SYNTHESIS OF TWO MECHANISMS WHICH GENERATE LUNULES OVER AN EQUILATERAL TRIANGLE'S SIDES

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Abstract. A synthesis for a mechanism generating lunules across the sides of an equilateral triangle is performed. The mechanism consists of an articulated quadrilateral, prolonged with a RRR dyad. It is proved that lunules can be generate, but the speeds of plotting points are neither equal, nor constant. The synthesis of another mechanism, composed of two articulated parallelograms is afterward performed. Delayed cranks are used this time and consequently equal speeds are obtained.

Keywords: Hyppocrates's lunules, mechanism to plot lunules

1. Introduction

A lunule represents a plane geometric surface with a half-moon shape. It is built across a chord on a circle and it is delimited by the arc from a circle and another arc of circle, built across that chord, now representing the diameter for the new arc.

Hyppocrates from Chios (430-380 B.C.) has built lunules across the sides of a rectangular triangle. The small surfaces from the basis of people fingers' nails are also called lunules.

[3] presents an analysis of lunules and surfaces at isosceles triangles. Lunules generated along the sides of a trapezoid are studied in [4]. A history of Hyppocrates's lunules is presented in [5], along with their analysis when considering a isoscel rectangular triangle and a trapezoid. The lunules' aria is computed in [6] for a regular triangle. Geometrical aspects of lunules are approached in [1], along with details on the modality to construct. A synthesis of a mechanism used to trace lunules for a rectangular triangle can be found in [1]. Here is also provided a description for a mechanism used to plot lunules along the sides of a square. No other similar information was found with respect to lunules' plotting in the specialty literature.

2. Synthesis of the first mechanism

An equilateral triangle GHK was firstly built (Fig. 1). The angles which present interest for our paper are emphasized. The generated lunules are between the circle of radius OH and centre in O - described by the road OH, and arcs of circle whose diameters are the sides of the equilateral triangle. OH is a separate leading element, not related to the mechanism which generates the other arcs of lunules.

The mechanism's synthesis is made by noticing the following particular movements. If the road AB is rotating and the point B is plotting the arc corresponding to the lunule from the side GH, starting from G, then the point C, starting from H is plotting the arc corresponding to the lunule of the side HK. The articulated quadrilateral mechanism ABCD is built as described above.

For the lunule corresponding to the side KGM, a tracing point F is used. F belongs to the element EF which belongs to the dyad BFE of type RRR, added to the quadrangle.

In this way a mechanism DCBAFE was obtained. It consists of a leading element and two dyads.

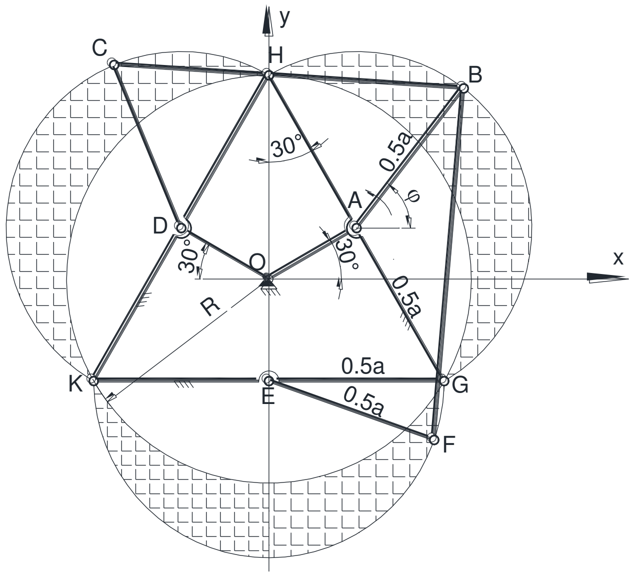


Fig. 1. Mechanism with two dyads tracing lunules

The following equations can be written:

