ALGORITHM TO DETERMINE THE EQUATIONS OF MOTION FOR PLANAR MULTIBODY SYSTEMS WITH LINKS AND WITHOUT JOINT CLEARANCE

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Abstract: *The purpose of this paper is to present an algorithm to assist in obtaining the equations of motion of plane mechanisms with joints with and without clearance*

Key words: algorithm, equation, plane mecanisms

INTRODUCTION

Based on previous works [1], using theorem of momentum [1] and from the theorem of moment of momentum [2] and graphical models shown in Figure. 1 and Figure. 2, obtained the equations of motion.

DYNAMICAL EQUATIONS OF THE MOTION

1.1. Case of the rotational kinematical joints without clearance

Isolating the element j, keeping into account the principle of action and reaction, denoting by H_k, V_k, H_h, V_h the reactions from the points O_k, O_h we obtain the representation drawn in the Fig. 1.

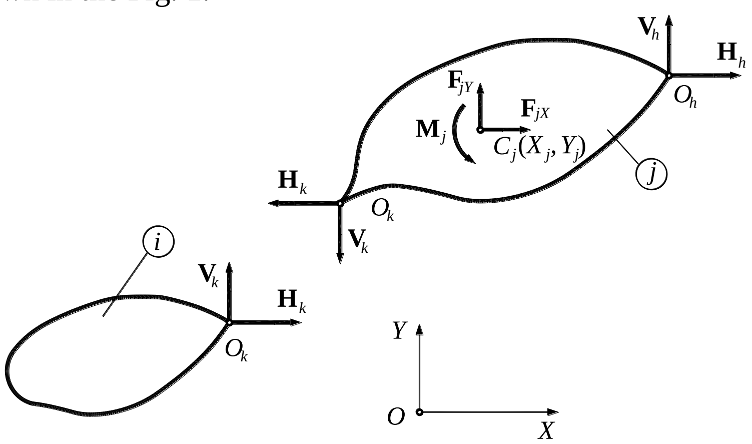

Figure. 1. Isolation of the element j with rotational kinematical joints with clearance.

Considering the notations m_j, mass of the element; J_j, moment of inertia relative to the centre of weight C_j; F_{jX}, F_{jY}, projections onto the axes OX, OY of the resultant of the external forces that act the element; M_j, resultant moment relative to the point C_j, from the theorem of momentum [1] one obtains the equations

$$m_j \ddot{X}_j = F_{jX} - H_k + H_h,\ m_j \ddot{Y}_j = F_{jY} - V_k + V_h, \tag{1.1}$$

and from the theorem of moment of momentum [1] relative to the point C_j one gets the equation

$$J_j \ddot{\theta}_j = M_j + H_k U_{kY}^{(j)} - V_k U_{kX}^{(j)} - H_h U_{hY}^{(j)} + V_h U_{hX}^{(j)}. \tag{1.2}$$

The equations (1.1), (1.2), using the notations $[\mathbf{m}_j] = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & J_j \end{bmatrix}$, $\{\mathbf{F}_j\} = [F_{jX}\ \ F_{jY}\ \ M_j]^T$, $\{\mathbf{R}_k\} = [H_k\ \ V_k]^T$, $\{\mathbf{R}_h\} = [H_h\ \ V_h]^T$, and keeping into account the relation $[\mathbf{B}_k^{(i)}] = \begin{bmatrix} 1 & 0 & -U_{ky}^{(i)} \\ 0 & 1 & U_{kx}^{(i)} \end{bmatrix}$ reads in a matrix form

$$[\mathbf{m}_j]\{\ddot{\mathbf{q}}_j\} = \{\mathbf{F}_j\} - [\mathbf{B}_k^{(j)}]^T \{\mathbf{R}_k\} + [\mathbf{B}_h^{(j)}]^T \{\mathbf{R}_h\}. \tag{1.3}$$

