THE DEVELOPMENT OF A ROTATION SURFACE GENERATED BY HYPOCYCLOID WITH 4 BRANCHES

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Abstract. *This paper proposes a method for determining a development of the rotation surface, which is the rotation of hypocycloids with 4 branches. It has been chosen a four-cusped hypocycloid called astroid, because, due to its aesthetic shape, this could be used in industrial design. This paper proposes getting the development for a rotation surface generated by astroid. It is applied the graphical method of descriptive geometry by analogy with the developed by the sphere. This approximate graphical method is applied relatively simple and can be used when a high precision is not required for the product.*

Keywords: development, graphical method, hypocycloid, rotation surface

1. INTRODUCTION

Parts, subassemblies, machines and installations designed and redesigned to be drawn to be achieved their manufacturing. Technical drawings are executed based on the rules that must be known by those who design and implement. Such objects in three-dimensional space are represented in two-dimensional space on the drawing board.

Some machines and installations have parts which are obtained by wrapping in foil. The deploying is currently applied in practice to construction elements made of thin pieces (tin, plastic, etc.), such as envelopes, sheet metal parts, tanks etc. For their manufacturing, first it is cut off the material after the drawing of the respective unfolded surface and then it is assembled the blank. In most cases the deployable surfaces and rarely the undeploayable surfaces intersect themselves. The developed area may be conducted through accurate methods. Unfolded surfaces can be carried out by approximate methods.

The use of surfaces of revolution is essential in many fields in physics and engineering. A surface of revolution is a surface in Euclidean space created by rotating a curve around an axis of rotation. The representation of geometric surfaces is absolutely necessary in the industry of technical documentation. Geometric shapes of bodies are very important for the technical domain as they are used to manufacturing of pieces and industrial equipment. It is necessary the knowledge of the principles and methods required to turn them from the three-dimensional space into the two-dimensional space of a sheet of paper on which a drawing is done.

Some parts are obtained by winding sheet, after previously was designed the piece's development. Some bodies have developable geometric surfaces, others have undevelopable surfaces, for these being obtained approximate developments using approximate methods.
Descriptive Geometry provides for rotational deployment surfaces approximate graphical methods, which were dealt with in the literature only for commonly used surfaces.

This paper proposes a method for determining a development of the rotation surface, which is the rotation of hypocycloids with 4 branches around a vertical axis represents the axis of symmetry of the curve. It has been chosen a four-cusped hypocycloid called astroid, because, due to its aesthetic shape, this could be used in industrial design and other domains.

2. THE ROTATION SURFACE DEVELOPMENT

The curve hypocycloid produced by one fixed point on the circumference of a small circle (generator circle) of r radius rolling around the inside of a large circle (directing circle) of radius R > r. The parametric form of the curve is given as follows:

$$x(t)= R \cos^3 t \quad (1)$$

$$y(t)= R \sin^3 t \quad (2)$$

where R is the directing circle radius, t belongs to the range [0, 2π].

The length of the γ curve in plan, which is also named the road curve, is:

$L(\gamma)= 6 R$, where R is the bg circle radius (directing circle) .

It starts from a four-cusped hypocycloid called astroid, for which the rate of the director circle rays and generator circle is R / r = 4, figure 1. The astroid rotates around its vertical axis which is the axis of symmetry. Similarly with the ellipsoid which is obtained by rotating the rolling surface of the ellipse, it is proposed in this paper one rotation surface obtained by rotating astroid.

The proposed rotation surface is part of the involute surfaces, in the way that it can not be developed by accurate methods such as the polyhedra, conical surfaces, cylindrical surfaces, etc. The proposed rotation surface can only be developed only by approximate methods. It is given in figure 2 the rotation surface obtained by rotating the astroid around the vertical axis, noted AB. In Figure 2 it is represented a spindle (Gore), which is defined by the rotation surface area, which is included between two vertical section planes passing through the AB vertical axis.

