TRAJECTORIES GENERATED BY MECHANISMS CONTAINING THE CHEBYSHEV DYAD

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Abstract. We study the trajectories generated by the articulated quadrangle, crank-rod and articulated pentagon mechanisms, each containing the Chebyshev dyad, characterized by the equality of three segments (two welded together), forming a RRR-type dyad. We chose mechanism sizes not mentioned by Chebyshev. We obtain symmetrical trajectories for the articulated quadrangle, trajectories formed of lines and circle arcs for the crank-rod mechanism and incomplete trajectories for the articulated pentagon because of not observing the geometrical conditions for a full rotation of the cranks.

Key-words: Chebyshev dyad, trajectories, fourbar, crank-rod, pentagon.

1.**Introduction**

The Chebyshev dyad consists of a bent rod, with the angle γ among the two bars and another part (figure 1), all these segments of length "a". It is given in [3], and in [2] it is shown how with this dyad, connected to a mechanism, we may create symmetrical curves. This dyad was studied thoroughly by Prof. Dijskman within various papers, connecting it to other observations on the mechanism geometry. So, in [4] the author connects it to a symmetrical Watt mechanism, with 6 bars, we may generate an 8-degree curve. In [1] it is synthesized a spherical quadrangle mechanism using functions and the superposition method, and finally reaching Chebyshev-type approximation functions. In [6] it is shown how the Chebyshev dyad is used for aeolian turbines. The Chebyshev dyad mechanism insures the regularization of the turbine paddle position depending on the wind force and direction. In [5] it is shown how to generate a symmetrical curve using the Chebyshev dyad (figure 2), with the side sizes calculated by Chebyshev and given in [2].

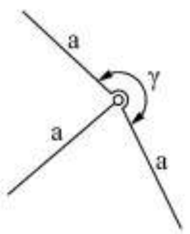

Fig. 1

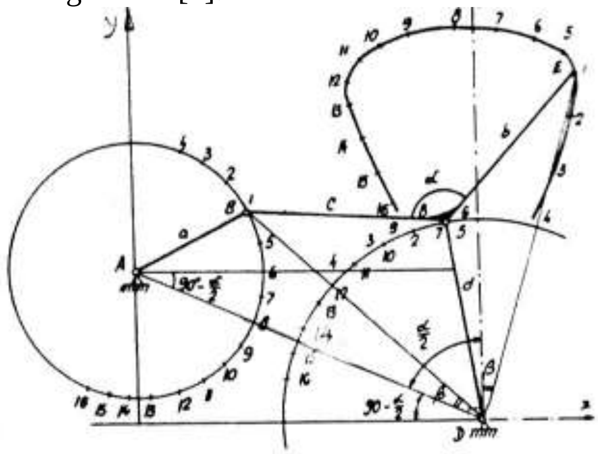

Fig. 2

2. Chebyshev dyad within the articulated quadrangle mechanism

This time we did not use mechanism sizes given by Chebyshev. We considered the mechanism in figure 3 where we see the BCDE Chebyshev dyad, have sides equal to “a”. We look for the trajectories generated by the point E.

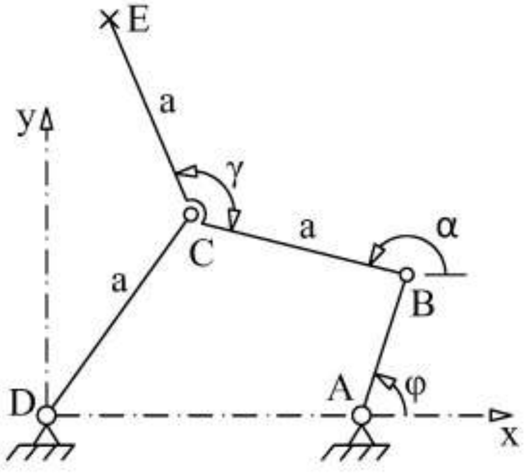

Fig. 3

We write the following equations:

$X_B = X_A + AB\cos\varphi;\ \ Y_B = Y_A + AB\sin\varphi;$ (1)

$X_E = X_C + a\cos(\gamma-\pi+\alpha);\ Y_E = Y_C + a\sin(\gamma-\pi+\alpha).$ (2)

We created a computer program where we used these equations and the RRR dyad subroutine. We used the following values, determined by attempts: XA=163, AB=25, a=48, γ=variable. We give beneath curves obtained for various values of γ, mentioned under the images.