$$KH = KG = GH = BC = BF = a \tag{1}$$

$$EH = GH\cos 30^o = a\cos 30^o \tag{2}$$

$$EG = a\sin 30^O \tag{3}$$

$$R = \frac{0,5a}{\cos 30^o} \tag{4}$$

$$OA = OD = OE = R\sin 30^o \tag{5}$$

$$\begin{cases} x_A = OA\cos 30^o \\ y_A = OA\sin 30^o \end{cases} \tag{6}$$

$$\begin{cases} x_D = -x_A \\ y_D = y_A \end{cases} \tag{7}$$

$$\begin{cases} x_E = 0 \\ y_E = -OE \end{cases} \tag{8}$$

$$AB = CD = EF = 0,5a \tag{9}$$

$$\begin{cases} x_B = x_A + \frac{a}{2}\cos(\varphi) \\ y_B = y_A + \frac{a}{2}\sin(\varphi) \end{cases} \tag{10}$$

$$\begin{cases} x_G = \frac{a}{2} \\ y_G = -OA \end{cases} \tag{11}$$

$$\begin{cases} x_H = 0 \\ y_H = a\cos 30^o - OE \end{cases} \tag{12}$$

$$\begin{cases} x_K = -\frac{a}{2} \\ y_K = y_G \end{cases} \tag{13}$$

A program was built to trace the mechanism for $\varphi = 55^o$ (Fig. 2).

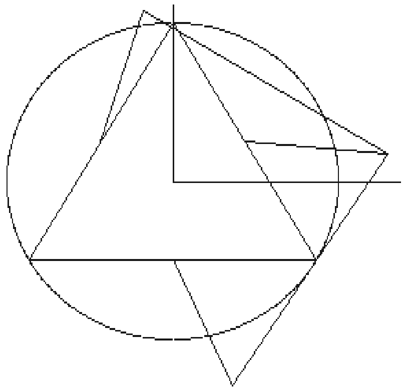


Fig. 2. The mechanism in a certain position

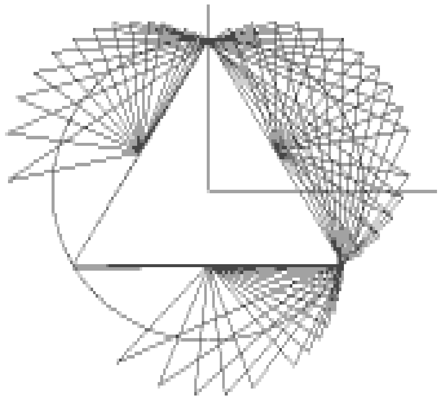
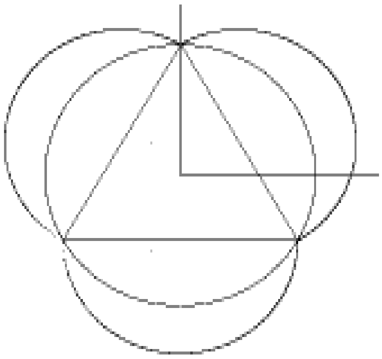
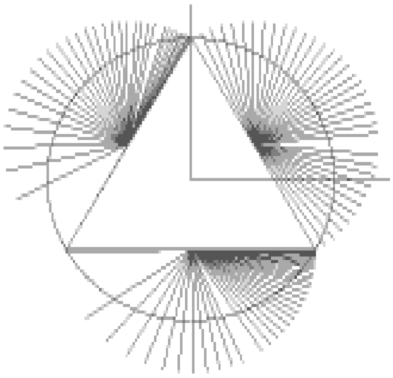


Fig. 3. Successive positions

Certain successive positions of the mechanism are depicted by Fig. 3. As long as the quadrilateral mechanism does not provide constant angles of the rocker CD, when rotating the road AB at constant angles one can notice crescent values for the angle CDH toward the zone neighboring K. When the cycling step is adopted to get more clarity for the drawing, a higher angle is obtained toward extremity. The same phenomenon is noticed in the zone EK.

One can see that for small cycling steps, these areas are swept too. Fig. 4 depicts the lunules traced with this mechanism across the three sides of the triangle.





