THE INFLUENCE OF ROAD DECK BRIDGE THICKNESS ON THE DYNAMIC DISPLACEMENT RESPONSE

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Abstract: *The article presents a method to obtaining the dynamic response bridge-vehicle system in displacements, for sustainable dimensioning of the road decks bridge. The method used is the numerical finite strips method (FSM) associated with the Rayleigh-Ritz variation principle. The road deck bridge was considered to be made from reinforced concrete and is considered to be modelled by isotropic plate with varying boundary conditions. The vehicle is represented as a concentrated mobile force moving with speed V on the deck bridge. We will analyze the influence of slab thickness on the dynamic response of the system introduced.*

Keywords: Deck bridge, dynamic response, mobile load

1. INTRODUCTION

The evolution of merchandise traffic on the national road system and on the highways is increasing in the last years. For optimising the transportation, the vehicles carry bigger and bigger loads and the moving speed is faster. At the same time, the road administrators limited the load weight on the axle.Therefore, the number of axles has increased. Also, the distribution of the load based on the mobile forces on the road structures and bridge decks changes and the structure response can difere in time, considering that the life span of a bridge is between 50 and 100 years.

The bridge structures vibrations created by the passage of the mobile loads appear also from the bridge deformations, even if the surface is plane. These vibrations are amplified by the bumps in the road surface and the vibration of the vehicles. To analyze the vibrations effects we need to modelled the decks bridge, the vehicles and the road surface.

2.THE THEORETICAL PRESENTATION OF THE PROBLEM

The deck bridge can be assimilate by orthotropic or isotropic plates with different forms and different boundary conditions. The method used to determine the normal modes of vibration is the Finite Strip Method (FSM) associated with the Rayleigh - Ritz variational principle. This method has the advantage of a low number of finite elements, which leads to a smaller matrix band width for both the stiffness and inertia, and quick convergence of the solution.

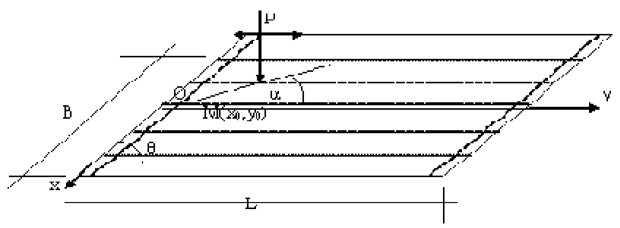

Fig. 1

In this article, the vehicle was considered as a concentrated mobile force but it can also be considered as a system with one, two or more degrees of freedom in which it is taken into account the elasticity and cushioning. The road surface was thought to be flat but the surface bumps can be modelled as sinusoidal functions for surface creases or cosine functions for potholes.

The displacement equation of the bridge-vehicle ensemble is:

$$M\ddot{w}+C\dot{w}+Kw=S_{Pd}\,\mathrm{f(t)} \tag{1}$$

Where:

M - masses matrix

C – damping matrix

K – stiffness matrix

$\mathbf{S_{pd}}$**-** load matrix

The general solution of the equation:

$$\mathbf{w}(x,y,t) = \mathbf{\Phi\eta}(t) \tag{2}$$

$$\mathbf{\Phi} =[\ \mathbf{\Phi_1}, \mathbf{\Phi_2}, \dots, \mathbf{\Phi_n}] \tag{3}$$

$\mathbf{\Phi}$ is the modal matrix independent of time and $\mathbf{\eta}$(t) is

$$\mathbf{\eta}(t)=\begin{bmatrix}\eta_1(\mathbf{t})\\ \eta_2(\mathbf{t})\\ .\\ .\\ .\\ \eta_{\mathbf{n}}(\mathbf{t})\end{bmatrix} \tag{4}$$

Using (2), (3) and (4) expression, the matrix equation becomes:

$$\mathbf{M\Phi}\ddot{\eta}(t)+\mathbf{C\Phi}\dot{\eta}(t)+\mathbf{K\Phi}\eta(t)=\mathbf{S_{pd}}f(t) \tag{5}$$

