

ANALYSIS AND SYNTHESIS OF PARALLEL MANIPULATORS

Drnd. Eng. Constantin BREZEANU* Silcotub S. A. (Tenaris Group) Călărași,
e-mail: br79c@yahoo.com

Cătălina ROBU (NAN), prof. "Tudor Vianu" High School, Giurgiu
Prof. em. PhD. Eng. Păun ANTONESCU, University POLITEHNICA Bucharest,
e-mail: panton38@hotmail.com

***Abstract:** Paper presents a new way of the structural - topological analysis for the parallel manipulators, with referring to the calculation of mobility (degree of freedom) of complex mechanisms. The parallel manipulators are complex topological structure, having the joints (as kinematical pairs) of various types. The thirty six formulas known in the literature for mobility calculation are grouped in two categories: those for a quick calculation of mobility from the visual inspection of kinematical schema, and the approaches based on rank of the kinematical constraint equations. For the parallel manipulators we present a new formula for mobility calculation in function of the kinematical chains number, considered as the identical legs.*

Keywords: structural analysis, parallel manipulator, mobility, kinematical schema, actuator

1. Introduction

Mobility or d.o.f. is the main structural – topological parameter of a mechanism and manipulator-robot, this influences the kinematical and dynamic modeling of mobile mechanical systems [1, 2].

The kinematical chains mounted in parallel between two platforms are composed of two or more kinematical elements, these being realized as open chains or as complex chains with closed and open contours [3, 13].

Gough's machine (1947) and the Stewart's platform (1965) are constituted as spatial mechanisms with six identical kinematical chains (KC) [4]. These mobile platforms represented the initial structural-topological model, from which subsequent research was started, using the name of parallel manipulator (PMP). The PMP-s can be connected in series, obtaining overlapping PMP-s, similar to the structural-topological of the serial elephant trunk type robots. Unlike the serial manipulators, at PMP-s practically all types of kinematical pairs (joints) are used [1, 5], what allows a larger diversity of kinematical schemas.

The parallel manipulator-robot is a mobile mechanic structure, having one or more levels, with closed kinematical chains, which are driven by actuators. In the last period, since 1990, the attention of more researches [5-9] has oriented to approaching the potential possibilities of PMP-s. The results of numerous studies [4, 14] show that the concept of the parallel manipulator has been generalized and can be used as a model of geometrical and kinematical analysis and synthesis in diverse domains, from the simulation platforms (in the aeronautic and automotive industries) and drilling petroleum platforms to industrial robots and mobile robots of type walking machine.

Defining PMP, J.P. Merlet [4] him compares with a terminal organ with n degrees freedom and a fixed base, linked together by at least two independent kinematical chains, which are actuated by n simple actuators. In the invention and design of parallel robots, the contriving of the new types of robotic manipulators is one of the most important activities [9]. The forward geometric-kinematic problem of parallel robots consists in determining the position and the orientation of the platform when the joints variables are known [10].

G. Gogu [11] makes a critical review on the calculation of the mobility for mechanisms and manipulators, presenting a critical analysis of various methods presented in the literature in the last 150 years.

2. New General Formula for Mechanism Mobility

We note with r the rank of the space associated to an independent loop and with N_r the number of independent closed contours (of the same rank: $2 \leq r \leq 6$). The number of loops (N_r) is function of the number n of links and the number (C_m) of all joints of various classes having the function class m (Euler formula adapted for KC):

$$N_r = \sum_{m=1}^5 C_m - n + 1 \quad (2.1)$$

In the articulated manipulators (having $r=3$) the maximal function class of the joints is $m=2$, such as with planar manipulators. The minimal function class ($m=1$) is obtained for $r=2$, like in the case of manipulators with screw. It is known [1, 2, 3] that the formula of geometrical mobility – d.o.f. - for the three dimensional manipulators, having one or more independent closed contours of the different rank r is of the form [1,2]:

$$M = \sum_{m=1}^5 (m \cdot C_m) - \sum_{r=2}^6 (r \cdot N_r) \quad (2.2)$$

In formula (2.2) it was noted with N_r the number of closed contours (of the r rank), which is calculated depending of C_m , the number of the joints of functional class $m \in [1, (r-1)]$, and of the number of links (n_r) from the topological structure of the r – rank *closed contour*, according to the actualized formula (2.1):