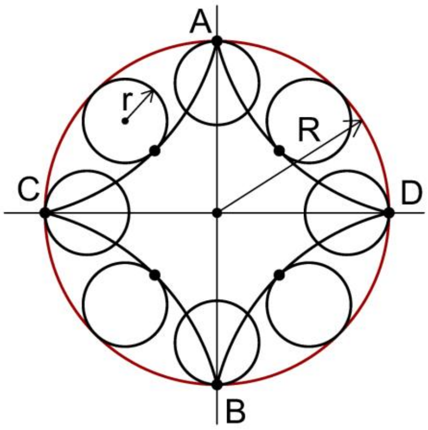

Fig. 1. The hypocycloid

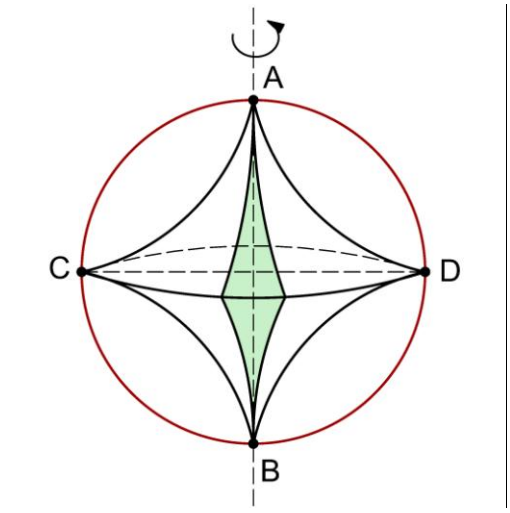

Fig. 2. The rotation surface

It is applied the graphical method of descriptive geometry by analogy with the sphere development by approximate graphical method.

There are used auxiliary plans: six level plans and six vertical plans.

For clarity of the graphical representation it is proposed the cutting of this surface with a number of six equidistant vertical plans (P1, P2, P3, P4, P5, P6) and six level plans (N1, N2, N3, N4, N5, N6).

With these auxiliary plane, the developed surface consists of 12 gores. In Figure 3 it is shown in draught an rotation surface, by two screenings in the vertical projection and in the horizontal projection plan and the construction of one gore.

The sectioning level surface plans N1, N2, N3, N4, N5, N6, determine circles that are represented in a [H] projecting plan. In the vertical projecting plan [V] the auxiliary planes are noted as follows: n_1', n_2', n_3', n_4', n_5', n_6'. The vertical section plans P1, P2, P3, P4, P5 and P6 is represented by their tracks in the [H] projecting plan. The vertical auxiliary plans divide the circles corresponding to auxiliary plans and the base circle in equal parts by 12 to be taken into account in the construction of one gore.

There are noted the points determined by N1, N2, N3, in the projecting plan [H] as 1,2,3,4,5,6,7 and 8. It is considered a line segment of length equal to the length of an astroid cusp (L = 3R= 2 x Lra= 12 r), placed horizontally, noted A*D*B*.

