ANALYSIS OF POINT CONTACTS SUBJECTED TO A TANGENTIAL CONCENTRATED FORCES

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Abstract.*To "non-compliant" contacts in which deformations are sufficiently small compared to the size of bodies, elasticity theory applies to closed contact defined by the contact area. Stresses and displacements in elastic semispaces can cause tractions of the surface, being deducted for the first time by Boussinesq (1885) and Cerruti (1882)*
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1. INTRODUCTION

To "non-compliant" contacts in which deformations are sufficiently small compared to the size of bodies, elasticity theory applies to closed contact defined by the contact area [1], [2]. Contact voltage is concentrated, registered in the contact region and decreasing rapidly with distance from the point of contact. Thus the region of interest is practically closed at the contact interface.

Thereby the size of the bodies, needs to be greater than the size of the contact area, the pressure of the contact region does not depend criticallyon the distance between the metallic surface and bodies. The voltage can be calculated by considering each as a solid body, almost infinit, limited by a flat surface, in other words an elastic semi-space. This idealization, where the bodies have arbitrary surface profile and are seen as an extended semiinfinit is almost universal for elastic contacts. [3]

2. POINT CONTACT CHARGED BY A TANGENTIAL CONCENTRATED FORCES

We consider elastic semispace shown in Figure 1. If you let C(ξ, η) an area within the loaded area S, and A (x, y, z) represents a general point located inside the solid body, the distance CA will be:

$$CA = \rho = \left\{(\xi - x)^2 + (\eta - y)^2 + z^2\right\}^{1/2} \tag{1}$$

Stresses acting on the surface S to be p(ξ, η), $q_x(\xi, \eta)$ şi $q_y(\xi, \eta)$.

The stresses satisfy the Laplace equation and can be determined on the basis of their potential functions.

$$\begin{aligned} F_1 &= \int\!\!\int_S q_x(\xi, \eta)\Omega \, d\xi d\eta \\ G_1 &= \int\!\!\int_S q_y(\xi, \eta)\Omega \, d\xi d\eta \\ H_1 &= \int\!\!\int_S p(\xi, \eta)\Omega \, d\xi d\eta \end{aligned} \tag{2}$$

where:

$$\Omega = z \cdot \ln(\rho + z) - \rho \tag{3}$$

Love (1957) indicates that the components of the elastic displacements ux, uy, use a point A (x, y, z) of the solid are given by the expressions[4], [5]:

$$u_x = \frac{1}{4\pi G}\left\{2\frac{\partial F}{\partial z} - \frac{\partial H}{\partial x} + 2\nu\frac{\partial \psi_1}{\partial x} - z\frac{\partial \psi}{\partial x}\right\} \quad (4.a)$$

$$u_y = \frac{1}{4\pi G}\left\{2\frac{\partial G}{\partial z} - \frac{\partial H}{\partial y} + 2\nu\frac{\partial \psi_1}{\partial y} - z\frac{\partial \psi}{\partial y}\right\} \quad (4.b)$$

$$u_z = \frac{1}{4\pi G}\left\{2\frac{\delta H}{\partial z} + (1-2\nu)\psi - z\frac{\partial \psi}{\partial z}\right\} \quad (4.c)$$

These relationships are decreasing by $\frac{1}{\rho}$ for great distances from loaded region.

Analysis of a nonconforming contact subjected to shear forces $q_x(\xi,\eta)$ allows determining the stresses and displacements of contact. Tangential traction parallel to the y axis, qy, and normal pressure p will be considered void.

So in (2) and (4) we will have: $G_1=H_1=G=H=0$

Therefore: $\psi_1 = \frac{\partial F_1}{\partial x}$, $\psi = \frac{\partial^2 F_1}{\partial x \partial z}$.

So:

$$u_x = \frac{1}{4\pi G}\left\{2\frac{\partial^2 F_1}{\partial z^2} + 2\nu\frac{\partial^2 F_1}{\partial x^2} - z\frac{\partial^3 F_1}{\partial y^2 \partial z}\right\} \quad (5.a)$$

$$u_y = \frac{1}{4\pi G}\left\{2\nu\frac{\partial^2 F_1}{\partial x \partial y} - \frac{z\partial^3 F_1}{\partial x \partial y \partial z}\right\} \quad (5.b)$$

$$u_z = \frac{1}{4\pi G}\left\{(1-2\nu)\frac{\partial^2 F_1}{\partial x \partial z} - \frac{z\partial^3 F_1}{\partial x \partial z^2}\right\} \quad (5.c)$$