If we multiply the equation with $\mathbf{\Phi_j^T}$ and we take in consideration the orthogonality of the vibration forms:

$$(\mathbf{\Phi_j^T M \Phi_j})\ddot{\eta}_\mathbf{j}(t) + (\mathbf{\Phi_j^T K \Phi_j})\eta_\mathbf{j}(t) = \mathbf{\Phi_j}^\mathbf{T}\mathbf{S_{Pd}}\, f(t) \tag{6}$$

noting $\mathbf{\Phi_j^T M \Phi_j} = \Re_j$ and if we take in account that $\mathbf{K\Phi_j} = \boldsymbol{\omega}^2\mathbf{M\Phi_j}$ the equation (6) becomes:

$$\ddot{\eta}_j(t) + \omega_j^2\, \eta_j(t)= \frac{\mathbf{\Phi_j^T S_{Pd}}}{\mathbf{\Re_j}}\, f(t) \tag{7}$$

$\mathbf{S}_{pd} =(\mathbf{S}_d + \mathbf{S}_P)(\mathbf{d})^T$ is a column matrix and can be obtained as a sum of the stripes of the exterior action matrix $\mathbf{S}_d^e$ (given by action of the dead weight) and $\mathbf{S}^e_P$ (the exterior load action on every stripe).

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The dynamic response associated with the differential equation (7) attached to the j vibration mode is:

$$\eta_j\ (t)=\frac{\mathbf{\Phi_j^T S_{Pd}}}{\mathbf{\Re_j}}\ \frac{1}{\omega_j}\ \int_0^t \mathbf{f(\tau)\,sin}\,\omega_j\mathbf{(t-\tau)d}\tau \tag{8}$$

where

$$\mathbf{\Phi_j} = \mathbf{d_j}\ \mathbf{G_j}\ F_j \tag{9}$$

Where

G$_\mathbf{j}$ - matrix of polynomial functions ensuring the displacement compatibility from the border line between the stripes

F$_\mathbf{j}$ - matrix of characteristic functions corresponding to the longitudinal direction of the strip for the j mode (ensure the compatibility displacement strip whit the bearing condition)

d$_\mathbf{j}$ - general displacement of each strip

With the notations (9) the equation (8) becomes:

$$\eta_j(t)=\frac{\mathbf{d_j^{(T)} S_p}}{\mathbf{\Re_j}\omega_j^2}\,\omega_j\int_0^t \mathbf{G}F_j f(\tau)\ \sin\omega_j(t-\tau)d\ \tau \tag{10}$$

where the ratio:

$$\gamma_j=\frac{\mathbf{d_j^{(T)} S_p}}{\mathbf{\Re_j}\omega_j^2} \tag{11}$$

represents the participation factor of the j mode at the total response of the plate.

$$\psi_j(t)=\omega\int_0^t \mathbf{G}F_j f(\tau)\ \sin\omega_j(t-\tau)d\ \tau \tag{12}$$

is the dynamic multiplication function

Considering that the movement of the vehicle is only on the y axis at $x= x_0$ the **G** matrix is independent of time (figure 6.2). In particular if the y axis overlaps with the middle of the plate and the vehicle moves on the middle of plate $x_0=0$.

F_j function (on the direction of y) for simply supported plate is:

$$F_j\ (y)=\sin\ \frac{j\pi y}{L} \tag{13}$$

We also take into account that in any moment, the position of the force on the plate may be established using $y= y_0+vt$ ($y_0=0$ if the force begins from O)

In these conditions, the dynamic multiplication function given by (12) becomes:

$$\Psi_j\ (t)=\ \omega_j\int_0^t \sin j\Omega_j \sin\omega_j(t-\tau)d\tau \tag{14}$$

Where

$\Omega_j = \frac{j\pi v}{L}$ represents the load pulsation

ω_j - represents the plate pulsation

3. NUMERICAL APLICATION

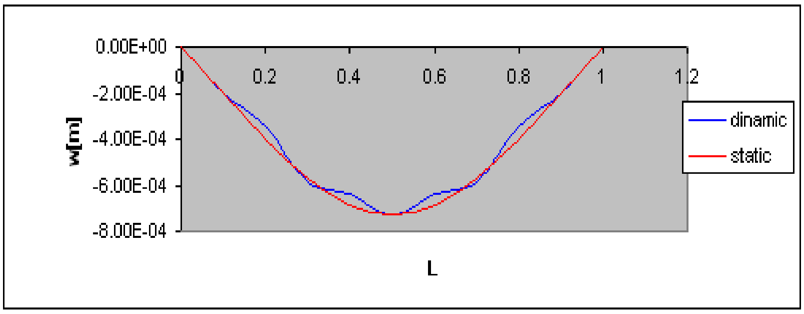
To test a section of the superstructure where dynamic multiplier has the highest value, we considered the action of a single vehicle (modelled as a mobile constant load) running in y direction, to the middle of the bridge. The ratio between the static and dynamic displacement in points at the 0.1L, 0.2L, ..., 0.9L towards the fixed bearing was calculated.