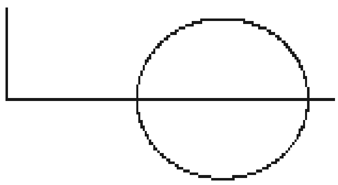

Fig. 4 (γ =0)

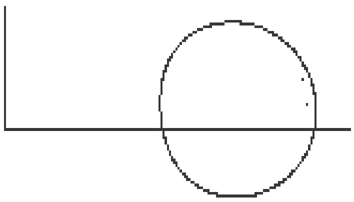

Fig. 5 (γ =10)

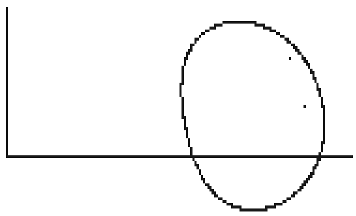

Fig. 6 (γ =20)

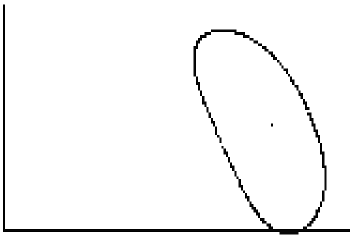

Fig. 7 (γ =45)

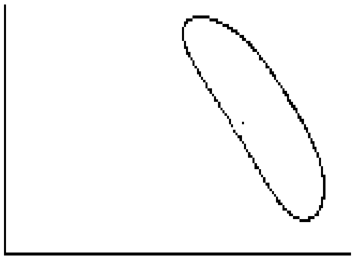

Fig. 8 (γ =60)

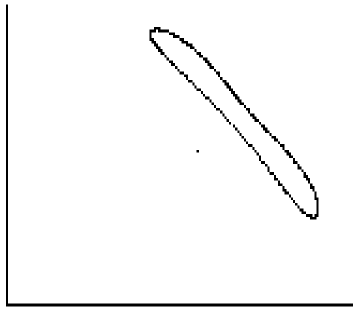

Fig. 9 (γ =80)

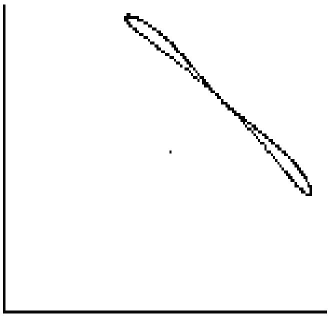

Fig. 10 (γ =90)

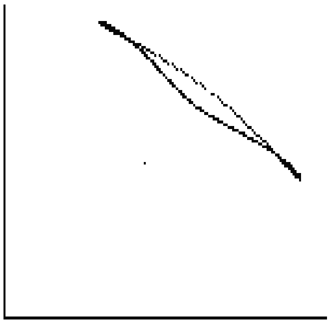

Fig. 11 (γ =100)

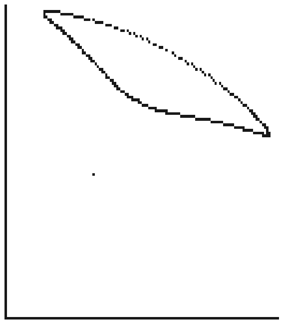

Fig. 12 (γ =120)

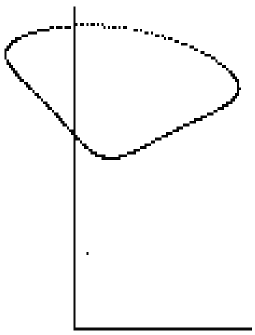

Fig. 13 (γ =160)

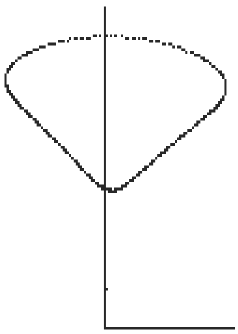

Fig. 14 (γ =175)

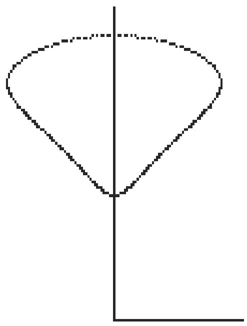

Fig. 15 (γ =180)