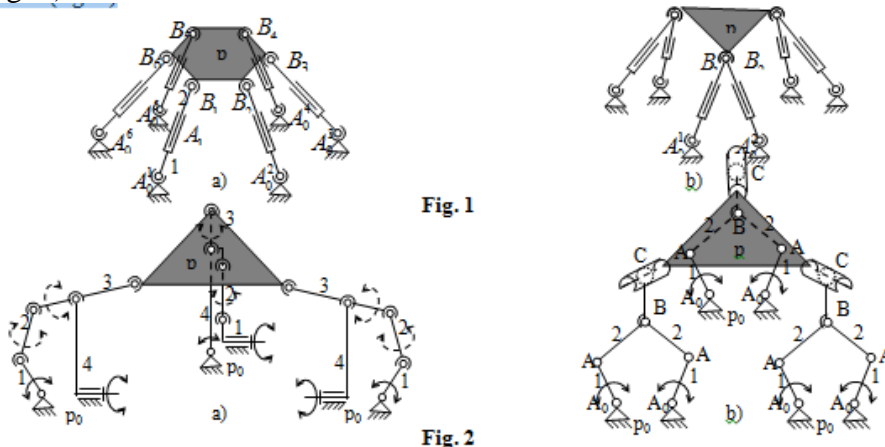
$$N_r = \sum_{m=1}^5 C_m - n_r + 1 \quad (2.3)$$

In the particular case of manipulators having only open contours, the number of closed contours is $N_r=0$, so formula (2.2) becomes:

$$M_0 = \sum_{m=1}^5 (m \cdot C_m) \quad (2.4)$$

3. New Formula for Calculation of PMp Mobility

If we note with N_l the number of kinematical chains, which are considered as legs for PMp, then the number of independent closed contours is $N_l - 1$. It is mentioned that the kinematical chain of a leg can be the open contour (fig. 1) or the complex contour, with open and closed contours (fig. 2).



Formula (2.2) is writes for PMp-s from first category (fig. 1) keeping account of (2.4) in the following form:

$$M = (M_0 - r_c) \cdot N_l + r_c \quad (3.1)$$

where r_c is the rank of one closed kinematical contour with two legs.

For example, for PMp shown in figure 1, a leg is composed from two joints of class $m=3$ and a joint of class $m=2$, therefore them mobility is: $M_0 = 2 \cdot 1 + 3 \cdot 2 = 8$. The geometrical mobility of this PMp is calculations with formula (3.1), where we replace $r_c=6$ and $N_l=6$: $M = (8-6) \cdot 6 + 6 = 18$.

The 18 d.o.f. of PMp are 6 active d.o.f. (represented of 6 actuators, which effect six sliding motions between elements 1 and 2) and 12 passive d.o.f. (represented of the rotations of elements 1 and 2 about axis A_0B).

Formula (2.2) can be written, for PMp-s from second category (fig. 2), in the following form:

$$M = \left(\sum_{m=1}^5 (m \cdot C_m) - \left(\sum_{i=1}^{n_c} r_i + r_c \right) \right) \cdot N_l + r_c \quad (3.2)$$

where: $\sum_{m=1}^5 (m \cdot C_m)$ are number of a leg joints, r_i is the rank of closed kinematical contour from structure a leg, n_c is number of closed contours of the leg and r_c is the rank of closed contour including the mobile platform.

For example, for PMp shown in figure 2a, $N_l=3$ legs and a leg is composed from a closed contour type spatial pentagon (with the rank $r_1=6$) and an open contour (with the rank $r_2=6$), therefore $n_c=1$, while $r_c=6$. The geometrical mobility of this PMp (fig. 2a) is calculated with formula (3.2) thus: $M = [(1 \cdot 2 + 3 \cdot 4) - (6 + 6)] \cdot 3 + 6 = 12$

The 12 d.o.f. of PMp are 6 active d.o.f. (indicated of the circle arrows which with continuous line drawn) and 6 passive d.o.f. (indicated by circle arrows with dash line). In the case of PMp from figure 2b, $N_l=3$ legs and a leg is composed from a closed contour type planar pentagon (with the rank $r_1=3$) and an open contour (with the rank $r_2=6$), therefore $n_c=1$, while $r_c=6$. The geometrical mobility of this PMp (fig. 2b) is calculated with formula (3.4) thus: $M = [(1 \cdot 5 + 4 \cdot 1) - (3 + 6)] \cdot 3 + 6 = 6$. All the 6 d.o.f. of this PMp (fig. 2b) are active d.o.f. represented through the rotations of the 6 elements notated on kinematical scheme with 1.

