SYNTHESIS OF CERTAIN MECHANISMS WHICH GENERATE LUNULES OVER POLYGONS WITH 5 SIDES

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Abstract. A synthesis for a mechanism relying on a sequence of articulated quadrilaterals which plot lunules along a penthagon is made. The plotting is successful. The speeds of plotting points are neither equal, nor constant. Afterward the synthesis of another mechanism, relying on a *sequence or articulated parallelograms is performed. In this case the lunules' plotting was made* at constant speed.

Keywords: Hyppocrates's lunules, mechanism to plot lunules

1. Introduction

The lunule, similar to a half-moon, represents a surface between two arcs of circle: one belongs to a circle on which a chord AB is placed, the second one is built along that chord, with the radius AB/2 and the centre in the middle of the chord. Hyppocrates from Chios (430-380 B.C.) has tried to solve the squaring of a circle – that is to built (by using only a only the ruler and compass) a squre with aria equal to that of a circle [2]. Ferdinand von Lindemannn has proven in 1882 that this technique cannot be successful, because of the irrational character of the constant π.

[3] presents an analysis of lunules and surfaces at isosceles triangles. Other demonstrations are provided by [4]. The case of a rectangular triangle is also approached in [5]. Detailed calculations are given in [6] with respect to the values of some surfaces for lunules when dealing with rectangular, isosceles and equilateral triangles. The correlation between the surfaces of lunules and that of a triangle, by using the Pitagora's Theorem, is demonstrated in [7]. Lunules are also plotted for a hexagon, by computing the lunules' arias.

A synthesis of two mechanisms plotting lunules is made in [1]. One is plotting lunules along the sides of a rectangular triangle, the other is plotting lunules along the sides of a square. To our knowledge, no other information on mechanisms generating lunules could be found in the specialty literature.

2. Synthesis of a mechanism for plotting lunules

Fig. 1 represents the starting point. It depicts a pentagon with notations on the angles presenting interest. The point B from the crank AB is plotting the arc HBF. The lunule for this side is obtained as the surface between the traced arc of circle and the arc with the radius OF, generated by a leading crank, as independent leading element. In order to avoid useless elements on the drawing, only the lunules corresponding to two sides are studied. The mechanism for the other sides is conceived in a similar manner.

The solving was made by using an articulated quadrilateral mechanism (ABCD), where CD=AB=a/2, BC=a. The sides AB and CD are not parallel and „a” represents the length of the pentagon's side.

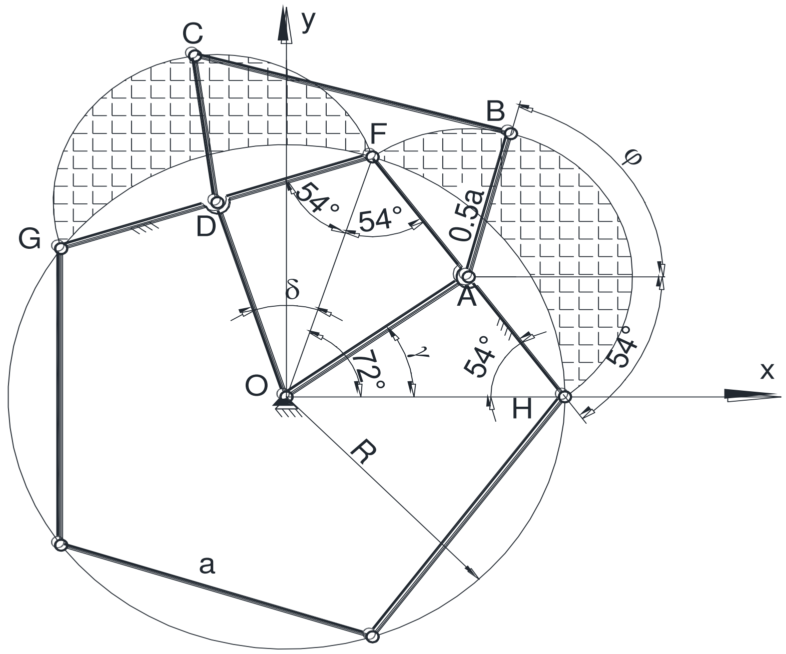


Fig. 1. Articulated quadrilateral mechanism used to plot lunules

The following equations can be written by using Fig. 1:

$$\frac{a}{\sin 72^o} = \frac{R}{\sin 54^o} \tag{1}$$

$$OA^2 = R^2 + \frac{a^2}{4} - 2R\frac{a}{2}\cos 54^0 \tag{2}$$

$$\begin{cases} x_A = R\cos\gamma \\ y_A = R\sin\gamma \end{cases} \tag{3}$$

$$OD^2 = \frac{a^2}{4} + R^2 - 2R\frac{a}{2}\cos 54^o \tag{4}$$

$$\begin{cases} x_D = OD\cos(72^o + \delta) \\ y_D = OD\sin(72^o + \delta) \end{cases} \tag{5}$$

$$\delta = \frac{\pi}{2} - 54^o \tag{6}$$

Calculations with respect to ABCD are made from this point on.