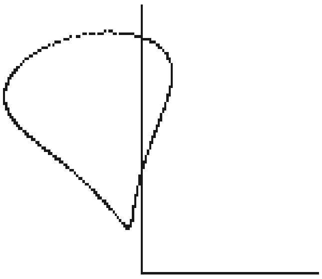

Fig. 16 (γ =210)

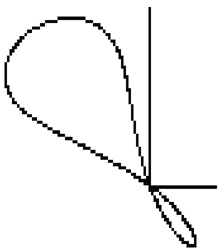

Fig. 17 (γ =250)

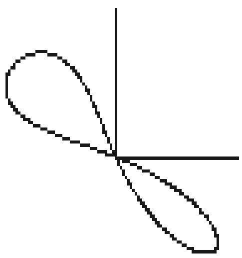

Fig. 18 (γ =270)

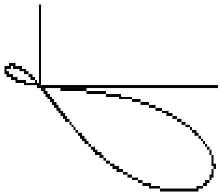

Fig. 19 (γ =300)

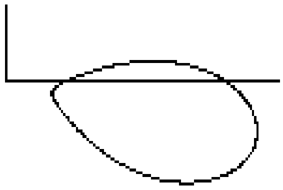

Fig. 20 (γ =330)

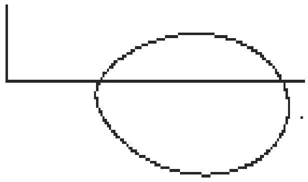

Fig. 21 (γ =345)

We notice that all curves generated are symmetrical, the symmetry axis varying from an angle to another.

3. Chebyshev dyad within the crank-rod mechanism

For the mechanism in figure 22, where AB=BC=BD=a, we write the equations:

$X_B = X_A + a\cos\varphi;\ \ Y_B = Y_A + a\sin\varphi;$ (3)

$X_B = S + a\cos\alpha;\ \ Y_B = a\sin\alpha;$ (4)

$X_D = X_B + a\cos(\gamma-\pi+\alpha);\ \ \ Y_D = Y_B + a\sin(\gamma-\pi+\alpha).$ (5)

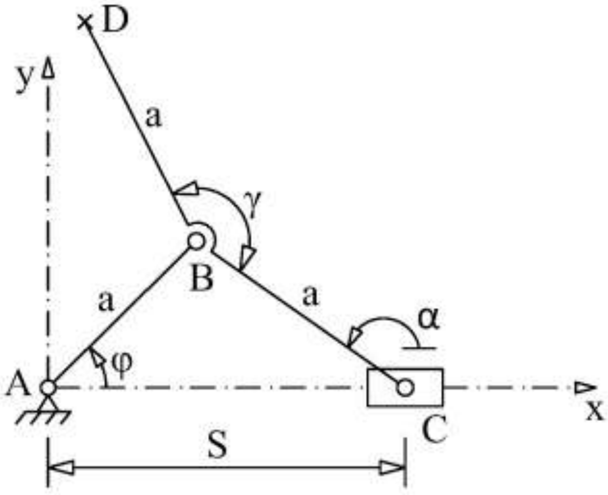

Fig. 22

Here we also look for the trajectories generated by a point, D. In figure 23 we show the mechanism generated for a position with the sizes: AB=a=60, γ=220.

Furthermore we worked with γ=120. In figure 24 we show the consecutive positions of the mechanism.

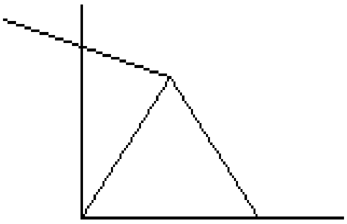

Fig. 23

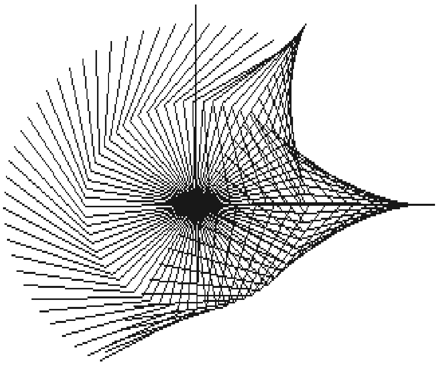

Fig. 24

We notice the generation of segments and of a circle arc; in figure 25 we may see this trajectory.