4. Verify of Mobility of Tsai's PMp [11]

The Tsai's PMp is a special PMp (fig. 3) which has three legs with four mono-mobile joints (A, B, C, D), while the three joints of the platform 4 have the perpendicular axes (x, y, z).

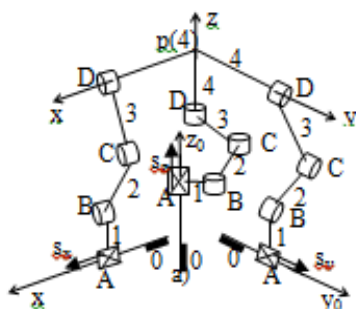


Fig. 3

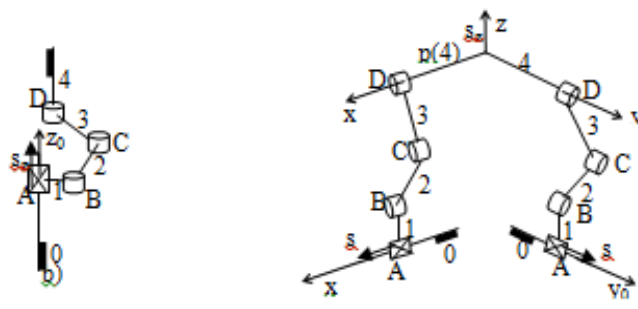


Fig. 4

The axes of the leg joints are parallel with the corresponding axis x, y and z. PMp has three legs (1, 2, 3) with the structure T+R+R+R and two independent contours with the same rank $r_c=5$.

For the calculation of mobility we use formula (3.1):

$$M = (M_0 - r_c)N_l + 6 = (4 - 5) \times 3 + 6 = 3 \quad (4.1)$$

By visual inspection of PMp (fig. 3a), we see that the mobility is $M=3$, all those three mobility being represented through three translation motions (s_x, s_y, s_z) are the partially mobility [11], since for each mobility the corresponding kinematical chain becomes a rigid body. Where is the mistake?

Let we analyze the PMp with only two legs (fig. 4), in this case with same formula (3.1) d.o.f. becomes:

$$M = (M_0 - r_c)N_l + r_c = (4 - 5) \times 2 + 5 = 3 \quad (4.2)$$

Those three mobility are given by the translation motions of elements noted with 1 (s_x, s_y) and of element 4 (s_z). Thus, the 3-th kinematical chain with same structure topological T+R+R+R (fig. 3b) is a passive open contour having the mobility null:

$$M_{kc} = C_1 - r_c = 4 - 4 = 0 \quad (4.3)$$

5. Verify of Mobility of one Special PMp with Screw

We analyze a special PMp having three screw actuators, with the non parallel coplanar axes, whose the kinematical schema is shown in figure 5.

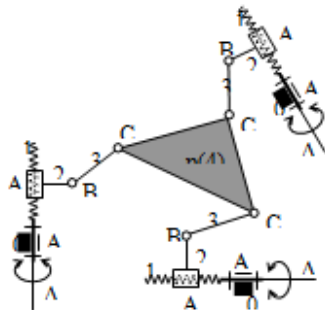


Fig. 5

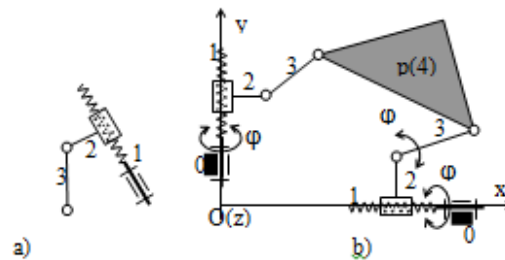


Fig. 6

If is calculated the mobility with formula (3.1) results:

$$M = (M_0 - r_c)N_l + 6 = (4 - 5) \times 3 + 6 = 3 \quad (5.1)$$

By visual inspection of PMp (fig. 5), we see that the mobility is $M=3$, all those three mobility being represented through three rotations around the axes Δ_1, Δ_2 and Δ_3 .

