ANALYSIS OF POINT CONTACTS USING THE COMBINED BOUSSINESQ-CERRUTI PROBLEM

Prof.dr.eng. Stefan GHIMISI, Constantin Brancusi University of Târgu Jiu,
ssghimisi@gmail.com

Abstract.*For "non-conforming" contact where deformations are small enough compared to body dimensions, the theoretical elasticity will apply to the closed contact defined by the contact area.The tension can be calculated by considering each body as a solid semiinfinite, limited by a flat surface, that is to say a semisphere of elasticity. This idealization, where the bodies have the surface of the arbitrary profile and seen as a semifinished extension is almost universal for the elastic contacts.Tensions and displacements in the elastic semisphere can determine surface tractions being deduced for the first time by Boussinesq (1885) and Cerruti (1882) who have made the theory of potential, and this approach is presented by Love as well (1957)*

Keywords: deformation, stress, displacements, semispaces.

1.Introduction

For "non-conforming" contact where deformations are small enough compared to body dimensions, the theoretical elasticity will apply to the closed contact defined by the contact area [1], [2]. The contact tensions is concentrated, being closed in the contact area and rapidly decreasing with the distance from the contact point. Thus, the region of practical interest is closed at the interface of the contact.

By conditioning the dimensions of the body as being larger than the dimensions of the contact area, the contact contact tensions is not critically dependent on the distance of the bodies from the contact area. The tension can be calculated by considering each body as a solid semiinfinite, limited by a flat surface, that is to say a semisphere of elasticity. This idealization, where the bodies have the surface of the arbitrary profile and seen as a semifinished extension is almost universal for the elastic contacts. [3]

2. Applying the Boussinesq-Cerruti problem

In the following, pressures and deformations produced in the elastic semi-stretch limited by a plane surface, z = 0, shall be considered under the action of a tangential and normal traction applied to the area S in the vicinity of the origin. Outside of the loading axis both tensions are null. Thus the problem of elasticity is one in which the tractions are not specified everywhere on the whole surface with z = 0.

In order to restrict the area where the load is applied, it will be considered the moment when all the pressure components reach zero. The loading is two-dimensional: the normal pressure p (x, y) and the tangential traction qx (x, y) and qy (x, y), generally varying in both directions x and y.

The two-dimensional tension system is that all six components: σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx}, tensions will appear.

A special case is when the load is asymmetric with respect to the z-axis. Considering that the polar coordinates (r,θ,z), the presussre p(r) and the tangential stress q (r) are independent of θ and q (r) if present is acting in the radial direction. The tensions components $\tau_{r\theta}$and $\tau_{\theta z}$disappear and the other components are independent of θ

Tensions and displacements in the elastic semisphere can determine surface tractions

being deduced for the first time by Boussinesq (1885) [4] and Cerruti (1882) who have made

the theory of potential, and this approach is presented by Love as well (1957).

The elastic semispace is shown in figure 1. If a C(ξ, η) is denoted a point on the surface inside the loaded area S, and A (x, y, z) represents a general point inside the solid body, the distance CA will be:

$$CA = \rho = \left\{(\xi - x)^2 + (\eta - y)^2 + z^2\right\}^{1/2} \tag{1}$$

Efforts that work on S surface will be p(ξ, η), q_x(ξ, η) şi q_y(ξ, η).

The unitary efforts satisfy the Laplace equations and we can determine the potential functions based on them.

$$\begin{aligned} F_1 &= \iint_s q_x(\xi, \eta)\Omega \, d\xi d\eta \\ G_1 &= \iint_s q_y(\xi, \eta)\Omega \, d\xi d\eta \\ H_1 &= \iint_s p(\xi, \eta)\Omega \, d\xi d\eta \end{aligned} \tag{2}$$

where:

$$\Omega = z \cdot \ln(\rho + z) - \rho \tag{3}$$

Result potential functions:

$$\begin{aligned} F &= \frac{\partial F_1}{\partial z} = \iint_s q_x(\xi, \eta) \cdot \ln(\rho + z) \, d\xi d\eta \\ G &= \frac{\partial G_1}{\partial z} = \iint_s q_y(\xi, \eta) \cdot \ln(\rho + z) \, d\xi d\eta \\ H &= \frac{\partial H_1}{\partial z} = \iint_s p(\xi, \eta) \cdot \ln(\rho + z) \, d\xi d\eta \end{aligned} \tag{4}$$

It can be written:

$$\psi_1 = \frac{\partial F_1}{\partial z} + \frac{\partial G_1}{\partial z} + \frac{\partial H_1}{\partial z} \tag{5}$$

$$\psi = \frac{\partial \psi_1}{\partial z} = \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} \tag{6}$$

Love (1957) shows that the components of the elastic displacements ux, uy, use in a point A (x, y, z) of the solid, will be given by the expressions of the terms from the above functions[5,6].

