THEORETICAL AND PRACTICAL ASPECTS FOR THE STRES STATE

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ABSTRACT:*In the present study we propose to determinethe knowing the state of stress at a point in a deformable geometrical solid involves knowing the unitary stresses acting on three surfaces (defined in three planes) perpendicular to each other that pass through the point considered. On each surface there acts a normal stress and a tangential stress, represented by the two components parallel to the common edges. Due to the duality of tangential tensions, they are equal two by two, so the stress state at one point is defined by six stress, three normal and three tangential.Applying the formulas corresponding to the state of tension for a given deformable geometrical solid to which known* σ_x, σ_y, τ_{xy} *determine the main stresses and the main directions that characterize the state of deformation.*

KEY WORDS:deformable geometrical solid, normal stress, tangential stress, main stresses, main directions.

1. INTRODUCTION

At a point in a deformable geometrical solid, the state of tension can be defined bythe tensions acting on three surfaces (defined in three planes) perpendicularbetween them, passing through the point considered. On each surface there acts a normal stress and a tangential stress, represented by the two components parallel to the common edges [1].Knowing the state of stresses and deformations in a required material involves knowing the unitary stresses and the specific deformations at each point of the material, as well as the relationships between stresses and specific deformations around a point.

The stress state at any point in the deformable solid is perfectly determined if the stresses are known on three coordinate planes that pass through that point[2]. If the position of the coordinate planes is arbitrary, each of these planes has, in the general case, both normal stresses and tangential stresses.

2. THE STATE OF TENSION

It considered a in theform of a plate of constant thicknessrequestedby a system of forces in equilibrium. Allforces act in the median plane of the plate. In thissituationthestressesappearing at anypoint in the body are paralleltothe plane of the plate, andtheplaquerequirementisidentical in allplanesparalleltothe median plane (3).

Thisrequestproduceswhatiscalledtheflattension state. Ifthe plate werefixedbetweentwo rigid walls, in ordertopreventtransversedeformations, sothatthedeformationsoccuronly in planesparalleltothe median plane, therecouldbe a flat state of deformation.Usingthreeplanes perpendicular tothe plane of the plate, a triangular prismisisolatedwiththebase of the OBC rectangular triangle OBC (fig.1).

Using a xOy rectangular axis system, normal stresses σ,willbearthe axis index withwhichthey are parallel, andthetangentialstressesτwillhavetwoindices, thefirstisthedirection of theirowndirection, andthesecondthe normal direction index tothe plane in whichthey act (3).

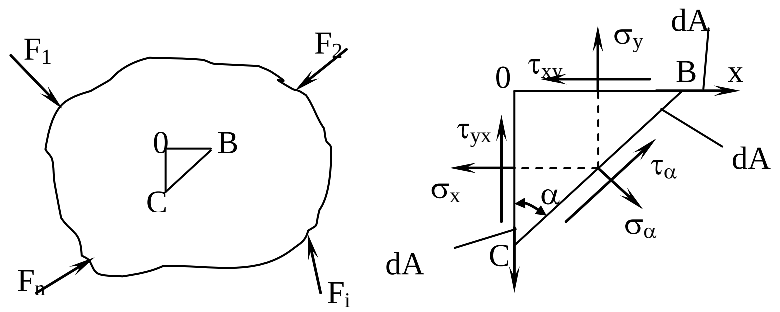

Fig.1. The state of tension

On the BC plane, makingtheangleαwiththe vertical plane OC, bothtensionswill carry theα index.

Writingwith dAthesurfacearea of theinclinedsurface BC of the element, the OB and OC surfaceareaswillbedA sin αanddA cos α.

The detached element on the plate willbe in balanceundertheaction of theforcesthat act as theplate.Iswritten a firstequation of equilibrium as thesum of momentscalculatedfromthepoint D, wheretheresultant of thethreetensions σ intersect:

$$\sum M_D = \tau_{xy} dA \sin\alpha \cdot \frac{BC}{2}\cos\alpha - \tau_{yx} dA \cos\alpha \frac{BC}{2}\sin\alpha = 0$$

Result: $\tau_{xy} = \tau_{yx}$, the law of parity or duality of tangential tensions.

