STRUCTURAL AND KINEMATICAL ANALYSIS OF THE COMPLEX PLANETARY MECHANISMS USED AS AUTOMATIC TRANSMISSIONS

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ABSTRACT: *It is considered the kinematic structure of a complex planetary mechanism, which operates by using three planetary units with cylindrical gearings, as automatic transmission on passenger cars. Knowing the kinematic conditions in all 4+1 gears, the paper deals with a method of synthesis of structural – topological and kinematical type by the calculation of the analytical relations of input–output transmission functions in each gear.*

1. Introduction

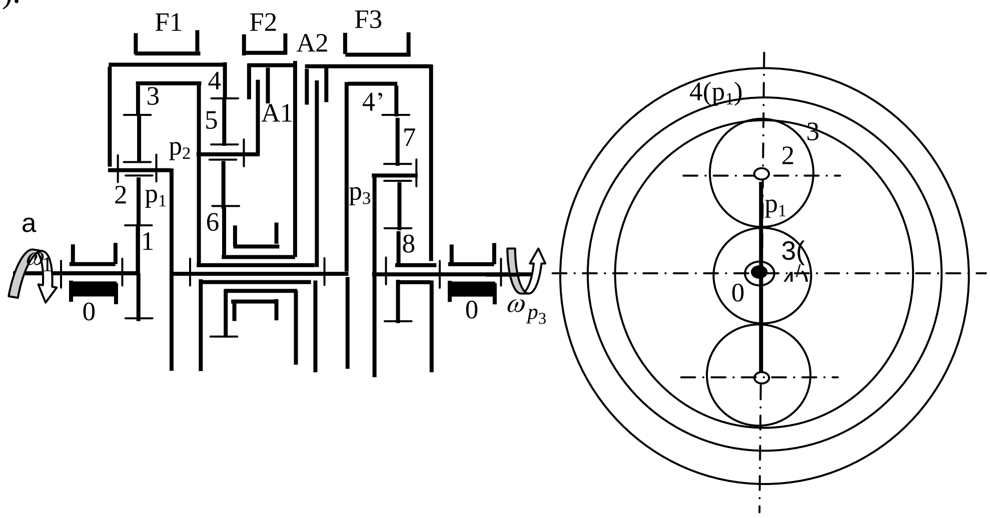
The cylindrical planetary mechanisms are used as the automatic with which are presented at the modern cars [1, 2]. Some interesting studies on the structure and kinematic of some planetary mechanisms are also presented in the papers [3], [4], [5]. Because of the multiple advantages of the automatic transmissions, in last period is ascertained a rise of the preoccupations concerning scientific research of the cylindrical planetary mechanisms with the complex variable structure [6, 7].

For the achieve of geometric – kinematical and dynamic synthesis of the cylindrical planetary mechanisms with multi-mobile variable structure, as the mobile systems which are associated of automatic transmissions, is traversed follow principal stages [7]:

- The establish in function of the car type of the maxim number of gears, simultaneously with the specifying of numerical values of the transmission ratios from each gear;
- The choice the geometrical structure and of the number of bi-mobile cylindrical planetary units, which follow to be connected serial, parallel and serial-parallel;
- The structural – topological analysis of the complex planetary mechanism, which is capable to achieve 3, 4, 5 or more gears, through the specify kinematical conditions to each gear;
- The geometric – kinematical synthesis of the multi-mobile planetary mechanism in the neutral position, through the imposed kinematical conditions, thus that, in function of the imposed values of the transmission ratios from each gear, is determined the transmission ratios which are interior of each the planetary units and of the teeth numbers for each the component gear;
- The dynamic analysis of the complex planetary mechanism, following the calculation of forces and moments of each the kinematic element (central / satellite gears and port – satellite arms), as well as the power flow from the mechanism and the calculus of mechanical efficiency in each gear.

2. Kinematic schema of complex planetary mechanism

Considering the kinematic schema of the planetary mechanism with cylindrical gearings, used as the automatic transmission with 4+1 gears, in axial and transversal views (fig. 1a, b).



a) b)

Fig. 1. The 4+1 gear automatic transmission

In the transversal projection (fig. 1b) are drawn the pitch circles of the central gear 1 (with external teeth), of the satellite gear 2, of the central gear 3 (with internal teeth) and the port-satellite arm p_1, which is solidarity with the central gear 4 with the internal teeth (fig. 1a).