Because from an equation already mentioned we calculate sin α, and furthermore we need too the value of cos α, we use the sign in front of the radical, which may change. The trajectory in figure 25 connects to Se=-1, and the one in figure 26 to Se=+1, the same, but displaced in the base part of the image.

In order to explain the way to obtain this trajectory, we checked the movement in the trigonometrical dials: figure 27 – dial I, figure 28 – dial II, figure 20 – dial III, figure 30 – dial IV. For dial I we notice that the trajectory is a segment.

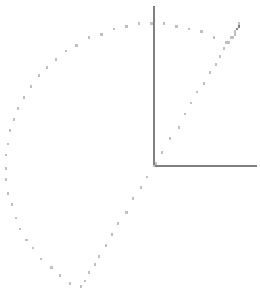

Fig. 25

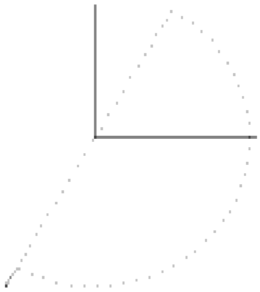

Fig. 26

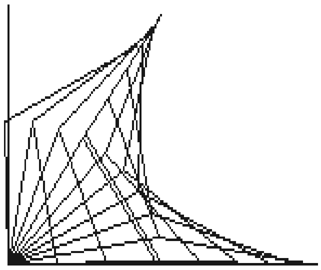

Fig. 27

For dial II the trajectory is a circle arc. The cause consists of that φ=90 CB segment is superposed on AB segment, furthermore acting as AB rod, meaning S=0, and point C remains in the center of the axes system. The same applies to dial III.

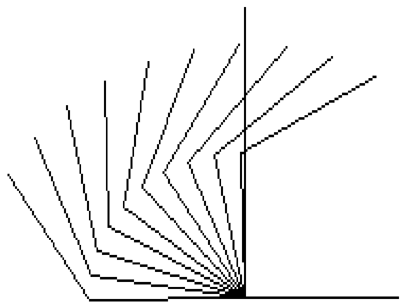

Fig. 28

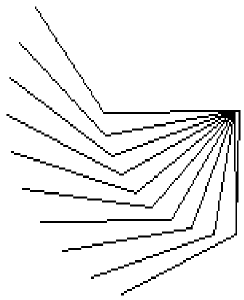

Fig. 29

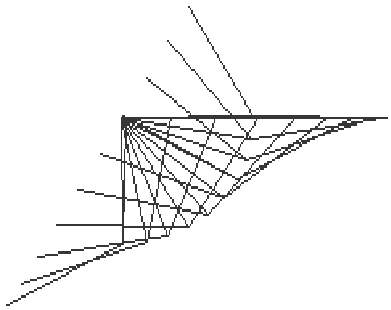

Fig. 30

In dial IV, after the rod passed φ=270 degrees, we come back to the trajectory after a segment, with S not zero and larger than zero.

In conclusion, for the crank-rod mechanism we do not obtain spectular trajectories.

4. Chebyshev dyad within the articulated pentagon mechanism

We consider the articulated pentagon mechanism in figure 31, similar to the quadrangle in figure 3, with the GD part besides. The Chebyshev dyad is BCDE. The new conductor part GD may rotate independently of the AB conductor, or dependently by a linear equation $\psi = c.\varphi$, modifying "c" from a case to another.

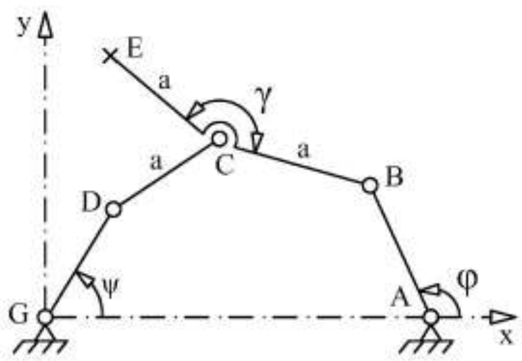

Fig. 31

We used the following input: XA=63, AB=25, a=48, GD=30, γ=156. Beneath we give the trajectories generated for various values of "c", values mentioned under the images.