where: $F_1 = \int\int_s q_x(\xi,\eta)\{z\ln(\varphi+z) - \rho\}\, d\xi d\eta$

and: $\rho^2 = (\xi - x)^2 + (\eta - y)^2 - z^2$

Replacing the derivatives in equation (5) we get:

$$u_x = \frac{1}{4\pi G}\int\int_s q_x(\xi,\eta)x\left\{\frac{1}{\rho} + \frac{1-2\nu}{\rho+z} + \frac{(\xi-x)^2}{\rho^3} - \frac{(1-2\nu)(\xi-x)^2}{\rho(\rho+z)^2}\right\} d\xi d\eta \quad (6.a)$$

$$u_y = \frac{1}{4\pi G}\int\int_s q_x(\xi,\eta)x\left\{\frac{(\xi-x)(\eta-y)}{\rho^3} - (1-2\nu)\frac{(\xi-x)(\eta-y)}{\rho(\rho+z)^2}\right\} d\xi d\eta \quad (6.b)$$

$$u_z = -\frac{1}{4\pi G}\int\int_s q_x(\xi,\eta)x\left\{\frac{(\xi-x)z}{\rho^3} + (1-2\nu)\frac{(\xi-x)}{\rho(\rho+z)}\right\} d\xi d\eta \quad (6.c)$$

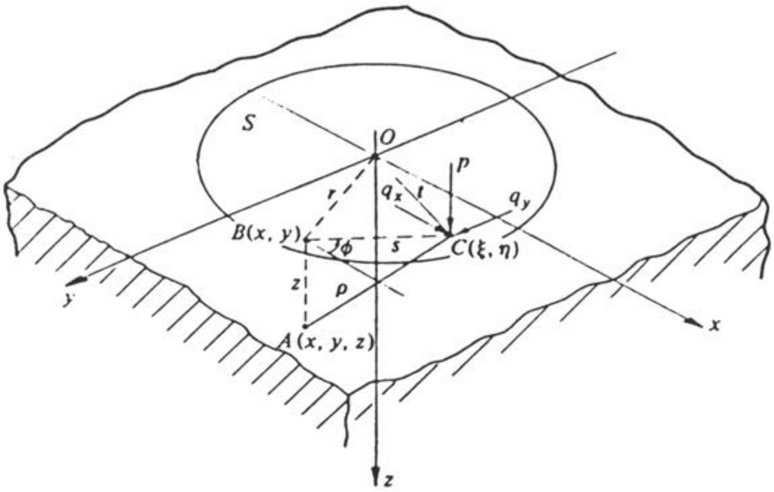

Fig. 1. *Elastic semispace [6]*

Tangential function will be deemed to be concentrated on a very small area near the origin, so that $\iint_S q_x(\xi,\eta)\, d\xi d\eta$ reduces the force concentrated Qx at a force acting in origin ($\xi=\eta=0$) in the parallel direction to the x-axis, the equation (6) for journeys solid is reduced to:

$$u_x = \frac{Q_x}{4\pi G}\left[\frac{1}{\rho} + \frac{x^2}{\rho^3} + (1-2\nu)\left(\frac{1}{\rho+z} - \frac{x^2}{\rho(\rho+z)^2}\right)\right] \tag{7.a}$$

$$u_y = \frac{Q_x}{4\pi G}\left[\frac{xy}{\rho^3} - (1-2\nu)\frac{xy}{\rho(\rho+z)^2}\right] \tag{7.b}$$

$$u_z = \frac{Q_x}{4\pi G}\left[\frac{xz}{\rho^3} + (1-2\nu)\frac{x}{\rho(\rho+z)}\right] \tag{7.c}$$

where: $\rho^2=x^2=y^2=z^2$.