As well as in the previous case (fig. 3) is separated a kinematical chain having elements 1, 2 and 3 (fig. 6a) thus PMp remain with only one closed kinematical contour (fig. 6b). Let we analyze the PMp with only a closed kinematical contour (fig. 6b); in this case with same formula (3.1) is obtained:

$$M = (M_0 - r_c)N_l + r_c = (4 - 5) \times 2 + 5 = 3 \quad (5.2)$$

Those three mobility are given by the rotation motions of elements noted with 1 (φ_x , φ_y) and element 3 (φ_z).

Thus, the 3-th kinematical chain with same structure topological R+R/T+R+R (fig. 6b) is a passive open contour having the mobility null:

$$M_{kc} = C_1 - r_c = 4 - 4 = 0 \quad (5.3)$$

6. Mobility of Planar PMp-s

We consider a planar PMp with the rotative joints (fig. 7), having three actuators.

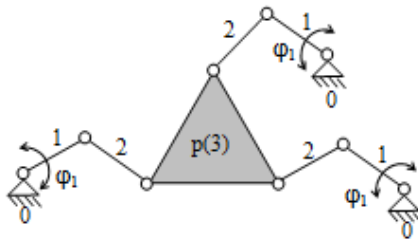


Fig. 7

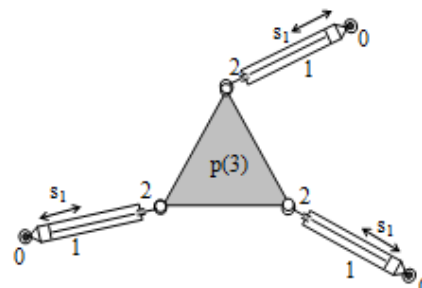


Fig. 8

It is used formula (3.1) and is calculated the mobility of this PMp (fig. 7) thus results:

$$M = (M_0 - r_c)N_l + r_c = (3 - 3) \times 3 + 3 = 3 \quad (6.1)$$

Through the rotative motions φ_1 of elements 1 about of fixed point, those three mobility of PMp are realized.

In the case of the actuators with sliding joints (fig. 8), are realized those three mobility of PMp through the rectilinear translations s_1 . Analyzing the topological structure of these PMp-s (fig. 7, 8), is observed this type of PMp can be considered an open kinematical chain $(0+1+2+3)$, which is constrained by addition of two kinematical chains dyadic type [1] CDD_0 and EFF_0 (with mobility null), such how is shown in figures 7a, 8a.

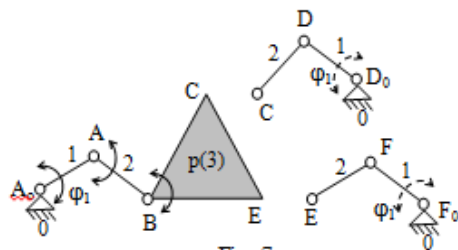


Fig. 7a

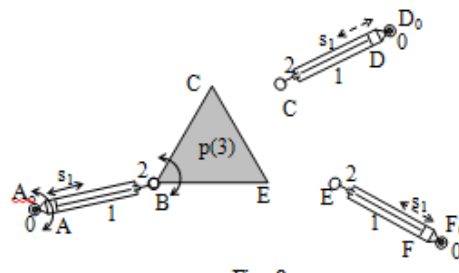


Fig. 8a

Mobility of open kinematical chain (fig. 7a, 8a) is deduced with formula (2.7) and results $M_0 = 3$.

7. Mobility Calculation of Planar Simplest PMp

The simplest PMp have only the sliding (prismatic) joints, in which case the mobile platform p is connected to fixed platform p_0 from two elements 1 and 2 (fig. 9).