The 4+1 gears are obtained by clutches A1 and A2, and by locking devices F1, F2 and F3, which are actuated according to some specified kinematical conditions (tab. 1).

It is followed the calculation of geometrical mobility, the drawing of structural-topological schema [8, 9] of the complex planetary mechanism in neutral position; the kinematic analysis of the planetary mechanism through the determinate of the linear velocities distribution and the transmission ratio in each gear; the kinematic synthesis of complex mechanism, concerning the calculation of the specify transmission ratios of those three component planetary units.

The study can be continued with the calculation of the cylindrical engagement, through the determination of the engagement factor through the graphical and analytical methods; the dynamic analysis of complex planetary mechanism, through the calculation of the mechanical efficiency of the complex planetary mechanism for each the speed stage and the determination of power to the driven shaft.

Table 1

Gear	Clutch A1	Clutch A2	Clutch F1	Clutch F2	Clutch F3	Kinematical Conditions	Transmission Function
I		*			*	$\omega_3 = \omega_8 = 0$	$i_I = i_{1p_3} = f_I(z)$
II				*	*	$\omega_6 = 0; \omega_8 = 0$	$i_{II} = i_{1p_3} = f_{II}(z)$
III	*				*	$\omega_1 = \omega_6; \omega_8 = 0$	$i_{III} = i_{1p_3} = f_{III}(z)$

IV	*	*				$\omega_3 = \omega_6 = \omega_8$	$i_{IV} = i_{1p_3} = f_{IV}(z)$
V(R)		*	*			$\omega_4 = 0; \omega_6 = \omega_8$	$i_V = i_{1p_3} = f_V(z)$

2. The mobility and structural-topological schema of mechanism

The complex planetary mechanism (fig. 1) with the cylindrical gearings, which is moved in parallel planes, is formed from three modules „planetary units" of cylindrical planetary mechanisms.

A cylindrical planetary unit (CPU) is formed from two central gears (with the fixed rotation axes), a planetary gear (satellite) with the mobile rotation axis and a crank – arm which is named the port-satellite arm (with the rotation fixed axis which is coaxial with the rotation axis of the central (sun) gears.

First cylindrical planetary unit (CPU1) is formed from the central cylindrical gears 1 (with the exterior teeth) and 3 (with the interior teeth), from the satellite cylindrical gear 2 (with the exterior teeth) and from the port-satellite arm p_1. In the figure 2a is shown kinematic schema of CPU1.

Second cylindrical planetary unit (CPU2) has in the make-up the central cylindrical gears 4 (p_1) with the interior teeth and 6 (with the exterior teeth), from the satellite cylindrical gear 5 (with the exterior teeth) and from the port-satellite arm p_2(3). In the figure 2b can be follow the kinematic schema of CPU2.

Third cylindrical planetary unit (CPU3) is formed from the central cylindrical gears 4' (p_1) with the interior teeth and 8 (with the exterior teeth), from the satellite gear 7 (with the exterior teeth) and from the port-satellite arm p_3, which is solid with the driven shaft.

In the figure 2c can be followed the kinematic schema of CPU3. In each of three kinematic schemas (fig. 2a, b, c) the kinematic elements that control the gears are shown with dashed line.