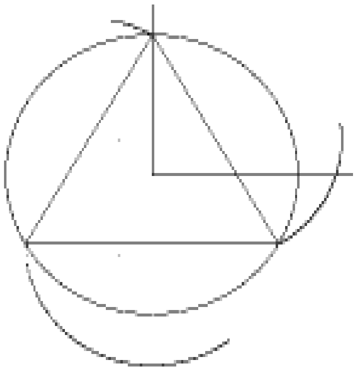


Fig. 4. Traced lunules Fig. 5. Successive positions of radii Fig. 6. Arcs at a certain moment

Fig. 5 depicts the successive positions of the elements AB, CD and EF for an operational cycle.

Variable angles can also be noticed at the road CD. The different speeds of the tracing point are emphasized by Fig. 6. Here, for the same range of variation of the generalized coordinate, the traced curves have different lengths. Only at the end they get fulfilled. The arcs traced at a given moment are presented here.

3. Synthesis of the second mechanism

The second step was to realize the synthesis of another mechanism, relying on a sequence of articulated parallelograms, each having two welded cranks (Fig. 7). The mechanism is represented by dashed lines. The lunule across the side GH is plotted by the point P belonging to the crank AB, welded to AB_1. The lunule across the side KH is traced by the point P_1 from the crank DP_1, welded to the crank DC, and the lunule across the side KG is traced by the point C_2 from the crank EC_2.

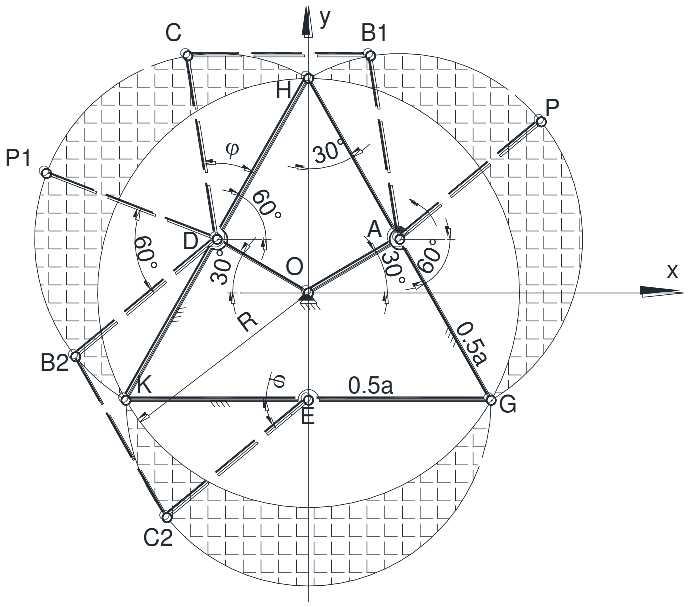


Fig. 7. The second mechanism, built across a sequence of parallelograms

Based on the angle φ between the crank AB_1 and the abscissa, on the coordinates of the points A, D, E (provided by the corresponding equations) and on the equations (6), (7) and (8), we get:

$$B_1C = AD \tag{14}$$

$$\begin{cases} x_C = x_D + 0{,}5a\cos(60^o + \varphi) \\ y_C = y_D + 0{,}5a\sin(60^0 + \varphi) \end{cases} \tag{15}$$

$$\begin{cases} x_{B1} = x_A + 0{,}5a\cos(\varphi + 60^0) \\ y_{B1} = y_A + 0{,}5a\sin(\varphi + 60^o) \end{cases} \tag{16}$$

$$\begin{cases} x_P = x_A + 0{,}5a\cos(\varphi - 60^o) \\ y_P = y_A + 0{,}5a\sin(\varphi - 60^o) \end{cases} \tag{17}$$

$$\begin{cases} x_{P1} = x_D + 0{,}5a\cos(120 + \varphi) \\ y_{P1} = y_D + 0{,}5a\sin(120 + \varphi) \end{cases} \tag{18}$$

$$\begin{cases} x_{C2} = x_E + 0{,}5a\cos(\pi + 2\varphi) \\ y_{C2} = y_E + 0{,}5a\sin(\pi + 2\varphi) \end{cases} \tag{19}$$

$$\begin{cases} x_{B2} = x_D + 0{,}5a\cos(\pi + \varphi) \\ y = y_D + 0{,}5a\sin(\pi + \varphi) \end{cases} \tag{20}$$

The resulting mechanism is given by Fig. 8, which also presents both shifted cranks. Fig. 9 depicts the successive positions of the mechanism.

