\partial H_{1}}{\partial r}+ \\
+6\left(H_{2}-H_{1}\right) \frac{\partial}{\partial \theta}\left[\left(v_{1}^{\theta}+v_{2}^{\theta}\right)\right]+6\left(v_{1}^{\theta}+v_{2}^{\theta}\right) \frac{\partial\left(H_{2}-H_{1}\right)}{\partial \theta}-12 r v_{2}^{\theta} \frac{\partial H_{2}}{\partial \theta}+12 r v_{1}^{\theta} \frac{\partial H_{1}}{\partial \theta}++12 r\left(v_{1}^{z}-v_{2}^{z}\right)
\end{gathered}
$$

So, the generalized Reynolds equation is:

$$
\begin{gather*}
\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left(H_{2}-H_{1}\right)^{3} \frac{\partial p}{\partial r}\right]+\frac{\partial}{\partial \theta}\left[\frac{1}{\mu r}\left(H_{2}-H_{1}\right)^{3} \frac{\partial p}{\partial \theta}\right]= \\
6\left(H_{2}-H_{1}\right) \frac{\partial}{\partial r}\left[r\left(v_{1}^{r}+v_{2}^{r}\right)\right]+6 r\left(v_{1}^{r}+v_{2}^{r}\right) \frac{\partial\left(H_{2}+H_{1}\right)}{\partial r}+ \\
+6\left(H_{2}-H_{1}\right) \frac{\partial}{\partial \theta}\left[\left(v_{1}^{\theta}+v_{2}^{\theta}\right)\right]+6\left(v_{1}^{\theta}-v_{2}^{\theta}\right) \frac{\partial\left(H_{2}+H_{1}\right)}{\partial \theta}++12 r\left(v_{1}^{z}-v_{2}^{z}\right) \tag{10}
\end{gather*}
$$

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