Two more equations of equilibrium are written, projecting the forces in the direction of the stresses$\sigma_\sigma$and τ_α(3):

$\sigma_\alpha dA - \sigma_\alpha dA \cos\alpha\cos\alpha - \sigma_y dA \sin\alpha\sin\alpha - \tau_{xy} dA \sin\alpha \cos\alpha - \tau_{yx} dA\cos\alpha \sin\alpha = 0$

$\tau_\alpha dA - \sigma_x dA \cos\alpha \sin\alpha + \sigma_y dA\ \mathrm{din}\alpha \cos\alpha - \tau_{xy} dA \sin\alpha\sin\alpha + \tau_{xy} dA \cos\alpha\cos\alpha = 0$

Taking into account the law of parity ($\tau_{xy} = \tau_{yx}$) in the equations obtained, we can express the stresses $\sigma_\alpha$and τ_α on the inclined face of the element, depending on the known stresses, on the perpendicular faces and according to the angle of obliquity α:

$\sigma_\alpha = \sigma_x \cos\alpha + \sigma \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha$;$\tau_\alpha = (\sigma_x - \sigma_y) \sin\alpha \cos\alpha - \tau_{xy} (\cos^2\alpha - \sin^2\alpha)$

Entering the double angle will result:

$$\sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$\tau_\alpha = \frac{\sigma_x - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha \qquad (1)$$

The stresses $\sigma_\alpha$and τ_α, as a function of the angle α (1), can take maximum or minimum values for certain α angle values. Such angular values define the so-called main directions of tension. Maximum and minimum stresses are called the main stresses.

The main directions are obtained mathematically by canceling the derivatives of relations (1):

$$\frac{d\sigma}{d(2\alpha)} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos 2\alpha = 0 \qquad (2)$$

$$\frac{d\tau}{d(2\alpha)} = \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha = 0 \qquad (3)$$

Relationship (2) will give:

$$tg\,2\alpha_{1,2} = \frac{2\tau_{xy}}{\tau_x - \tau_y} \qquad (4)$$

and the relationship (3):

$$tg\,2\alpha_{3,4} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \qquad (5)$$

Equations (4) and (5) have two solutions offset with 180 °:

$$2\alpha_2 = 2\alpha_1 + \pi; \qquad \alpha_2 = \alpha_1 + \frac{\pi}{2}; \qquad (6)$$

$$2\alpha_4 = 2\alpha^3 + \pi; \qquad \alpha_4 = \alpha_3 + \frac{\pi}{2} \qquad (7)$$

There are two main directions for each of the voltages σ and τ, the directions forming between them a right angle. After a main direction the voltage is the maximum value, and after the second the minimum value.

From the comparison of relations (1) and (2) we can deduce:

$$\frac{d\sigma}{d(2\alpha)} = -\tau = 0 \qquad (8)$$

which shows that in the planes where normal tensions are the main ones, tangential tensions are missing.

From the comparison of relations (4) and (5) we deduce that,tg2$\alpha_{1,2}$tg2$\alpha_{3,4}$ = -1

so, $2\alpha_{3,4} = 2\alpha_{1,2} + \frac{\pi}{2}$ or $2\alpha_{3,4} = 2\alpha_{1,2} + \frac{\pi}{4}$ (9)

for example, the planes in which the tangential stresses are maximum or minimum, make angles of 45 ° with the planes in which the normal stresses are the main ones.

By replacing the angles α given by the relations (4) and (5) in relations (1) the main stresses will result in the form (3),

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_v}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (10)$$

$$\sigma_{3,4} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (11)$$

Relationships (10) will,$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$ = constant,(12)

which shows that the sum of the normal tensions from two perpendicular planes, taken at one and the same point, does not depend on the angle α, is invariant.

According to relations (11) the maximum tangential tension is equal to the minimum. This is in accordance with the law of duality of tangential tensions.

Based on the relationship (10) it can be written,

$$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_2) \quad (13)$$

this means that the maximum tangential voltage is the half-dimension of the main normal stresses.

In fig. 2 are the main normal stresses and their main directions around an arbitrary point. Considering aelement in the vicinity of the contour, which is assumed to be unloaded, the element's faces will lack the tangential stresses. As a result, the main directions of contour stresses will be normal and tangential to contour respectively (3).