By differentiating the equation (7)corresponding to movements will be written as:

$$\frac{2\pi\sigma_x}{Q_x} = -\frac{3x^3}{\rho^5} + (1-2\nu)\left\{\frac{x}{\rho^3} - \frac{3x}{\rho(\rho+z)^2} + \frac{x^3}{\rho^3(\rho+z)^2} + \frac{2x^3}{\rho^2(\rho+z)^3}\right\} \tag{8.a}$$

$$\frac{2\pi\sigma_y}{Q_x} = -\frac{3xy^2}{\rho^5} + (1-2\nu)\left\{\frac{x}{\rho^3} - \frac{x}{\rho(\rho+z)^2} + \frac{xy^2}{\rho^3(\rho+z)^2} + \frac{2xy^2}{\rho^2(\rho+z)^3}\right\} \tag{8.b}$$

$$\frac{2\pi\sigma_z}{Q_x} = -\frac{3xz^2}{\rho^5} \tag{8.c}$$

$$\frac{2\pi\tau_{xy}}{Q_x} = -\frac{3x^2y}{\rho^5} + (1-2\nu)\left\{-\frac{y}{\rho(\rho+z)^2} + \frac{x^2y}{\rho^3(\rho+z)^2} + \frac{2x^2y}{\rho^2(\rho+z)^3}\right\} \tag{8.d}$$

$$\frac{2\pi\tau_{yz}}{Q_x} = -\frac{3xyz}{\rho^5} \tag{8.e}$$

$$\frac{2\pi\tau_{zx}}{Q_x} = -\frac{3x^2 z}{\rho^5} \tag{8.f}$$

$$\frac{2\pi}{Q_x}\left(\sigma_x + \sigma_y + \sigma_z\right) = \frac{-2(1+\nu)x}{\rho^3} \tag{8.g}$$

Tensions and surface movements (excluding the origin) are calculated replacing z=0 and ρ=λ.

2.1.Unidirectional tangential traction applied to the circular regions

A. If circular region $\left(n = -\tfrac{1}{2}\right)$

Consider a distribution of the form:

$$q_x(x,y) = q_0\left(1 - r^2/a^2\right)^{-1/2} \tag{9}$$

acting parallel to the axis Ox in the region of the radius of the circle shown in Figure 2.Constant pressure distribution produces normal displacement within the area of the circle.

By the above presented analogy for ν = 0, tangential traction given by (9) produces a uniform tangential displacement $\overline{u_x}$,at the surface, in the direction of traction.

It can be shown that nonzero values of νresult in uniform tangential deflections.

For a point inside the circle loaded (r≤a), equation (6) reduces to

$$\overline{u_x} = \frac{1}{2\pi G}\iint_s q_x(\xi,\eta)\left\{\frac{1-\nu}{s} + \nu\frac{(\xi - x)^3}{s^3}\right\} d\xi\, d\eta \tag{10.a}$$

$$\overline{u_y} = \frac{\nu}{2\pi G}\iint_s q_x(\xi,\eta)\left\{\frac{(\xi - x)(\eta - y)}{s^3}\right\} d\xi\, d\eta$$

(10.b)

$$\overline{u_z} = -\frac{1-2\nu}{4\pi G}\iint_s q_x(\xi,\eta)\frac{\xi - x}{s^2}\, d\xi\, d\eta \tag{10.c}$$

where: $s^2 = (\xi - x)^2 + (\eta - y)^2$

These expressions for surface movements can also be derived, obtaining from equation (7) the displacements to any point B (x, y) stemming from a concentrated tangential force $Q_x = q_x\, d\xi\, d\eta$ acting at a point C(ξ,η)[7], [8],[9],.

Making a change of variable, moving from coordinates (ξ,η) at (s,ϕ) where

$$\xi^2 + \eta^2 = (x + s\cos\phi)^2 + (y + s\sin\phi)^2$$

considering: $\alpha^2 = a^2 - x^2 - y^2$ şi

$$\beta = x\cos\phi + y\sin\phi$$

$$q_x(s,\phi) = q_0 a\left(\alpha^2 - 2\beta s - s^2\right)^{-1/2} \tag{11}$$

The equations become:

$$\overline{u_x} = \frac{1}{2\pi G}\int_0^{2\pi}\int_0^{s_1} q_x(s,\phi)\{(1-\nu)+\nu\cos^2\phi\}\, d\phi\, ds$$

(12.a)

$$\overline{u_y} = \frac{\nu}{2\pi G}\int_0^{2\pi}\int_0^{a} q_x(s,\phi)\sin\phi\cos\phi\; d\phi\, ds \qquad (12.b)$$

$$\overline{u_z} = -\frac{1-2\nu}{2\pi G}\int_0^{2\pi}\int_0^{s_1} q_x(s,\phi)\cos\phi\; d\phi\, ds$$

(12.c)