Based on the relation (1.3) it results that for a planar system, with the notations $[\mathbf{m}] = \begin{bmatrix} [\mathbf{m}_1] & [0] & \ldots \\ [0] & [\mathbf{m}_2] & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix}$, $\{\mathbf{q}\} = [\{q_1\}^T\ \ \{q_2\}^T\ \ \ldots]^T$, $\{\mathbf{F}\} = [\{\mathbf{F}_1\}^T\ \ \{\mathbf{F}_2\}^T\ \ \ldots]^T$, $\{\mathbf{R}\} = [\{\mathbf{R}_1\}^T\ \ \{\mathbf{R}_2\}^T\ \ \ldots]^T$, one gets the equation $[\mathbf{m}]\{\ddot{\mathbf{q}}\} = \{\mathbf{F}\} - [\mathbf{B}]^T\{\mathbf{R}\}$, where $[\mathbf{B}]$ is the matrix of constraints.

1.2. Case of the rotational kinematical joints with clearance

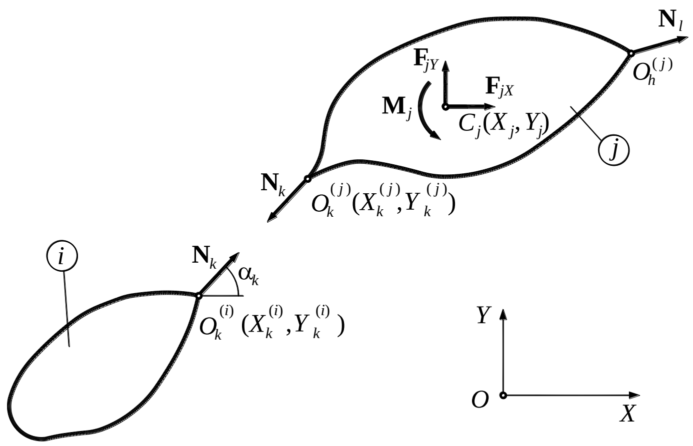

Figure. 2. Isolation of the element j, with rotational kinematical joints with clearance.

By the isolation of the element j, keeping into account the principle of action and reaction, and denoting by N_k, N_h the reactions at the points $O_k^{(j)}$, $O_h^{(j)}$, one obtains the representation from the Fig. 2.

From the theorem of momentum we obtain the equations

$$m_j \ddot{X}_j = F_{jX} - N_k D_{kX} + N_h D_{hX},\ m_j \ddot{Y}_j = F_{jY} - N_k D_{kY} + N_h D_{hY}, \tag{1.4}$$

and from the theorem of moment of momentum we get the equation

$$J_j \ddot{\theta}_j = M_j - N_k\left(-D_{kX}U_{kY}^{(j)} + D_{kY}U_{kX}^{(j)}\right) + N_h\left(-D_{hX}U_{hY}^{(j)} + D_{hY}U_{hX}^{(j)}\right). \tag{1.5}$$

The equations (1.4), (1.5) keeping into account the previous notations, reunite in the matrix equation

$$\left[\mathbf{m}_j\right]\left\{\ddot{\mathbf{q}}_j\right\} = \left\{\mathbf{F}_j\right\} - N_k\left[\mathbf{E}_k^{(j)}\right]^T + N_h\left[\mathbf{E}_h^{(j)}\right]^T \tag{1.6}$$

The matrix equation (1.6) is formally identical to the equation (1.3) in which one replaces the matrices $\left[\mathbf{B}_k^{(j)}\right]$ by the matrices $\left[\mathbf{E}_k^{(j)}\right]$.

1.3. Differential matrix equations of the motion of system

In the case when the point O_k is a rotational kinematical joint with clearance, and at the point O_h is a rotational kinematical joint without clearance, then, obviously, one obtains the equation $\left[\mathbf{m}_j\right]\left\{\ddot{\mathbf{q}}_j\right\} = \left\{\mathbf{F}_j\right\} - N_k\left[\mathbf{E}_k^{(j)}\right]^T + \left[\mathbf{B}_h^{(j)}\right]\left\{R_h\right\}$, and if, for instance, the point O_k has an imposed motion, and at the point O_h is a rotational kinematical joint with clearance, then we obtain the matrix equation $\left[\mathbf{m}_j\right]\left\{\ddot{\mathbf{q}}_j\right\} = \left\{\mathbf{F}_j\right\} - \left[\tilde{\mathbf{B}}_k^{(j)}\right]\left\{\mathbf{R}_k\right\} + N_h\left[\mathbf{E}_h^{(j)}\right]^T$.