The deck bridges was considered L = 10 m, B = 10 m and h = 0.50 m, 0.60 m and 0.70 m, with homogeneous structure and loaded by a constant force P = 30 moving at a speed of 60 km/h, 100 km/h and 120 km/h over the bridge. The amortization of the structure and the surface irregularities were not taken in consideration.

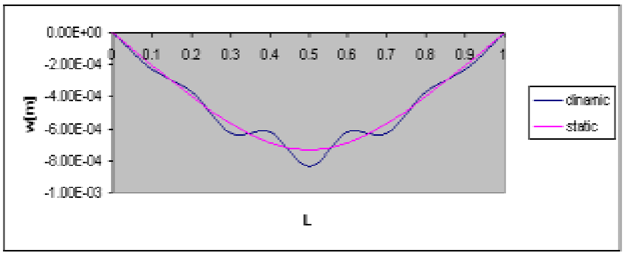
Table 1 Dynamic displacements of the bridge for different moving speeds

Thickness plate / speed	h=0.50 m	h=0.60 m	h=0.70 m
60 km/h	1.73 E-3	9.62 E-4	7.30 E-4
100 km/h	2.30 E-3	1.13 E-3	8.03 E-4
120 km/h	2.64 E-3	1.28 E-3	9.20 E-4
Static displacement	2.00 E-3	1.00 E-3	7.30 E-4

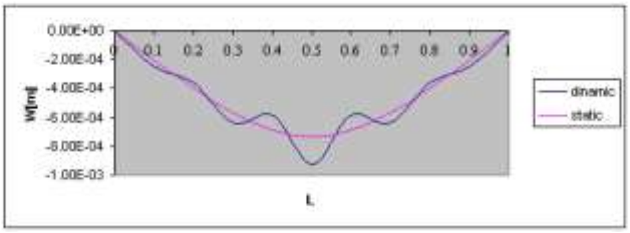
The graphical representation of the deck displacements with dimensions L = 10 m, B = 10 m and h = 0.65 m for the three circulation speeds is given in the following: *speed* v=60km/h



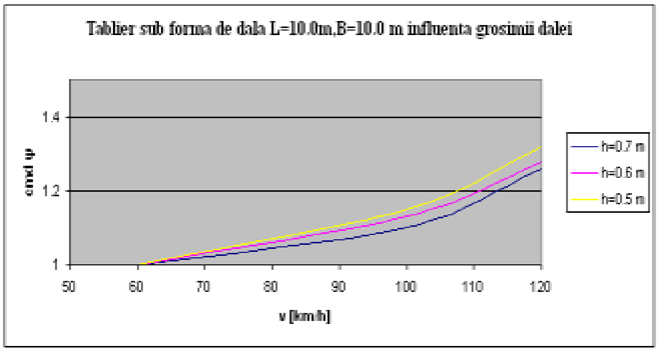
speed v=100km/h



speed v=120km/h



Also based on Table 1 we can draw the dependency of the multiplier dynamics and the thickness of the deck bridge with L=10 m, B=10 m and thicknesses (h) 0.5 m, 0.6 m and 0.7m



4. Conclusions

Dynamic analysis of road bridges is a priority for their sustainable composition. As was stated at the beginning of the paper, the carrying capacity of vehicles is in continuous growth. Also engine performance increases from one year to another which leads to higher speeds than those for which the bridge was designed. The speed of 100 km/h is now a usual speed of travelling outside localities. This results in the need for a dynamic study every time an intervention on a bridge is made.

The analysis shows that the dynamic multiplier ψ increases while the thickness of the deck is decreasing and the travelling speed is increasing. The opening for which the calculations were made is a relatively short length and the dynamic displacement is almost 30% bigger than the static, which means increasing the efforts in the deck elements. This is another reason for the dynamic calculations of bridges.

Using finite strips method is a very useful tool to perform such analyzes as it leads to very rapid results. Dividing the structure in finite strips is relatively simple and very easy to do.

The numerical example presented has the role to test the proposed method above and studies will continued with the consideration of amortization structure, modelling vehicles as systems with one and two degrees of freedom and with consideration of surface irregularities

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