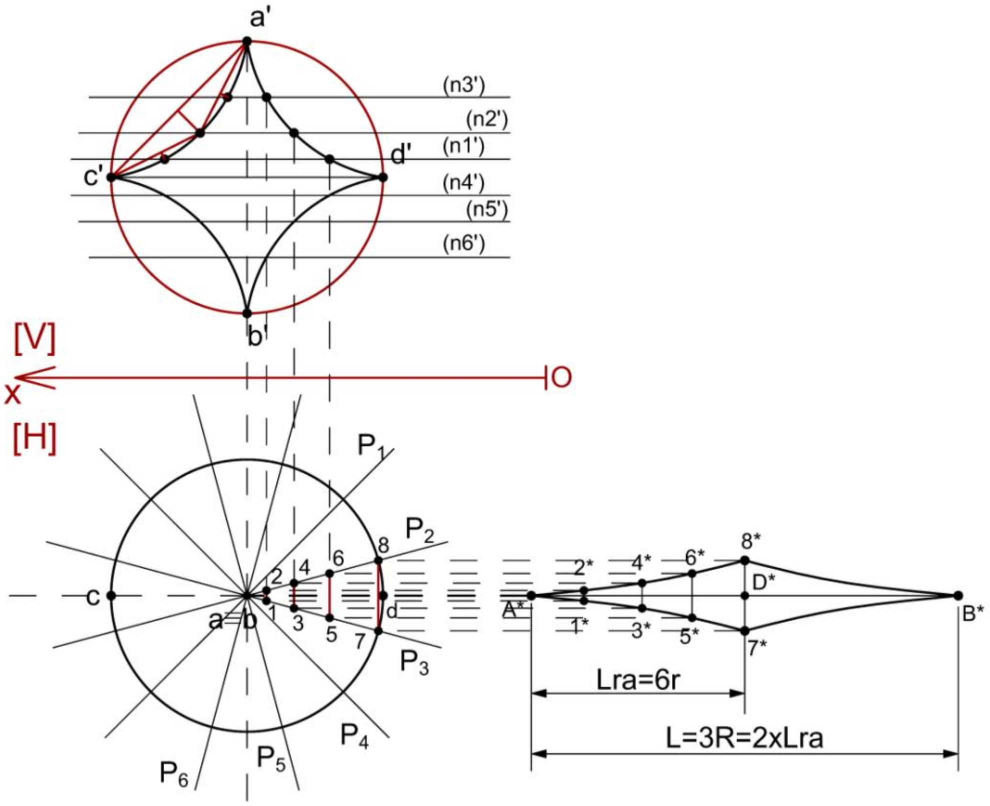

Fig. 3. The projection of rotation surface

This one is divided into four equal parts, there are drawn perpendicular lines in division points and on these are measured the appropriate sizes of the arcs 12, 34, 56 and 78. It is proposed here the approximation of a circular arc to the size of an arc with the size of a sub-stretched chord. There are obtained the points 1*, 2*, 3* ... 8 *, which define half spindle. There are joined the dots with the curved line and it results the profile of a semi-gore result.

Symmetrically, it is built the second semi-gore and it results a spindle with A * B *length. In figure 4 it is given the development of a rotation surface, consisting of 12 gores which are built next to each other, along a segment line 7*7*, whose length is S = 2π R.

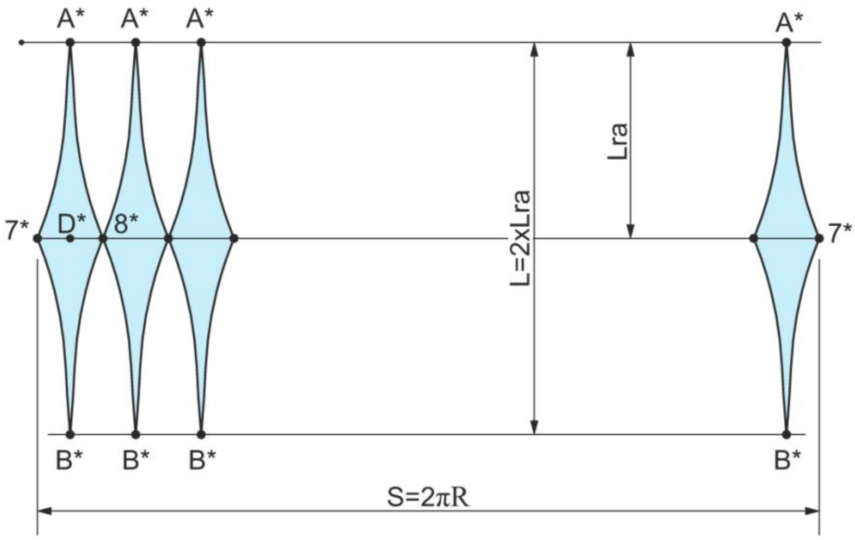

Fig. 4. *The development of rotation surface*

The lengths of the arcs of the curve A*7* B* and A*8* B* which define the shape of one gore, are greater than the length of the 2x Lra, a difference generating a portion of the errors of these approximate methods. Other errors of this method result from approximating arcs with sub-extended chords, but there are inherent errors of graphic construction. Errors are smaller as the area will be divided into a large number of gores (large number of vertical auxilliary planes) and with a large number of level planes, but the number of auxiliary plans is limited by the need for a clear graphical representation.

3. CONCLUSION

This approximate graphical method is applied relatively simple and it can be used when a high precision is not required for products execution. The proposed method can be used when it is desired a rigorous control over the errors by which it is realized the development of the rotation surface. By the graphics method cannot be built a large number of gores. If it is possible to choose a large number of gores, then a high precision is obtained.

The method proposed in the paper can be improved by a computerized solution utilizing computer programs.

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