Fig. 2 depicts the pentagon and the quadrilateral mechanism for a certain position, generated by a program relying on the above equations, which call a procedure dedicated to the quadrilateral mechanism.

The point B is running from H up to F whilst C is running from F up to G. It means that the crank AB will cover an angle equal to 180^o.

Fig. 3 depicts the successive positions of the mechanism for this range, corresponding to the plotting of two lunules.

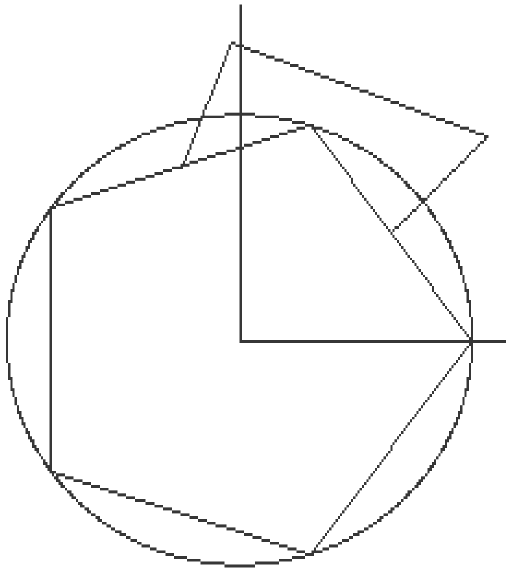


Fig. 2. The articulated quadrilateral mechanism in a certain position

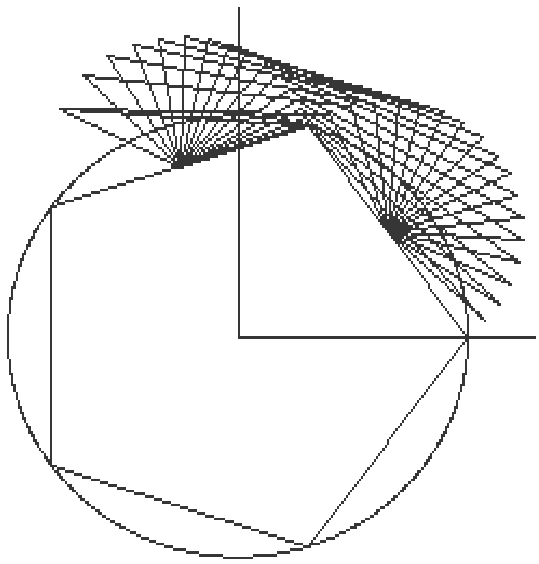
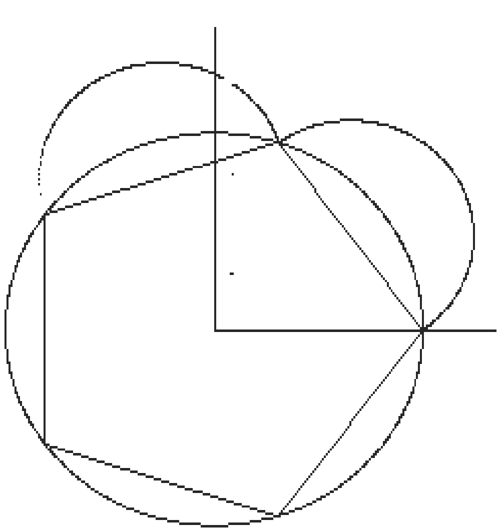


Fig. 3. Successive positions of the mechanism

It is easy to see that the rod BC is not parallel to previous positions of itself and the point C moves with a variable speed across the arc FCG. The angles FDC are variable (feature specific to an articulated quadrilateral). The point C reaches the position D. The figure represents a sub-range out of the rod's route – the explanation residing in the high value of the step φ chosen for cycling.

Fig. 4, representing the lunules plotted for both analyzed sides, reveals that C reaches the position G. Fig. 5 depicts the successive positions of the cranks plotting both lunules.