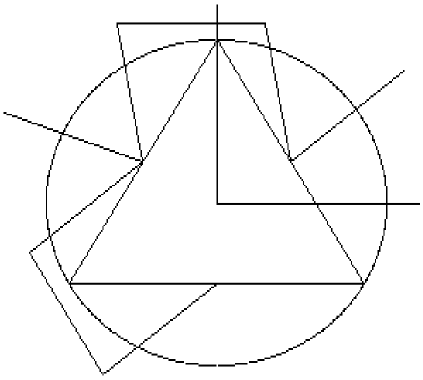


Fig. 8. The mechanism in a certain position

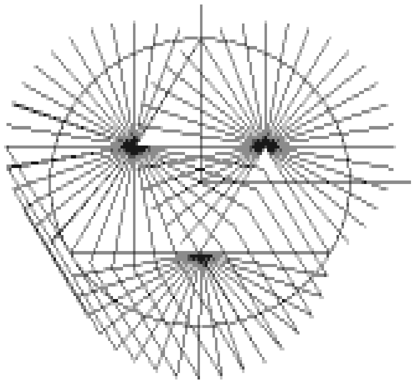


Fig. 9. Successive positions

The successive positions of the plotting cranks are presented by Fig. 10.

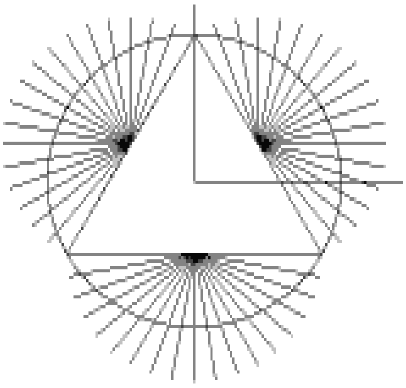


Fig. 10. Positions of plotting cranks

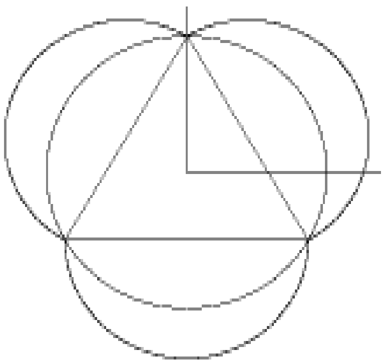


Fig. 11. Lunules

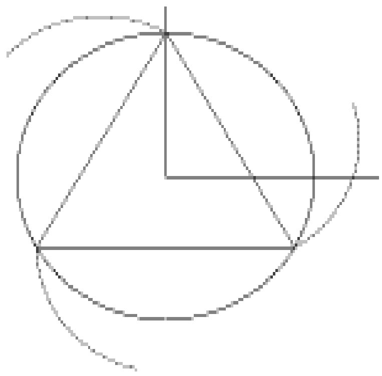


Fig. 12. Curves at a certain moment

Fig. 11 presents the lunules traced by this mechanism across the three sides of the equilateral triangle. This time the curves' plotting is performed at constant speeds (Fig. 12), unlike the case of articulated quadrilaterals. The curves plotted at a certain moment are also plotted and one can see that they are equal.

Conclusions

- The synthesis of two original mechanisms was performed. They plot lunules across the sides of an equilateral triangle.
- The first mechanism relies on an articulated quadrilateral, prolonged with a dyad of type RRR.
- The checking of the mechanism's successive positions demonstrated its correct operation.

- Because at this mechanism the last driven element has angles which are variable from one position to another (and therefore exhibits a non-uniform movement), the lunules, although correct, are plotted with variable speeds.

- The second mechanism relies on two articulated parallelograms, with shifted cranks, welded to real cranks from mechanism.
- In this last case the lunules' plotting speeds are equal and constant**.**

References

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