STUDY OF A PROBLEM OF GRAPHIC ENGINEERING

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Abstract. *The paper proposes an approximate graphical method for determining the development of the revolution surface obtained by means of the deltoid. The approximate development of the revolution surfaces are based on general methods of development, provided by Descriptive Geometry.*

Keywords: development, revolution surface, deltoid.

1. Introduction

Cycloidal curves have multiple applications in technics. Their study and graphic representating a great interest to many specialists in the field of machine building, decorative art and architecture, as well as for teaching (teaching staff). Cycloidal curves are classified into 3 main categories: cycloids, epicycloids and hypocycloids.

The cycloid is the curve described by a point that is on a circle that rolls without friction on a straight line. The epicycloid is the curve described by a point on a circle radius r that moves without friction outside the radius of circle R. Hypocycloid is the curve described by a point on a circle of radius r that moves without friction on the inside of a circle Of radius R. Some epicycloids and hypo-cycloids are known as having a nice shape.

In this paper we start from a particular case of hypocycloid, the curve called the deltoid, a beautiful curve. This curve rotates around a vertical axis that is also the axis of symmetry of the curve and so it is obtained a revolutionary surface. For the obtained geometric surface we study the determination of the approximate deployment by the graphic method. The graphic method adopted is similar to gores method presented in [1] and [6].

Studies about the development of rotation surfaces generated by astroid, hypocycloid with 4 branches and ellipsoid, are presented in the papers [2] , [3] and [4]. In [5] is presented aesthetic forms of hypocycloids and generator mechanisms.

2. Geometrical properties of deltoid

The deltoid is a hypocycloid of three cusps. In other words, it is the roulette created by a point on the circumference of a circle as it rolls without slipping along the inside of a circle with three or one-and-a-half times its radius. It is named after the Greek letter delta which it resembles [9]. A deltoid (or tricuspoid) is the locus of a point on the circumference of a circle rolling inside another circle with a radius three times larger in magnitude. It is a hypocycloid, a curve formed by a point on the circumference of a rolling circle internally tangent to another, larger circle [10]. The deltoid was studied by Euler in 1745. This curve was first considered by Euler in connection with an optical problem. It was also investigated by Steiner in 1856 and is sometimes called Steiner curve. Thus, in geometry the deltoid is called tricuspoid or Steiner curve.

The equation of the hypocycloid:

$$\begin{cases} x = (R-r)\cos\varphi + a\cos\dfrac{R-r}{r}\varphi \\ y = (R-r)\sin\varphi - a\sin\dfrac{R-r}{r}\varphi \end{cases} \qquad (1)$$

where : **a** - the distance of the generator point P to the center of the generator circle,

φ - the angle of position of the generating point to the vector radius ,

R, r - the radius of the base circle and the generator circle.

Depending on the value of parameter a, the hypocycloid may be normal, elongated or shortened. The graphical construction of the normal hypocycloid cell is given in figure 1.

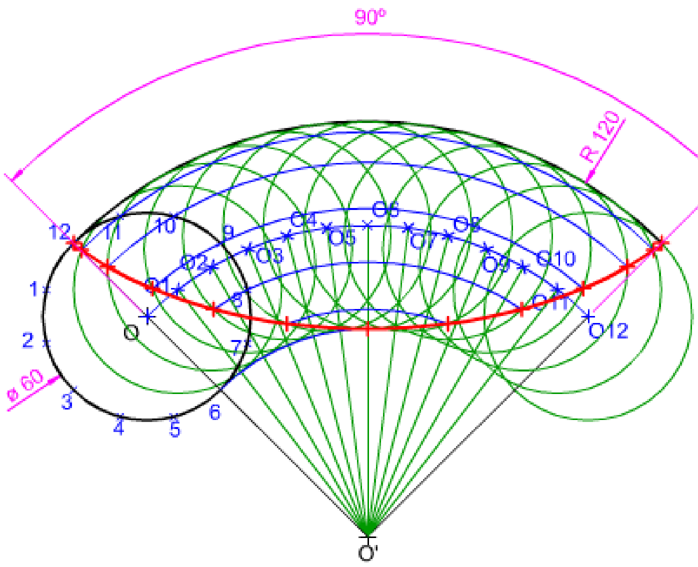

***Fig.1**. The hypocycloid, [7]*

The equation of the deltoid is obtained by setting R/r=3 in the equation of the hypocycloid, where R is the radius of the large fixed circle and r is the radius of the small rolling circle.
A normal deltoid can be represented by the following parametric equations:

$$\begin{cases} x = \dfrac{2R}{3}\cos\varphi + \dfrac{R}{3}\cos 2\varphi \\ y = \dfrac{2R}{3}\sin\varphi - \dfrac{R}{3}\sin 2\varphi \end{cases} \qquad (2)$$

Or,

$$\begin{cases} x = 2r\cos\varphi + r\cos 2\varphi \\ y = 2r\sin\varphi - r\sin 2\varphi \end{cases} \qquad (3)$$

The total arc length, from deltoid is : $S_3=16R/3$.