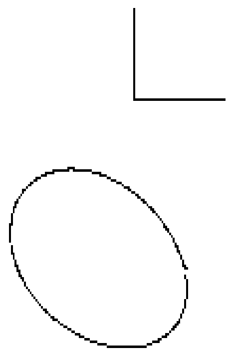

Fig. 32 (c=1)

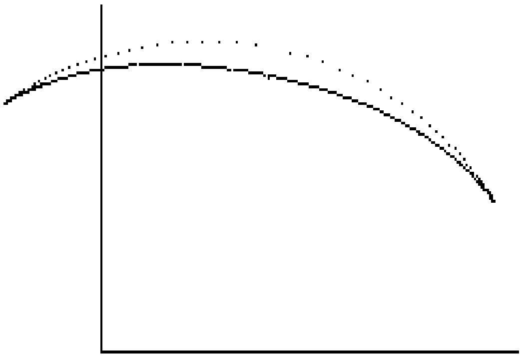

Fig. 33 (c=0)

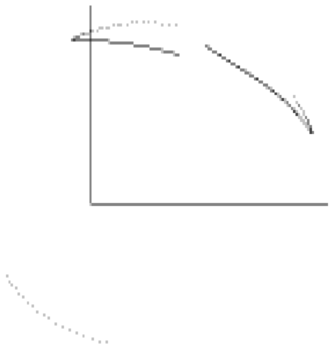

Fig. 34 (c=0,1)

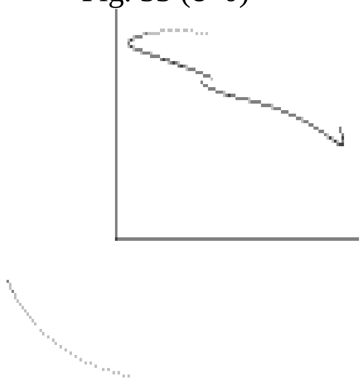

Fig. 35 (c=0,2)

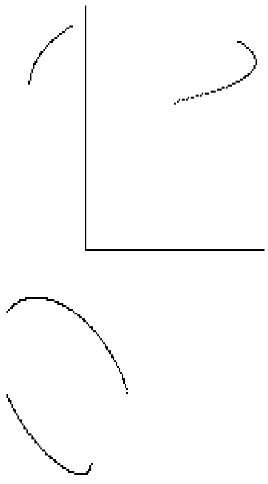

Fig. 36 (c=1,1)

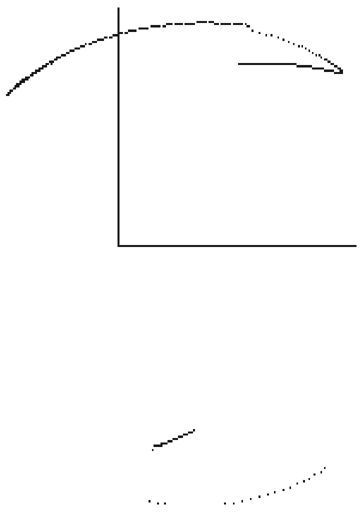

Fig. 37 (c= – 0,1)

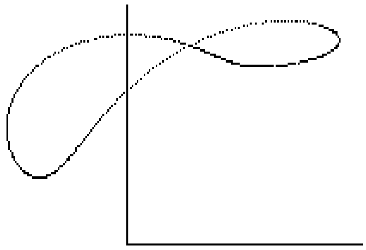

Fig. 38 (c= –1)

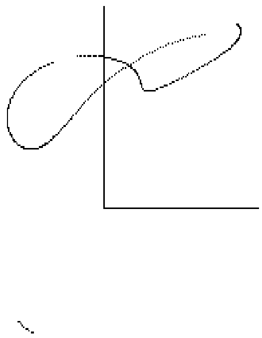

Fig. 39 (c= – 0,9)

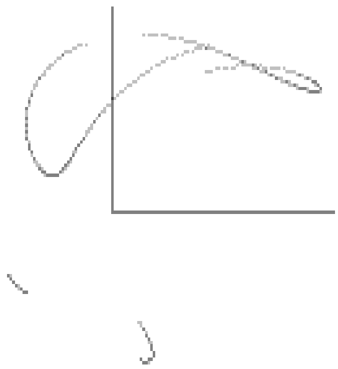

Fig. 40 (c= – 1,2)

We notice that some curves are symmetrical, and others are asymmetrical, some being incomplete, with different branches.