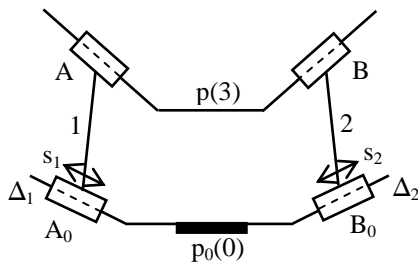


Fig. 9

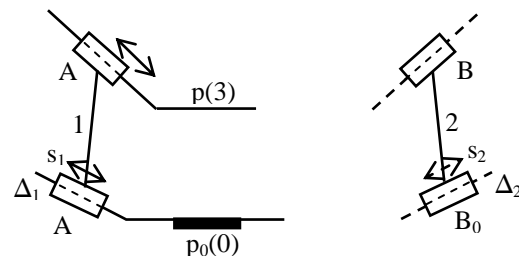


Fig. 9a

Mobility of this parallel manipulator (fig. 9) with formula (3.1) is calculation:

$$M = (M_0 - r_c)N_l + r_c = (2 - 2) \times 2 + 2 = 2 \quad (7.1)$$

This mobility is represented of the sliding motions s_1 and s_2 of elements 1 and 2, which are realized along the fixed axes Δ_1 and Δ_2 .

This planar PMp (fig. 9) must be considered as being obtained from an open kinematical chain (0+1+3) with two prismatic joints (fig. 9a), which is constrained through addition of one kinematical chain, which is represented of element 2 of mobility zero (in the space of rank 2).

8. Structural Synthesis of Spatial PMp-s

Unlike serial manipulators, at PMp-s all types of kinematical pairs are used. However, the most frequently utilized are the mono-mobile kinematical pairs of rotation (R) or translation (T), the bi-mobile cylindrical pairs (C) and tri-mobile spherical pairs (S). We follow the topological structure of kinematical chains with actuators (KCA), for the general case of spatial PMp-s (fig. 10a-d).

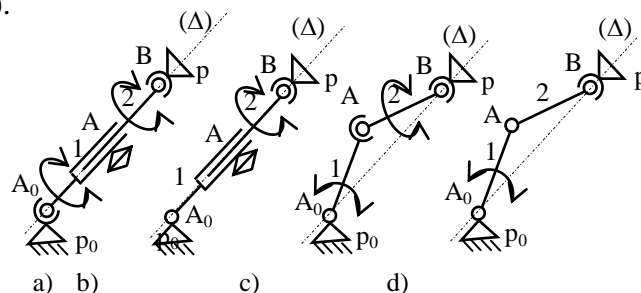


Fig. 10

KCA type SCS (fig. 10a) is an open kinematical chain KC ($p_0+1+2+p$) (formed from two elements 1 and 2) which link together two platforms, of which one - p_0 (with spherical articulation A_0) - can be fixed, while the other is the mobile platform p (with spherical articulation B). The leg has three d.o.f. but we impose that movement be controlled by one single actuator. If it is noted with M_Δ the mobility of KCA type SCS compatible with axis (Δ), this is equal to 3, being represented by the *active* sliding motion along the axis (of bar 2

relative to bar 1) and by two *passive* rotational movements (of bars 1 and 2) about the axis. The mobility of PMP with x KCA is calculated with the general formula of the manipulators with $(x-1)$ independent contours of rank 6 [1, 12]:

$$M = (2 \cdot 1 + 3 \cdot 2) \cdot x - 6 \cdot (x - 1) \quad (8.1)$$

We impose $M = xM_{\Delta}$ for this structural synthesis from (8.1) results

$$(2 \cdot 1 + 3 \cdot 2) \cdot x - 6 \cdot (x - 1) = 3x \quad (8.2)$$

From the equation (8.2) results $x = 6$, that leads to PMP with six actuators type SCS (fig. 1a), as the Gough - Stewart platform, with six active mobility (the translations of pistons 2 in cylinders 1) and 12 passive mobility (the non-controlled rotations of pistons 2 and of cylinders 1 about the axis (Δ)). The mobility of PMP (fig. 1a) can be immediately verified with formula (3.1) such was obtained $M = 18$. Indeed, the 18 mobility are identified from 6 active mobility (which are realized by the s_{21} translations) and another 12 passive mobility, these being the potential $\varphi_{1\Delta}$ and $\varphi_{2\Delta}$ rotations. Both the inferior fixed platform and the mobile superior one have, in general, a hexagonal form (fig. 1a) with the tips $A_0^i (i=1 \dots 6)$ and $B_i (i=1 \dots 6)$, respectively. In particular cases, the mobile platform can have a triangular form (fig. 1b) when the spherical articulations $B_i (i=1 \dots 6)$ coincide or are very close to one another (fig. 11).