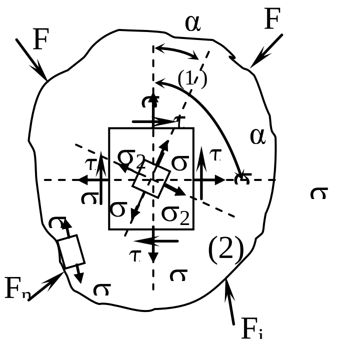

Fig.2.Main normal stresses and main directions

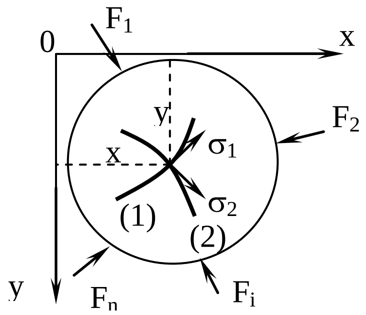

Fig.3. Maximum and Minimum Requests Direction

For a flat state of tension, a network of tangent lines can be drawn at all their points at the main stress tensions. These are called isostatic curves and they indicate the direction of maximum and minimum stresses (fig. 3).

The equation of the isostatic lines is easily obtained by the relationship (4):

$$tg\,2\alpha_{1,2} = \frac{2tg\,\alpha_{1,2}}{1 - tg^2\alpha_{1,2}} = \frac{2y'_1}{1 - y'_{1,2}} = \frac{2\tau_{xy}}{\sigma_x - \sigma_Y} \,(14)$$

from which it follows,

$$y'^2_{1,2} + \frac{\sigma_y - \sigma_x}{\tau_{xy}} y'_{1,2} - 1 = 0 \,(15)$$

and solutions,

$$y'^2_{1,2} = \frac{\sigma_x - \sigma_y}{2\tau_{xy}} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2\tau_{xy}}\right)^2 + 1} \qquad (16)$$

which represents two families of perpendicular curves, because, $y'_1 \cdot y'_2 = -1$

Tangential curves to maximum tangential stresses are called sliding lines. The differential equation of these lines will be obtained as before, in the next form (3):

$$y'_{3,4} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)^2 + 1} \qquad (17)$$

Relationships (1) allow the expression of the stresses$\sigma_\alpha$and $_\alpha$ from any section at any one point, depending on the main stresses.

For this you must as: $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, $\tau_{xy} = 0$:

$$\sigma_\alpha = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\alpha$$
$$\tau_\alpha = \frac{\sigma_1 + \sigma_2}{2}\sin 2\alpha \qquad (18)$$

By eliminating the angle 2α,

$$\left(\sigma_\alpha - \frac{\sigma_1 + \sigma_2}{2}\right)^2 + \sigma_\alpha = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 \qquad (19)$$

3. PRACTICALCONSIDERATIONS

This calculation can be applied to a parallelepiped shape volume element having known (4) (Fig. 3) σ_x=80MPa, σ_y=40MPa șіτ_{xy} =50MPa

The formula (10) is used to calculate the main stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_v}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{80+40}{2} \pm \sqrt{\left(\frac{80-40}{2}\right)^2 + 50^2}$$

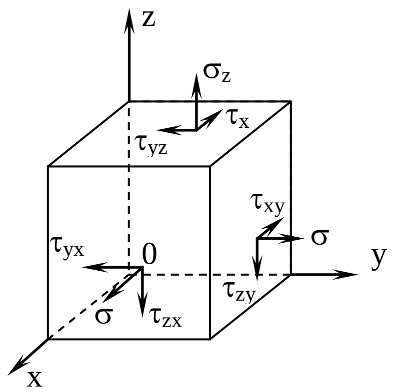

Fig. 4. Tension distribution

The result is σ1 = 113,851MPa and σ2 = 1,149 MPa

The main directions are determined by the formula (14):

Then $\alpha 1 = 34{,}1^0$and $\alpha 2 = 124{,}1^0$

3. CONCLUSIONS

For aplate form of constant thickness requested by a system of forces in equilibrium all forces act in the median plane of the plate.
The stresses appearing at any point in the body are parallel to the plane of the plate and the plaque requirement is identical in all planes parallel to the median plane.If the plate is fixed between two rigid wallsthere could be a flat state of deformation.
In this paper we did the calculation for a parallelepiped shape volume element and we determined the main direction and the main stresses that are oriented about α angle.

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