Furthermore we worked with GD=50, obtaining the images beneath.

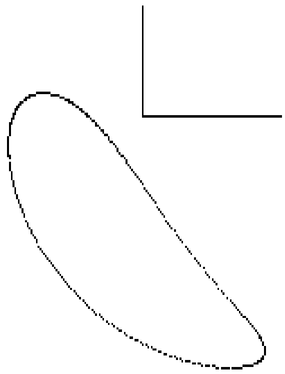

Fig. 41 (c=1)

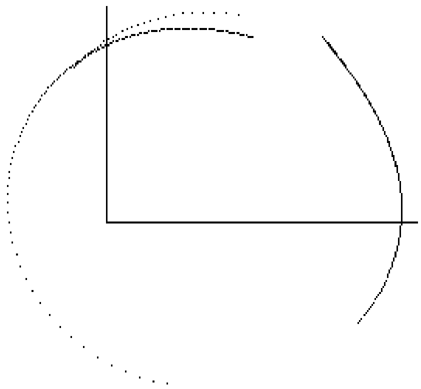

Fig. 42 (c=0,1)

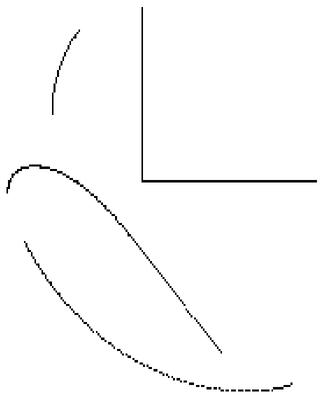

Fig. 43 (c=0,9)

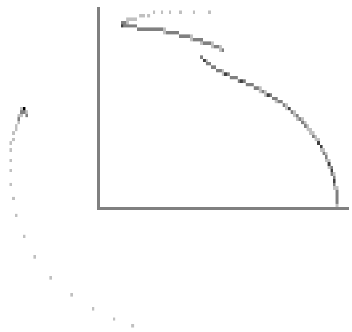

Fig. 44 (c=0,2)

We notice that other curves are generated, consequently the GD rod length influence the generated curves. And here, as in the examples above (still for the pentagon), there appear incomplete curves and more branches, caused by the Grashof-type conditions avoiding the complete rotation of both rods, for the adopted sizes. Furthermore the “c” value influences the position of the point D and of the rod GD related to AB, which modifies the conditions concerning the complete rotations.

5. Conclusions

It checked by many examples what trajectories are obtained for three types of mechanisms using the Chebyshev dyad, but with sizes not calculated by him. For the articulated pentagon mechanism we obtained symmetrical curves, but not spectacular. For the crank-rod mechanism the trajectories are not interesting because the rod and the crank are equal, they superpose within dials II and III, acting as a unique rod. For the articulated pentagon the curves are not special, generating many incomplete curves, or more different branches, caused by the values of “c” and by the part lengths, so they do not observe the complete rotation conditions of both rods.

References

1. Alizade, R., Can, F.C, Kilit, O. – Least square approximate motion generation synthesis of spherical linkages by using Chebyshev an equal spacing. Mechanism and Machine Theory. 61, 2013, pg. 123-135.
2. Artobolevskii, I.I, Leviţki, N. I., Cerkudinov, S.A. – Sintez ploskih mehanizmov. Izd. GIFML, Moskva, 1959.
3. Cebâşev, P.L. – Izbrannâe trudî. Izdatelstvo Akademii Nauk SSSR, Moskva, 1955.
4. Dijksman, E.A. – Watt-1 linkages with shunted chebyshev-dyads, producing symetrical 6-bar curves. Mechanism and Machine Theory, Volume 16, Issue 2, 1981, pag. 153-165.
5. Popescu Iulian -Sinteza mecanismului patrulater articulat pentru trasarea unor curbe simetrice In: "Lucrarile simpozionului de mecanisme si transmisii mecanice", Reşiţa, 1976, vol. I, pg. 291-298.
6. Shrestha, K. – Development of vertical axis flap type wind turbine with Chebyshev-Dyad link Mechanism. Nepal Engineers`Association Japan Center (NEA-JC). Research Digest, aprilie 2013, pg. 7.