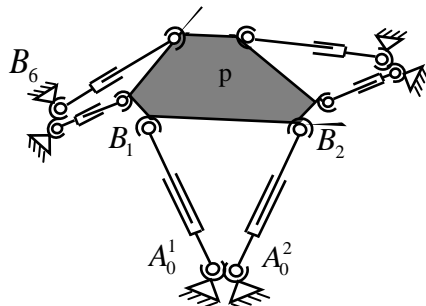


Fig. 11

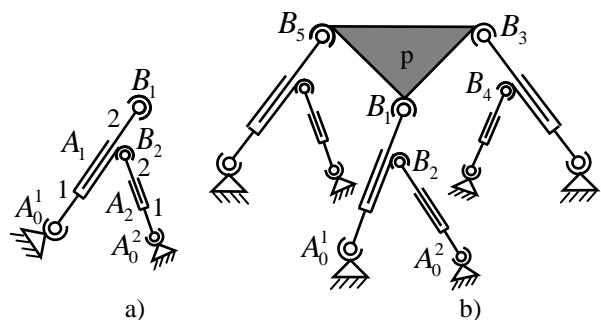


Fig. 12

Since the solution of locating of these two spherical articulations (fig. 1b) is difficult to realize in practice. The optimal solution is connection of one of the two spherical articulations (B_2) to the cylinder of a actuator (1), such shown in figure 12a (in detail) and in figure 12b in ensemble (similar to Stewart's platform).

If the mobile platform p (fig. 1a) is reduced to a bar with two spherical articulations $B_I (B_1 \equiv B_2 \equiv B_3)$ and $B_{II} (B_4 \equiv B_5 \equiv B_6)$, respectively (fig. 13), the PMP allows the bar movement $B_I B_{II}$ in any position inside the working space. In this case another passive mobility results, represented by the p bar rotation about axis (Δ) that connects points B_I and B_{II} .

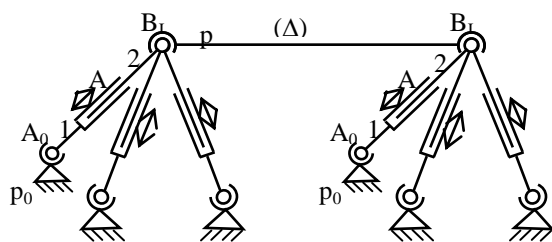


Fig. 13

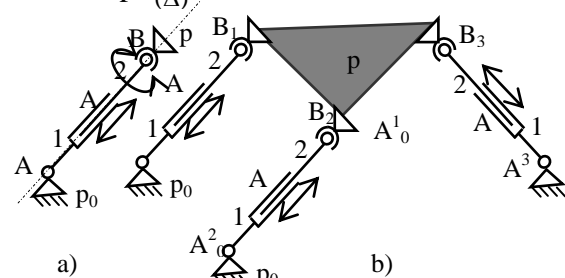


Fig. 14

In the particular case when the mobile platform (p) is reduced to a point B is obtained the manipulator type Mianowski [4], which has three parallel KC-s of type SCS.

With the help of this manipulator, which has three active d.o.f. notated s_{2l} and six passive d.o.f. (fig. 14b) point B can be locating in any position within the working space. The three independent linear displacements (s_{2l}) correspond to the three Cartesian coordinates of point B in the three-dimensional space.

KCA type RCS (fig. 10b) has mobility $M_{\Delta} = 2$, which means an active linear displacement of piston 2 in cylinder 1, as well as a passive rotation of piston 2 about axis Δ (fig. 14a). To find the number x of actuators for a PMp, the formula $M = xM_{\Delta}$ is applied:

$$M = (1 \cdot 1 + 2 \cdot 1 + 3 \cdot 1) \cdot x - 6 \cdot (x - 1) = 2x \quad (8.3)$$

from which the solution $x = 3$ is obtained, that corresponds to the kinematical schema of figure 14b.