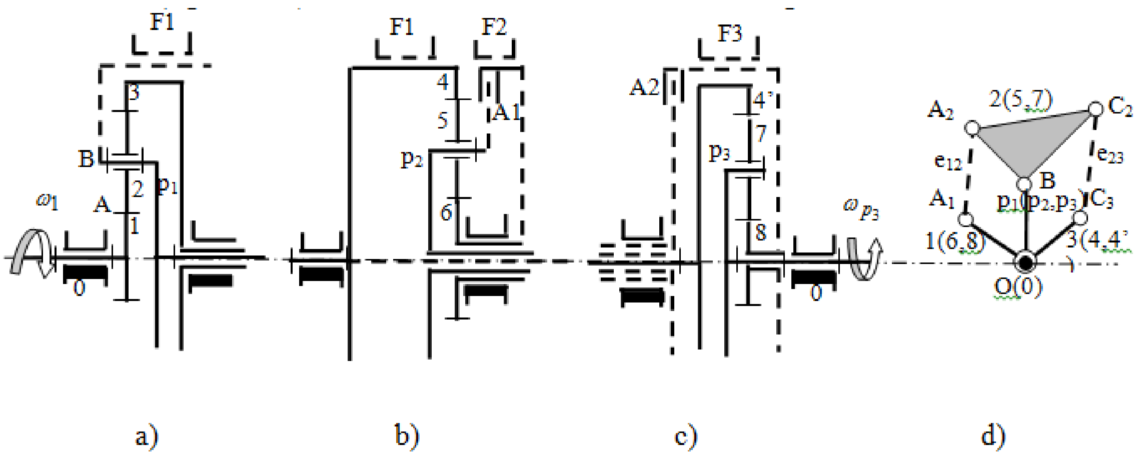


Fig. 2. The cylindrical planetary units (CPU) and topological schema

The mobility of the CPU (fig. 1), in the neutral position, is calculated with formula:

$$M_3 = 3.n - 2C_5 - C_4 \tag{1}$$

In the formula (1) are introduced the numerical values for the number n of the kinematic elements, the number C_5 are the rotation joints and the number C_4 are the rotation-translation joints: $n = 9; C_5 = 9; C_4 = 6.$ With these data results the numerical value of the geometrical mobility from formula (1):

$$M_3 = 3\times 9 - 2\times 9 - 6 = 3 \tag{2}$$

First from these three potential mobility of the mechanism (fig. 1) is the revolute motion of the driving shaft (input shaft) on which is fixed the central gear 1. Those two potential mobility can be the rotation motions of others two the kinematic elements, as the gears 3(p_2), 4(p_1), 6, 8.

It is underlined that all fixed axes dovetails, the fixed element being represented of this fixed axis, which is common of all the central gears and of the port-satellite arms.Because is a single driving element 1, for to transmission the motion univocal determined from the driving shaft (1) to the driven shaft (p_3), others two mobility must canceled.

The cancel those two mobility, which not correspond with the driving element, is achieved with help some couplings type clutches (A1 and A2) respectively disc brakes (F1, F2 and F3).

It is notated that a clutch solider two the mobile elements, which will turn as au single body, while a disc brake solider a mobile element with the fixed element, what involves the blocking it and the cancel of the respectively motion.

The structural-topological schema is achieved in the transversal plane through the equivalence joints of rotation - translation (engagements) through all a element type bar and two the revolute joint (articulation).

We begin from the fixed element which is reduced to a fixed point in which meet more mobile elements articulated at the base, as the central gears and the port-satellite arms (fig. 2).

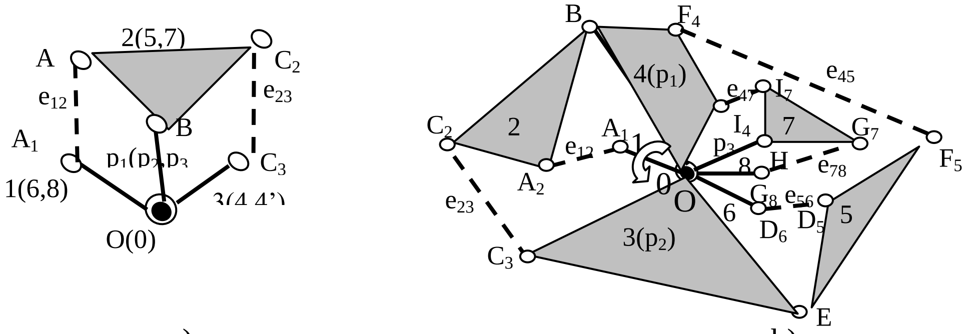
It is demonstrated that all those three cylindrical planetary units (fig. 2a, b, c) have same the structural-topological schema of type simple (fig. 2d) with two mobility:

$$M_3 = 3.n - 2C_5 - C_4 = 3\times 6 - 2\times 8 = 2 \tag{3}$$

In reality the port-satellite arm p_1 is stiffened of the gears 4 and $4^{/}$, that is $p_1 \equiv 4 \equiv 4'$, while the gear 3 is stiffened with the port-satellite arm p_2, that is $3 \equiv p_2$. Thus are introduced three structural conditions. The mobility of the complex mechanism can be calculated and with the formula of connection of mechanisms [4]:

$$M = M_{(1)} + M_{(2)} + M_{(3)} - M_l = 2 + 2 + 2 - 3 = 3 \tag{4}$$

where $M_{(1)} = M_{(2)} = M_{(3)} = 2; \quad M_l = 3$.