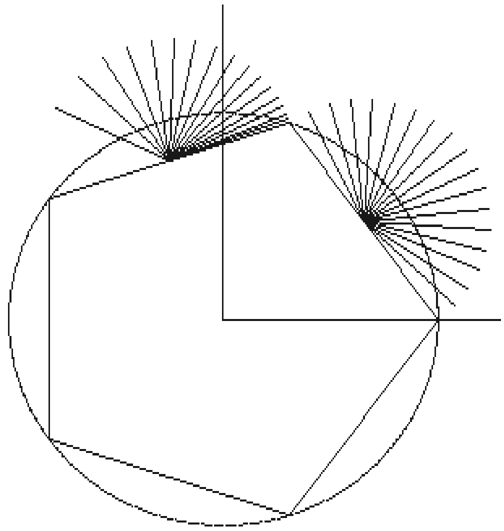


Fig. 4. Lunules plotted across both analyzed sides of the pentagon

Fig. 5. Successive positions of the cranks AB and CD which plot both lunules

Providing that an articulated parallelogram mechanism is used (Fig. 6), the deficiencies occurred when using an articulated quadrangle are avoided. In this case the lunule from the side HF, instead of being plotted by the point B, is plotted by the crank AP, made rigidly secured with AB, but shifted to it by the angle between the pentagon's sides.

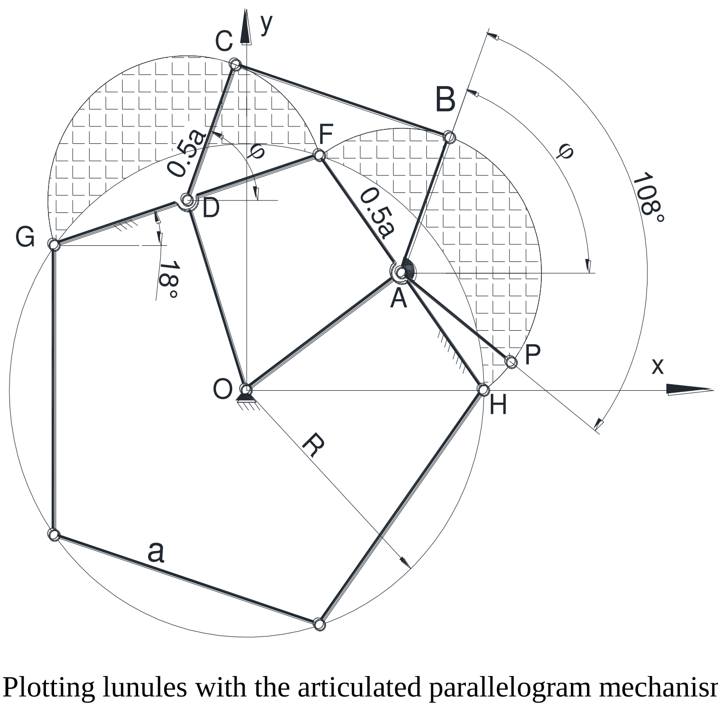


Fig. 6. Plotting lunules with the articulated parallelogram mechanism

The following equations can be written:

$$AB = CD = AP = \frac{a}{2} \tag{7}$$

$$\begin{cases} x_B = x_A + AB\cos\varphi \\ y_B = y_A + AB\sin\varphi \end{cases} \tag{8}$$

$$\begin{cases} x_C = x_D + CD\cos\varphi \\ y_C = y_D + CD\sin\varphi \end{cases} \tag{9}$$

$$\begin{cases} x_P = x_A + AP\cos\varepsilon \\ y_P = y_A + AP\sin\varepsilon \end{cases} \tag{10}$$

$$\varepsilon = 2\pi - \left(108^{o} - \varphi\right) \tag{11}$$

A program was conceived and used to plot the mechanism for a certain position (Fig. 7).

The successive positions of the mechanism are depicted by Fig. 8. One can see that the point B, which does not plot lunules, overcomes the point F. The crank AP stops in F – at the end of race. This represents a drawback for this mechanism.