So:

$$u_x = \frac{1}{4\pi G}\left\{2\frac{\partial F}{\partial z} - \frac{\partial H}{\partial x} + 2\nu\frac{\partial \psi_1}{\partial x} - z\frac{\partial \psi}{\partial x}\right\} \tag{7.a}$$

$$u_y = \frac{1}{4\pi G}\left\{2\frac{\partial G}{\partial z} - \frac{\partial H}{\partial y} + 2\nu\frac{\partial \psi_1}{\partial y} - z\frac{\partial \psi}{\partial y}\right\} \tag{7.b}$$

$$u_z = \frac{1}{4\pi G}\left\{2\frac{\delta H}{\partial z} + (1-2\nu)\psi - z\frac{\partial \psi}{\partial z}\right\} \tag{7.c}$$

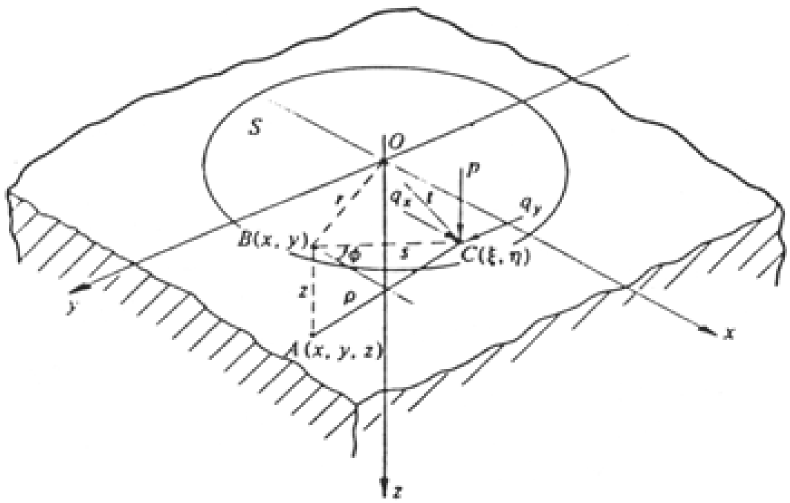

Fig. 1. *Elastic semispace [4]*

These relationships decrease by $\frac{1}{\rho}$ for long distances to the loaded region.

Thus, the elastic movements, of the closed points in the loaded region, relative to the solid points distant from the loaded region ($\rho \to \infty$) where the elastic elastic is located rather than fixed, corresponds to a two-dimensional load.

The displacements will be calculated by allowing stress determination based on Hooke's law:

$$\sigma_x = \frac{2\nu G}{1-2\nu}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + 2G\frac{u_x}{\partial x} \tag{8.a}$$

$$\sigma_y = \frac{2\nu G}{1-2\nu}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + 2G\frac{\partial u_y}{\partial y} \tag{8.b}$$

$$\sigma_z = \frac{2\nu G}{1-2\nu}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + 2G\frac{\partial u_z}{\partial z} \tag{8.c}$$

$$\tau_{xy} = G\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \tag{8.d}$$

$$\tau_{yz} = G\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \quad (8.e)$$

$$\tau_{zx} = G\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \quad (8.f)$$

For a normal (pure) p(ξ, η) pressure that exists for non-friction contacts the above equations can be written[7]:

$F = F_1 = G = G_1 = 0$

So:

$$\psi_1 = \frac{\partial H_1}{\partial z} = H = \iint_s p(\xi, \eta) \cdot \ln(\rho + z)\, d\xi d\eta \quad (9)$$

$$\psi = \frac{\partial H}{\partial z} = \frac{\partial \psi_1}{\partial z} = \iint_s p(\xi, \eta) \cdot \frac{1}{\rho}\, d\xi d\eta \quad (10)$$

$$u_x = -\frac{1}{4\pi G}\left\{(1-2\nu)\frac{\partial \psi_1}{\partial x} + z\frac{\partial \psi}{\partial x}\right\} \quad (11.a)$$

$$u_y = -\frac{1}{4\pi G}\left\{(1-2\nu)\frac{\partial \psi_1}{\partial y} + z\frac{\partial \psi}{\partial y}\right\} \quad (11.b)$$

$$u_z = \frac{1}{4\pi G}\left\{2(1-o)\psi - z\frac{\partial \psi}{\partial z}\right\} \quad (11.c)$$

withψand ψ_1 – Harmonic functions of x, y and z, thus satisfying the Laplace equations:

$\nabla^2\psi = 0$

$\nabla^2\psi_1 = 0.$

The value ofΔis given by :

$$\Delta \equiv \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = \frac{1-2\nu}{2\pi G} \cdot \frac{2\psi}{\partial z} \quad (12)$$

By replacing equations (11) and (12) in equations (8) the expression for the tensions components in a solid point is expressed.