a) b)

Fig. 3. Structural-topological schemas in neutral position

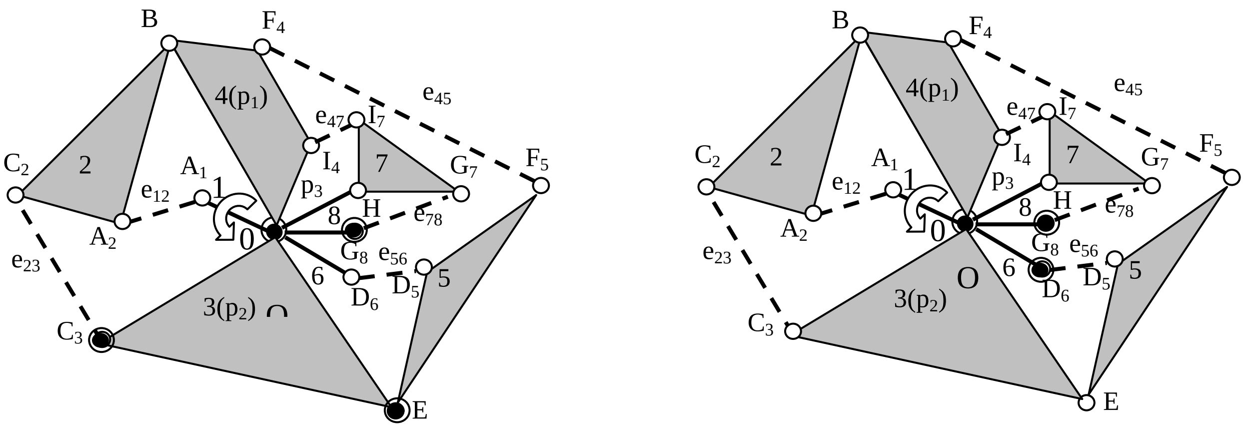
Whether the three conditions of coupling and blocking of some kinematic elements are introduced, by overlapping of the three structural schemas (fig. 2a, b and c $\Rightarrow$ fig. 2d) corresponding with the three planetary units CPU1 (fig. 2a), CPU2 (fig. 2b) and CPU3 (fig. 2c), it can be obtained the complex structural-topological schema (fig. 3b) of whole complex planetary mechanism (fig. 1) in the neutral position, in which are identified those three identical structural schemas (fig. 2d) which were reconsidered in fig. 3a.

4. Kinematics of complex planetary mechanism

4.1. Distribution of linear velocities in first gear

In the gear I, conform of the table 1, are activated the clutch A2 and disc-blocking device F3 (fig. 1a), that introduce the kinematic conditions which are mentioned in table: $\omega_3 = \omega_8 = 0$. This means that the elements 3 and 8 are coupled by A2 and blocked by F3.

Through introducing those two kinematic conditions, the kinematic elements 3(p_1) and 8 are stiffened and blocked to base 0. From the structural – topological schema in neutral position of mechanism (fig. 1) obtained the structural schema in gear I (fig. 4a) with 4 fixed articulations (O, C_3, E, H).



a) b)

Fig. 4. Structural-topological schema in first and second gear

The structural-topological formula of the mono-mobile mechanism, at which element 1 is driving (fig. 4a) written thus:

$$MM_I = FM(0+1) + TKC_1(e_{12} + 2 + e_{23} + p_1) + TKC_2(e_{4'7} + 7 + e_{78} + p_3) +$$
$$+ DKC_1(e_{45} + 5) + DLc_2(e_{56} + 6) \quad (5)$$

By the **F**undamental **M**echanism $\mathrm{FM}(0+1)$ is identified [6, 7] two **T**riadic **K**inematic **C**hain (TKC) respectively chains $TKC_1(e_{12} + 2 + e_{23} + p_1)$, $TKC_2(e_{4'7} + 7 + e_{78} + p_3)$ and others **D**yadic **K**inematic **C**hain (DKC) as $DKC_1(e_{45} + 5)$ and $DKC_2(e_{56} + 6)$. Is observed that those two dyadic chains are not on power flow from the driving element (1) until the driven element (p_3).