Thus, for the system captured in the Fig. 9, at which the element 1 has a rotational motion, the elements being acted only by their own weights, and the rotational kinematical joint O_3 being with clearance, one gets the differential matrix equation

$$[\mathbf{m}]\{\ddot{\mathbf{q}}\} = \{\mathbf{F}\} - [\mathbf{B}]^T\{\mathbf{R}\}, \tag{1.7}$$

where $\{\mathbf{F}\} = \begin{bmatrix}0 & -m_2 g & 0 & 0 & -m_3 g & 0 & 0 & -m_4 g & 0\end{bmatrix}^T$, $\{\mathbf{R}\} = \begin{bmatrix}H_2 & V_2 & N_3 & H_4 & V_4 & H_5 & V_5\end{bmatrix}^T$,

$$\{\mathbf{q}\} = \begin{bmatrix}\{\mathbf{q}_2\}^T & \{\mathbf{q}_3\}^T & \{\mathbf{q}_4\}^T\end{bmatrix}^T,\ [\mathbf{m}] = \begin{bmatrix}[\mathbf{m}_2] & [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{m}_3] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{0}] & [\mathbf{m}_4]\end{bmatrix},\ [\mathbf{B}] = \begin{bmatrix}\left[\mathbf{B}_2^{(2)}\right] & [\mathbf{0}] & [\mathbf{0}] \\ -\left[\mathbf{E}_3^{(2)}\right] & \left[\mathbf{E}_3^{(3)}\right] & [\mathbf{0}] \\ [\mathbf{0}] & -\left[\mathbf{B}_4^{(3)}\right] & \left[\mathbf{B}_4^{(4)}\right] \\ [\mathbf{0}] & [\mathbf{0}] & -\left[\mathbf{B}_5^{(4)}\right]\end{bmatrix}.$$

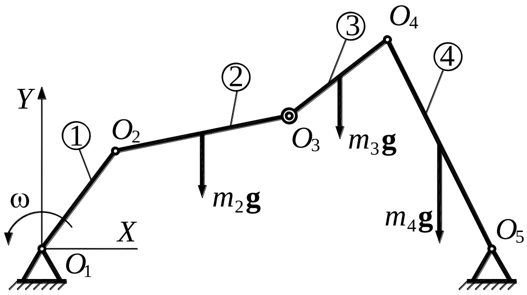

Figure. 3. Planar system of bars at which the element 1 has uniform rotational motion, the joint O_3 is with clearance, and the elements are acted only by their own weights.

At the matrix equation (1.7) we add the equality

$$[\mathbf{m}]\{\dot{\mathbf{q}}\} = \{\mathbf{C}\}, \tag{1.8}$$

where $\{C\} = l_1\omega[-\sin\omega t \;\; \cos\omega t \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$.

By derivation of the relation (1.8) with respect to time and from the equation (1.7) one gets [1, 13] the general matrix equation for the motion of the multibody systems

$$\begin{bmatrix} [\mathbf{m}] & [\mathbf{B}]^T \\ [\mathbf{B}] & [\mathbf{0}] \end{bmatrix} \begin{bmatrix} \{\ddot{\mathbf{q}}\} \\ \{\mathbf{R}\} \end{bmatrix} \{\dot{\mathbf{q}}\} = \begin{bmatrix} \{\mathbf{F}\} \\ \{\dot{\mathbf{C}}\} - [\dot{\mathbf{B}}]\{\dot{\mathbf{q}}\} \end{bmatrix}, \tag{1.9}$$

where the matrix $\{\mathbf{R}\}$ of the reactions is, in fact, the Lagrange multipliers' matrix with minus sign.

CONCLUSIONS

The establishing in the paper of a new form of the matrix of constraints made possible the elaboration of the multibody method that permits both the numerical study of the general motion of the planar jointed systems with and without clearances

The numerical applications solved here confirm the statements mentioned above.

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