These are:

$$\sigma_x = \frac{1}{2\pi}\left\{2\nu\frac{\partial \psi}{\partial z} - z\frac{\partial^2 \psi}{\partial x^2} - (1-2\nu)\frac{\partial^2 \psi_1}{\partial x^2}\right\} \quad (13.a)$$

$$\sigma_y = \frac{1}{2\pi}\left\{2\nu\frac{\partial \psi}{\partial z} - z\frac{\partial^2 \psi}{\partial y^2} - (1-2\nu)\frac{\partial^2 \psi_1}{\partial y^2}\right\} \quad (13.b)$$

$$\sigma_z = \frac{1}{2\pi}\left\{\frac{\partial \psi}{\partial z} - z\frac{\partial^2 \psi}{\partial z^2}\right\} \quad (13.c)$$

$$\tau_{xy} = -\frac{1}{2\pi}\left\{(1-2\nu)\frac{\partial^2 \psi_1}{\partial x \partial y} + 2\frac{\partial^2 \psi}{\partial x \partial y}\right\} \quad (13.d)$$

$$\tau_{yz} = -\frac{1}{2\pi} z \frac{\partial^2 \psi}{\partial y \partial z} \quad (13.e)$$

$$\tau_{zx} = -\frac{1}{2\pi} z \frac{\partial^2 \psi}{\partial x \partial z} \quad (13.f)$$

It can be noted that the tensions σ_z, τ_{yz} and τ_{zx} depend on a single function ψ.

The tension components σ_x şi σ_y depend on the function ψ1 but their sum does not depend, therefore:

$$\sigma_x + \sigma_y = \frac{1}{2\pi}\left\{(1+2\nu)\frac{\partial \psi}{\partial z} + z\frac{\partial^2 \psi}{\partial z^2}\right\} \quad (14)$$

At the solid surface the normal tension will be:

$$\overline{\sigma_z} = \frac{1}{2\pi}\left(\frac{\partial \psi}{\partial z}\right)_{z=0} = \begin{cases} -p(\xi,\eta), \text{înside S} \\ 0, \text{outside of S} \end{cases} \quad (15)$$

And the displacements will be:

$$\overline{u_x} = -\frac{1-2\nu}{4\pi G}\cdot\left(\frac{\partial \psi_1}{\partial x}\right)_{z=0} \quad (16.a)$$

$$\overline{u_y} = -\frac{1-2\nu}{4\pi G}\cdot\left(\frac{\partial \psi_1}{\partial y}\right)_{z=0} \quad (16.b)$$

$$\overline{u_z} = \frac{1-\nu}{2\pi G}\cdot\left(\frac{\partial \psi_1}{\partial z}\right)_{z=0} = \frac{1-\nu}{2\pi G}(\psi)_{z=0} \quad (16.c)$$

Equations (15) and (16.c) show that normal pressure and normal displacement within the loaded area depend only on the potential function ψ.

If pressure distribution within S area is known then stresses and displacements at a point in the solid will be discovered. In practice, obtaining tension expressions at a point in the solid presents a number of difficulties.

For particular conditions, the coordinates of the rectangular points in the ellipsoidal can be changed, thus solving the problems in which the contact area is limited by an ellipse (Lur'e - 1964 [135], Galin - 1953, de Pater - 1964)[8,9].

For circular contact areas, a special stress-function complex suggested by Rostovtzev (1953) (and also by Green and Zerna, 1959) can be used to detect tensions when the movements are specified within the loaded area.

For axial symmetry cases, the integrals transformation method discovered by Noble and Spence (1971) (and Gladwell 1980) can be used.[10]

Another way to determine stresses and deformations at a point within the load area is to overlap the effects of a normal load and tangential force, solving the problems based on numerical analysis.

3.Conclusions

Applying the Boussinesq-Cerruti problem allows the determination of the pressures and deformations produced in the elastic semi-suspension limited by a flat surface under the action of a tangential and normal traction applied to the S. surface area

Normal pressure and normal displacement within the loaded area depend only on the potential functionψ.

4.References

[1].**Archard J.F., Hirst W.**, *The Wear of Metals Scient*., Lubricat, Nov.1958,p.3-8.

[2].**Archard J.F.,***Wear*, Proc.NASA, 1967, Ed NASA, Wasinghton, 1968 p.267-333

[3].**Jhonson, K.L.**,*Contact Mecanics*,Cambridge,Cambridge University Press, 1985, 452p

[4].**Boussinesq, J.***Aplication des potential a l'etude de l'equilibre et du mouvement des solides elastiques*, Paris , Gauthier Villars, 1885,p.580

[5].**GhimişiŞtefan**, *Fenomenul de fretting,* Editura Sitech, Craiova, ISBN 973-746-422-2, ISBN 978-973-746-422-4, 2006, pag. 331;

[6].**Ghimişi Ştefan**,*Elemente de tribologie,* Editura MatrixROM, Bucureşti, 2005, ISBN 973-685-903-7, pag. 160;

[7].**Timoshenko, S.,Goodier, J.N.,***Theory of Elasticity*,New York,London et al,McGraw-Hill,1951

[8].**Ghimişi Ştefan**,*The displacements in the fretting phenomenon*,Annals of the „Constantin Brâncuşi" University of Târgu-Jiu - Engineering Series, ISSN 1842-4856, Nr.4/2014, pag. 25-29;

[9] **Ghimisi, S**. "*An elastic-plastic adhesion model for fretting,* 15th." Symposium "DanubiaAdria", Bertinoro, Italia.

[10] **Ghimisi, Stefan**. "*Analysis of point contacts subjected to a tangential concentrated forces*." *Fiability & Durability/Fiabilitate si Durabilitate* 2 (2016).