That is who drawn the velocities distribution only for the principal flow in which entered CPU1 through the characteristic points A, B, C and CPU3 through the characteristic points G, H, I (fig. 6a).

Because the gears 3 and 8 are blocked, the points C and G are fixed, while the velocity of each is zero, as instantaneous rotation center of the satellite gears 3 respectively 7.

For ears 1, 2 and 3 (CPU1) and gears $4^{/}$, 7 and 8 (CPU3) calculated the radii of pitch circles with formulas:

$$r_1 = r_8 = \tfrac{1}{2} m \cdot z_{1(8)}; \quad r_2 = r_7 = \tfrac{1}{2} m \cdot z_{2(7)}; \quad r_3 = r_{4'} = \tfrac{1}{2} m \cdot z_{3(4')}. \tag{6}$$

For numerical values: $z_{1(8)} = 18;\ z_{3(4')} = 50;\ z_{2(7)} = 16;\ m = 2mm$

results

$$r_{1(8)} = 18mm;\ r_{2(7)} = 16mm;\ r_{3(4')} = 50mm\,.$$

On the reference line Δ_I (fig. 6b) is measured at scale following segments:

$$\mathrm{OA} = \mathrm{OG} = 18\mathrm{mm};\ \mathrm{AB} = \mathrm{BC} = \mathrm{GH} = \mathrm{HI} = 16\mathrm{mm}.$$

4.2. Distribution of linear velocities in second gear

In gear II the disc locking devises F2 and F3 blocked gears 6 and 8, thus points D_6 and G_8 are fixed, while

the structural-topological schema (fig. 4b) permitted the writing of the component equations:

$$\begin{aligned} MM_{II} = FM(0+1) + HKC(e_{12} + 2 + e_{23} + 3 + 5 + e_{56} + e_{54} + 4) + \\ + TKC(e_{4'7} + 7 + e_{78} + p_3) \end{aligned} \tag{7}$$

Now all mobile kinematic elements are in the power flow on way from the driving shaft (central gear 1) to the driven shaft (port-satellite arm p_3).

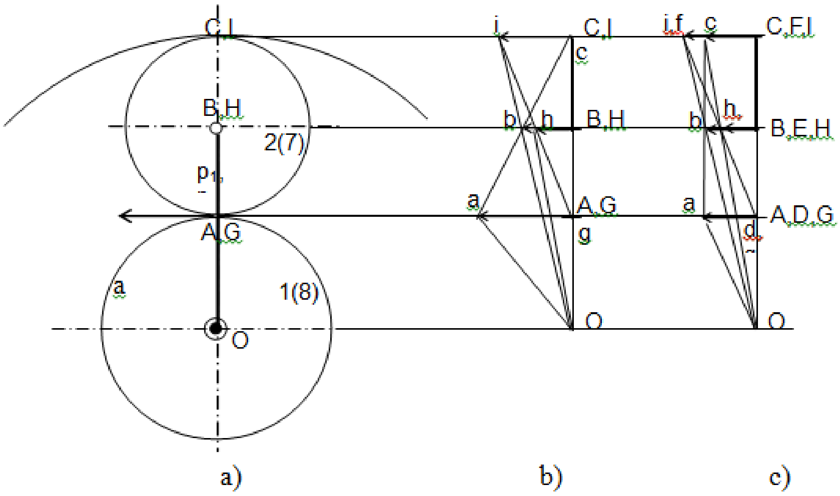


Fig. 5. Velocity distribution in the first and second gear

In structural-topological schema identified conform with equation (7) a first **H**exagonal **K**inematic **C**hain (HKC) with 8 elements $HKC(e_{12}+2+e_{23}+3+5+e_{56}+e_{54}+4)$ and a second **T**etradic **K**inematic **C**hain (TtKC) with 4 elements $TtKC(e_{4'7}+7+e_{78}+p_3)$, which contained the output element p_3.

In this case, the velocity distribution can be obtained through the method of changing of driving element, thus so that the structural schema to contain at most TKC. Thus, whether started from element p_3 as being driving (fig. 5b), formula of structural-topological composing becomes:

$$MM_{II}^{*} = FM(0+p_3)+DKC_1(7+e_{78})+DKC_2(e_{47}+4)+ \\ +TKC(e_{45}+5+e_{56}+3)+DKC_3(e_{23}+3)+DKC_4(e_{12}+1) \qquad (8)$$

The velocities graphical distribution shown in fig. 5c, where it is started from point H, in which was chooses vector $\overrightarrow{Hh}$ orientated to left. In the final of graphical drawing is resulted vector $\overrightarrow{Aa}$ orientated to left, what verified that the transmission ratio in gear II is positive.