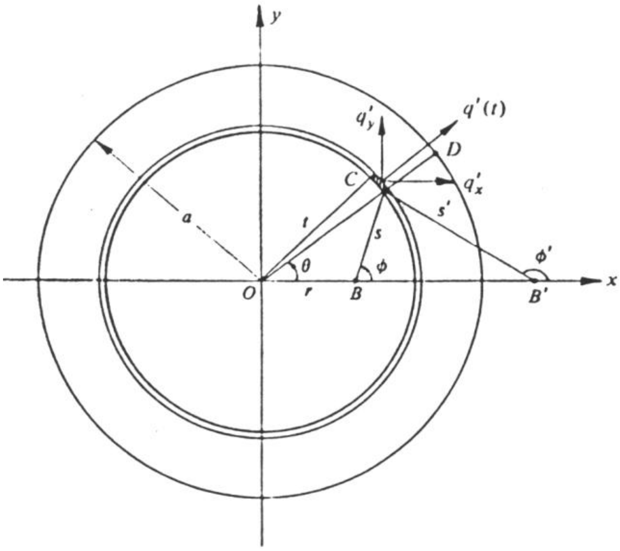

Fig.2*.Tractions distribution for a circular region*

S1 limit is given by point D on the edge of the circle to which:

$$s_1 = -\beta + \left(\alpha^2 + \beta^2\right)^{1/2}$$

By integrating withϕ between 0 and 2π, noting $\beta(\phi) = -\beta(\phi+\pi)$, we get ($r \le a$).

$$u_x = \frac{q_0 a}{4G}\int_0^{2\pi}\{(1-\nu)+\nu\cos^2\phi\}\, d\phi = \frac{\pi(2-\nu)}{4G} q_0 a = cons\tan t \qquad (13.a)$$

$$u_y = 0 \qquad (13.b)$$

$$u_z = \frac{(1-2\nu)q_0 a}{4\pi G}\int_0^{2\pi}\cos\phi\tan^{-1}\left(\beta/\alpha\right) d\phi = -\frac{(1-2\nu)q_0 a}{2G}\left\{\frac{a}{r} - \frac{\left(a^2 - r^2\right)^{1/2}}{r}\right\} (13.c)$$

b.***The case of circular region*** $\left(n = \frac{1}{2}\right)$

For a traction distribution like:

$$q_x = q_0\left(1 - \frac{r^2}{a^2}\right)^{1/2} \quad (14)$$

acting in a circular region, can be treated in the same manner by substituting equation (14) into the equation (12)[10],.
By integrating in compliance with s we have:

$$\overline{u_x} = \frac{\pi q_0}{32G \cdot a}\left\{4(2-\nu)a^2 + (4-\nu)x^2 + (4-3\nu)y^2\right\} \quad (15.a)$$

Similary: $$\overline{u_y} = \frac{\pi q_0}{32G \cdot a} 2\nu xy \quad (15.b)$$

In this case $\overline{u_x}$ is not constant at any point in the circle loaded and $\overline{u_y}$ is not canceled.

It may be noted that normal movements are not zero, but can not be expressed explicitly. Loaded tangential movements outside the circle (r> a) given by the thrust (14) were investigated by Illingworth with the results:

$$\overline{u_x} = \frac{q_0}{8G \cdot a}\left\{(2-\nu)\left[\left(2a^2 - r^2\right)\sin^{-1}\left(\frac{a}{r}\right) + ar\left(1 - \frac{a^2}{r^2}\right)^{\frac{1}{2}}\right] + \frac{1}{2}\nu\left[r^2 \sin^{-1}\left(\frac{a}{r}\right) + \left(2a^2 - r^2\right)\cdot\left(1 - \frac{a^2}{r^2}\right)^{\frac{1}{2}}\left(\frac{a}{r}\right)\right]\left(x^2 - y^2\right)\right\} \quad (16.a)$$

$$\overline{u_y} = \frac{q_0 \nu}{8G \cdot a}\left\{r^2 \sin^{-1}\left(a/r\right) + \left(2a^2 - r^2\right)\left(1 - a^2/r^2\right)^{1/2}\left(a/r\right)\right\}xy \quad (16.b)$$

Investigation of tensions within the solid given by the thrust (14) was treated by Hamilton and Goodman (1966).

3.CONCLUSION

The fact that elastic semispace stresses and displacements can cause traction of the surface was deducted for the first time by Boussinesq and Cerruti.

For a point contact loaded by a tangential force concentrated on issues Boussinesq-Cerruti we can deduce displacements and stresses at the contact level.

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