The geometrical mobility of this PMp (fig. 14b) is calculated with formula [1]:

$$M = (1 \cdot 1 + 2 \cdot 1 + 3 \cdot 1) \cdot 3 - 6 \cdot 2 = 6$$

The six mobility are three active translations (s_{2l}) of pistons 2 about cylinders 1 and three mobility are passive rotations of pistons 2 about the Δ axis. This axis is a straight line drawn from the spherical joint center B , having the direction perpendicular to the axis of rotation joint in A_0 (fig. 14a).

9. Conclusion

Structural analysis and synthesis of parallel manipulator with respect to rank of closed contours is an actual problem. For the parallel manipulators we present a new formula for mobility calculation in function of the kinematical chains number, considered as the identical legs. The particular cases of mechanisms must to be analyzed with attention for the correct determination of rank (spatiality) of each independent contour.

References

- [1] **Antonescu P.**, *Mechanism and machine science*, Printech Publishing House Bucharest, 2005.
- [2] **Antonescu P.**, *General formula for the d. o. f. of complex structure manipulators and robots*, Proceedings of "Tenth world Congress on the TMM" Oulu, Finland, Vol. 3, pp.1067-1072, 1999.
- [3] **Antonescu P., Gâlmeanu C.**, *Contributions to the calculus of mobility of mechanisms and manipulators*, Review Constructia de masini, Vol. 50, No. 7, pp. 5-8, 1998.
- [4] **Merlet J. P.**, *Les robots paralleles*, Hermes, Paris, 1990.
- [5] **Gosselin C. M., Wang J., Laliberte T., Ebert – Uphoff I.**, *On the design of a statically balanced 6-d.o.f. parallel manipulator*, Proceedings of "Tenth world Congress on the TMM" Oulu, Finland, Vol. 3, pp.1045-1050, 1999.
- [6] **Laliberte T., Gosselin C. M., Cote G.**, *Rapid prototyping of mechanisms*, Proceedings of "Tenth world Congress on the TMM" Oulu, Finland, Vol. 3, pp. 959-964, 1999.
- [7] **Yang G., Chen I. M., Yeo S. H.**, *Design consideration and kinematic modeling for*

modular reconfigurable parallel robots, Proceedings of “Tenth world Congress on the TMM” Oulu, Finland, Vol. 3, pp. 1079-1084, 1999.

[8] **Tsai L-W.**, *The enumeration of a class of three-d.o.f. parallel manipulators*, Proceedings of “Tenth world Congress on the TMM” Oulu, Finland, Vol. 3, pp. 1121-1126, 1999.

[9] **Gao F., Li W., Zhao X., Jin Z., Zhao H.**, *New kinematic structures for 2-, 3-, 4-, and 5-DOF parallel manipulator designs*, Mechanism and Machine Theory 37 (2002) pp. 1395-1411, 2002.

[10] **Tancredi L., Teillaud, M.**, *Application de la geometrie synthetique au probleme de modelisation geometrique directe des robots paralleles*, Mechanism and Machine Theory 34 (1999) pp. 255-269, 1999.

[11] **Gogu G.**, *Mobility of mechanisms: a critical review*, Mechanism and Machine Theory 40 (2005) pp 1068 – 1097, 2005.

[12] **Antonescu O., Antonescu P.**, *Contributions to topologic structural synthesis of the parallel manipulators*, Conf. of Robotics, Iasi, 2006.

[13] **Luca L., Popescu I.**, *Paths and Laws of Motion of a Mechanism with Two Successive Conductive Elements and a Triad*. Applied Mechanics and Materials Vol. 772 (2015) © (2015) Trans Tech Publications, Switzerland, pp 344-349;

[14] **Luca L, Popescu I.**, *Trajectories generated by mechanisms containing the chebyshev dyad*. Fiabilitate si Durabilitate, nr.1, 2016, Editura Academica Brancusi, pp.26-34.