The area from deltoid is: $A_3=2\pi R^2/9$.

The deltoid is presented in figure 2.

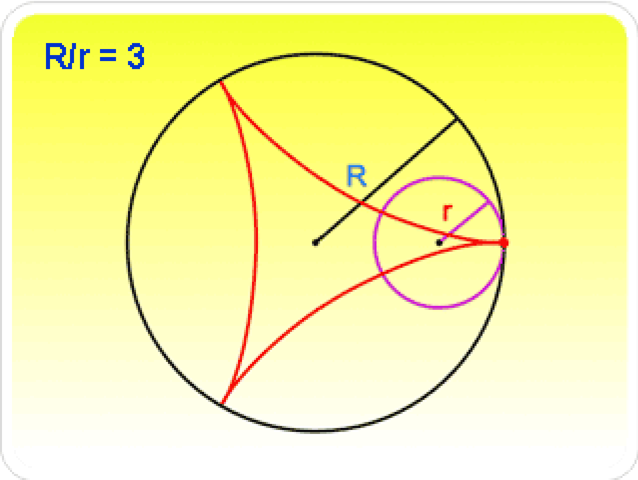

***Fig. 2**. The deltoid, [8]*

3. Approximate development of the revolution surface obtained by rotation of the deltoid

Figure 3 shows the revolutionary surface obtained by rotating the deltoid around the vertical axis zz ', the axis coinciding with the deltoid symmetry axis. The deltoid is defined by the fixed circle (the director circle) - Cf and the generator circle-Cg. The resulting surface consists of two parts: a surface having the V1 peak and the CD diameter circle, and the second surface having a pole in V2 and as a basis the circle of CD diameter.

Editura "Academica Brâncuşi" , Târgu Jiu, ISSN 1844 – 640X

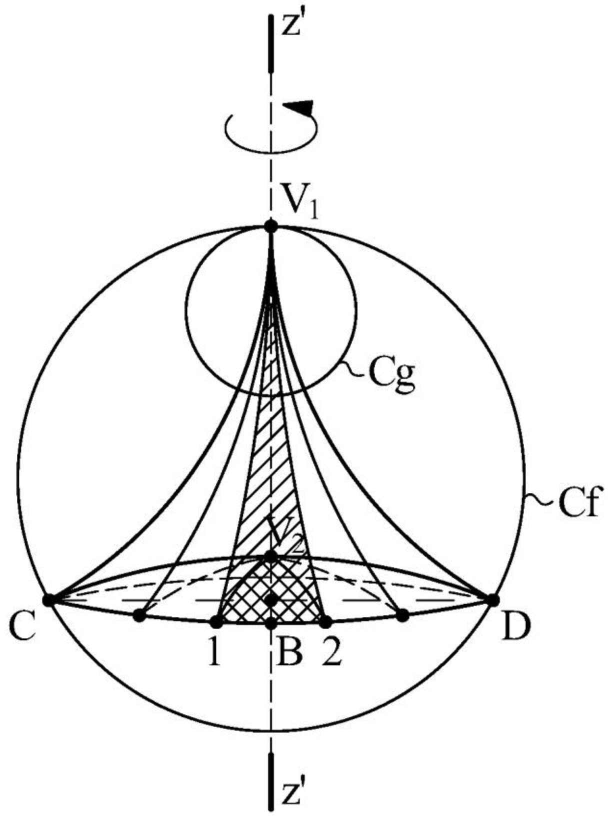

Fig. 3. *Surface generated by rotation of the deltoid.*

Figure 4 shows the projections of the surface in the orthogonal projection system, on the vertical projection plan and on the horizontal projection plan. The graphical construction of a spindle is also presented.

In order to find out, the surface with auxiliary plans is divided into two categories: horizontal plans and vertical plans. The vertical plans drawn equidistantly divide the surface into several equal parts called gores.

It is proposed the cutting of this surface with a number of 5 equidistant vertical plans (P1, P2, P3, P4, P5) and six level plans (N1, N2, N3, N4, N5, N6). Thus, the graphically drawn is made up of 10 gores and the drawing obtained for a gore has an acceptable clarity.

The sectioning level surface plans [N1], [N2], [N3], [N4], [N5], [N6], determine circles that are represented in a [H] project plan. Vertical traces of planes are noted: n_1', n_2', n_3', n_4', n_5', n_6'. The vertical section plans [P1], [P2], [P3], [P4] and [P5] is represented in the [H] project plan, through the horizontal tracks: p_1, p_2, p_3, p_4, p_5. The intersections of two consecutive vertical planes with the 6 planar planes and the revolutionary surface give the points describing a gore. These points are denoted in horizontal projection [H]: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 si 14.