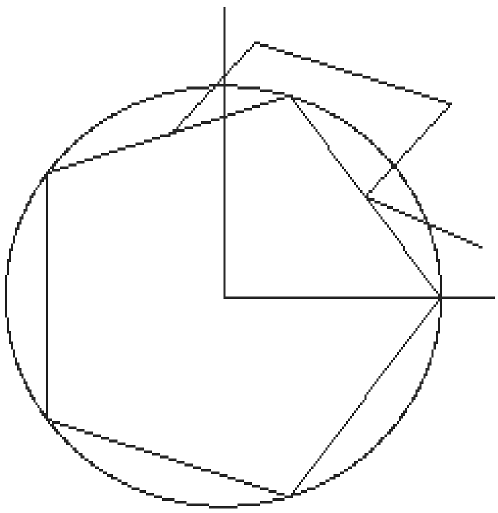


Fig. 7. Articulated parallelogram mechanism

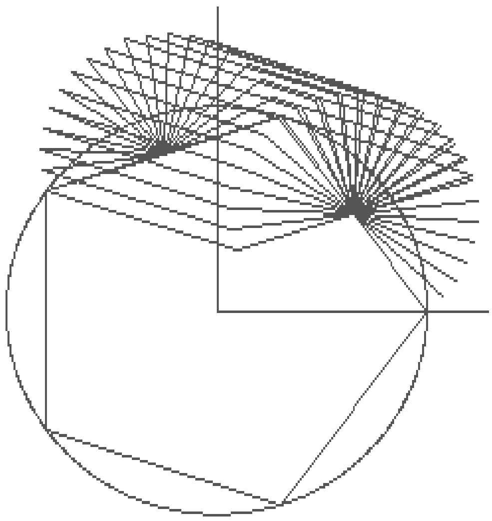


Fig. 8. Successive positions of the mechanism

Fig. 9 depicts the cranks AP and DC which plot lunules. The yielded lunules are given in Fig. 10.

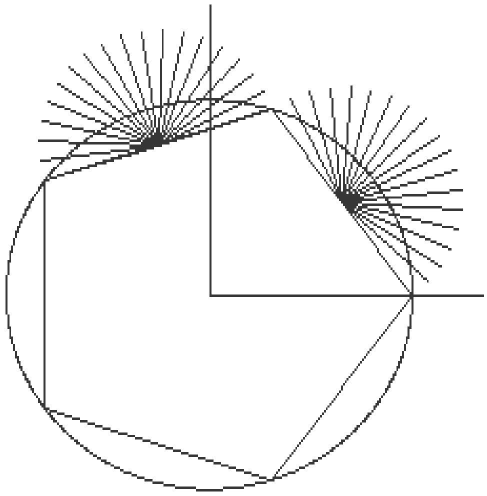


Fig. 9. The positions of the cranks AP and DC which plot lunules

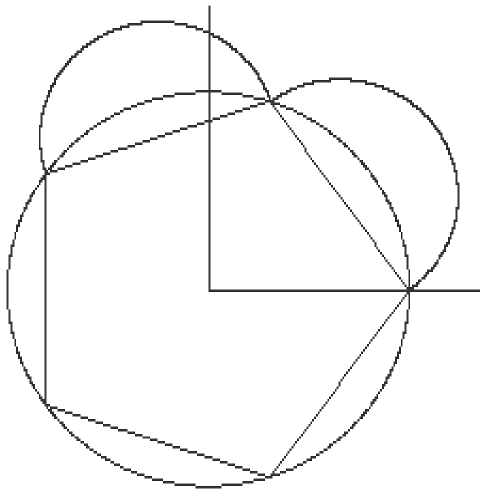


Fig. 10. Two lunules plotted along the sides HF and FG of the polygon

For the rest of the polygon's sides, shifted cranks (by an angle of 108^o) cranks and parallelogram are built, in a manner similar to that described above.

Conclusions

- Two mechanisms were conceived in order to generate lunules along the sides of a polygon with 5 sides.
- The first mechanism used to plot lunules was built by using a sequence of articulated quadrangles.
- The speeds of plotting points are not equal to each other.
- A second mechanism was built afterward. It relies on a sequence of some articulated parallelogram mechanisms. The cranks are shifter relative to the leading ones.
- The lunules are plotted with constant and equal speeds.

References

1. **Popescu Iulian, Sass, L**. - *Figuri geometrice ale antichității, generate cu mecanisme*. În: Vol. The VII-th Edition of the National Conference with International Participation, GRAFICA-2000, Craiova, oct. 2000, pg. 341-346.
2. https://en.wikipedia.org/wiki/Lune_of_Hippocrates] [Accessed: 15 th November 2015].
3. http://pro-didactica.ro/forum/index.php?forumID=8&ID=9435 [Accessed: 11 th November 2015].
4. https://tube.geogebra.org/material/simple/id/39636 [Accessed: 11 th November 2015].
5. https://tube.geogebra.org/m/45575 [Accessed: 12 th November 2015].
6. http://mathworld.wolfram.com/Lune.html] [Accessed: 16 th December 2015].
7. http://jwilson.coe.uga.edu/emt668/EMAT6680.F99/Williford/Lunes/lunes.html [Accessed: 15 th December 2015].