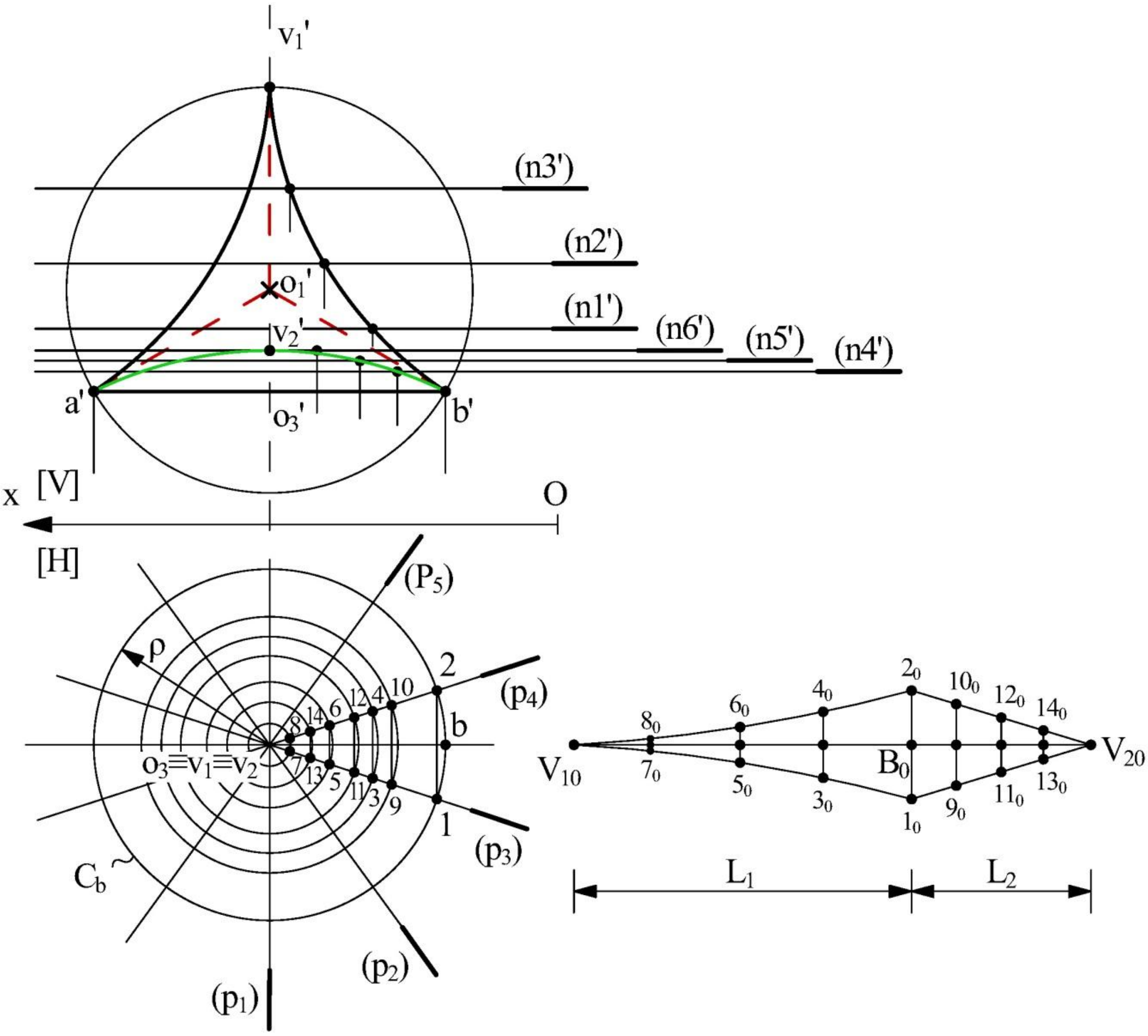

__Fig. 4.__ Projections of the revolution surface and drawing for a gore.

For tracing the evolute of a gore (picture 4), it is drawn a straight line segment V_{10} V_{20}. The segment has a length equal to L1 + L2. The length L1 is the length of the arc v1 'b' and is L1 = 16R / 9. The length L2 is the length of the arc v2 'b' and is L2 = L1 / 2 = 16R / 18.

Then the two lengths L1 and L2 are divided into four equal parts. There are drawn in the dividing points perpendicular segments, symmetrically arranged lengths equal to the strings underlined by the circular arcs that are defined by the level planes are plotted at the points of division. In the run of the gore the chords are noted: $1_0 2_0$, $3_0 4_0$, $5_0 6_0$, $7_0 8_0$, $9_0 10_0$, $11_0 12_0$, $13_0 14_0$. The points thus obtained are joined together and the profile of a gore is obtained.

In figure 5 it is given the development of a revolution surface , consisting of 10 identical gores. The gores are drawn adjacent to a straight segment 1_01_0, having the length L. The length L is calculated with the relation: L = 2π ρ, where ρ is the radius of the base of the revolution surface ($\rho = O_3'b'$).

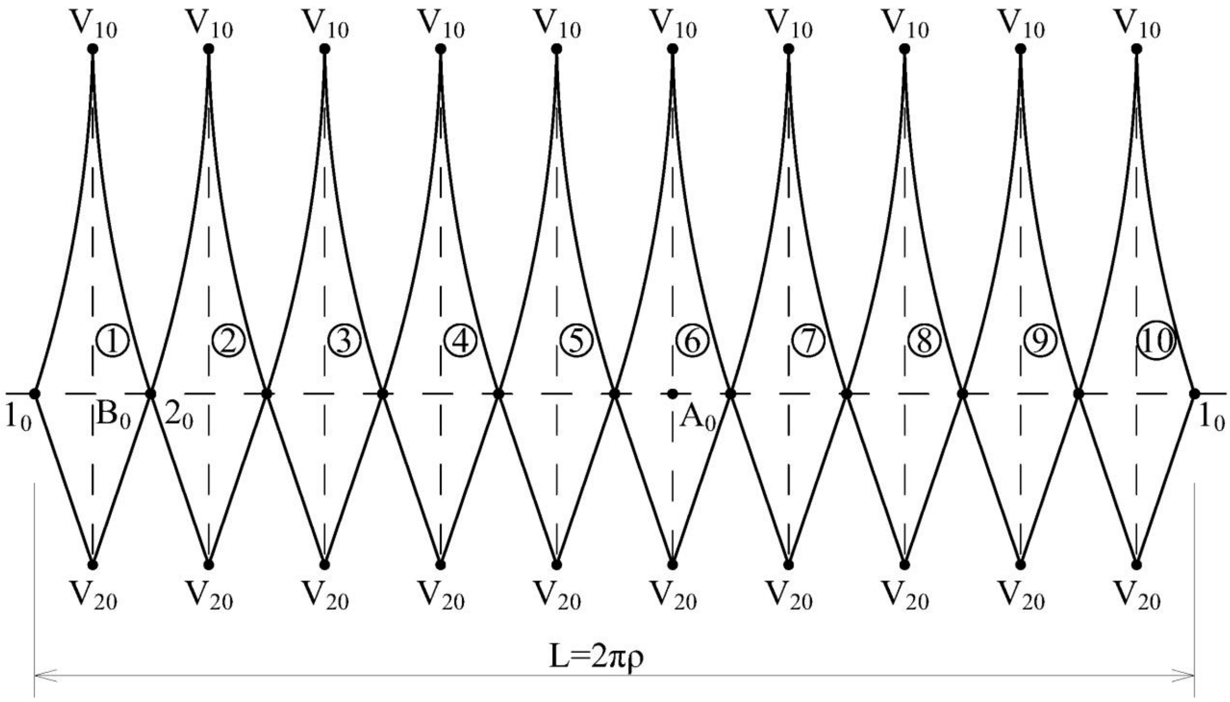

Fig. 5. *Approximate development of the revolution surface*

This evolute has errors due to the approximation of the circular arcs with underlined strings plus the errors specific to a graphical construct with geometry tools. Errors can be reduced if one runs with as many gores as possible, but in these cases the drawings are less clear.

4. Conclusions

Cyclic curves or cycloids are a category of curves of particular importance in mathematics, descriptive geometry, but especially in technique, being applied to many mechanisms. Cyclical curves have many applications in construction, architecture, ambient design, especially aesthetically pleasing cycloid variants.

The revolutionary surface obtained with deltoid is also a beautiful surface, with special aesthetics, which can find multiple uses. In some applications, it may be necessary to determine the surface to be developed.

The paper proposes an approximate graphical method for determining the development of the rotational surface obtained by means of the deltoid. The approximate development of the revolution surfaces are based on general methods of development, provided by Descriptive Geometry.

The proposed method is approximate, it has errors generated by the approximation of circular arcs with underlined strings as well as inherent errors of graphical methods. Errors can be reduced by using as many as possible auxiliary plans (horizontal and vertical plans), but here are limitations imposed by the clarity of the construction. A more accurate deployment, with much smaller errors, can be achieved with computer-assisted methods, using appropriate programs. It can be made a program for determining the errors.

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