5. Conclusions and specific transmission functions for gears

On the base the imposed kinematic conditions from each gear (table 1) we follow the power flow way (fig. 1) through each from CPU.

It is written the equation of transmission ratio between the central gears, in the hypothesis of immobilizing of the port-satellite arm (Willis's method):

$$\text{CPU1: } i_{13}^{p_1} = i_{01} = \frac{\omega_1-\omega_{p_1}}{\omega_3-\omega_{p_1}}; \ \text{CPU2: } i_{46}^{p_2} = \frac{1}{i_{02}} = \frac{\omega_4-\omega_{p_2}}{\omega_6-\omega_{p_2}}; \ \text{CPU3: } i_{4'8}^{p_3} = \frac{1}{i_{03}} = \frac{\omega_{4'}-\omega_{p_3}}{\omega_8-\omega_{p_3}}. \qquad (8)$$

in which the transmission ratios in hypothesis of fixed axes are expressed in function of teeth numbers of the corresponding central gears:

$$i_{01} = i_{13}^{p_1} = i_{12}^{p_1}\cdot i_{23}^{p_1} = \left(-\frac{z_2}{z_1}\right)\cdot\left(\frac{z_3}{z_2}\right) = -\frac{z_3}{z_1}; i_{02} = -\frac{z_4}{z_6}; i_{03} = -\frac{z_{4'}}{z_8}. \qquad (9)$$

The transmission ratio i_{1p_3} in each gear expressed the ratio between the angular velocities of the driving shaft (*1*) and the driven shaft (*p_3=9*) as function of teeth numbers *z*:

$$i_{1p_3(9)} = \frac{\omega_1}{\omega_{p_3}} = \frac{\omega_1}{\omega_9} = f(z) \qquad (10)$$

In gear *I* ($\omega_3 = \omega_8 = 0$) the formulas (8) written in this case:

$$i_{13}^{p_1} = i_{01} = \frac{\omega_1 - \omega_{p_1}}{-\omega_{p_1}} = 1 - \frac{\omega_1}{\omega_4};\ i_{46}^{p_2} = \frac{1}{i_{02}} = \frac{\omega_4}{\omega_6};\ i_{4'8}^{p_3} = \frac{1}{i_{03}} = \frac{\omega_{4'} - \omega_{p_3}}{-\omega_{p_3}} = 1 - \frac{\omega_{4'}}{\omega_9}. \qquad (11, 12, 13)$$

In formula (11) is made in evident the transmission ratio input – output (10):

$$i_{13}^{p_1} = i_{01} = 1 - \frac{\omega_1}{\omega_4} = 1 - \frac{\frac{\omega_1}{\omega_9}}{\frac{\omega_4}{\omega_9}} = 1 - \frac{i_{19}}{i_{49}}; \qquad (14)$$

from which deduced, taking account of formula (13):

$$i_{19} = (1 - i_{01}) \cdot i_{49} = (1 - i_{01}) \cdot (1 - \frac{1}{i_{03}});\ i_I = \left(1 + \frac{z_3}{z_1}\right) \cdot \left(1 + \frac{z_8}{z_{4'}}\right) \qquad (15, 16)$$

In gear *II* ($\omega_6 = 0; \omega_8 = 0$) formulas (8) written thus:

$$i_{01} = \frac{\omega_1 - \omega_4}{\omega_3 - \omega_4};\ \frac{1}{i_{02}} = \frac{\omega_4 - \omega_3}{-\omega_3} = 1 - \frac{\omega_4}{\omega_3};\ \frac{1}{i_{03}} = \frac{\omega_{4'} - \omega_9}{-\omega_9} = 1 - \frac{\omega_{4'}}{\omega_9}. \qquad (17)$$

From formulas (17) expressed the ratio input – output (10)

$$i_{01} = \frac{\omega_1 - \omega_4}{\omega_3 - \omega_4} = \frac{\frac{\omega_1}{\omega_9} - \frac{\omega_4}{\omega_9}}{\frac{\omega_3}{\omega_9} - \frac{\omega_4}{\omega_9}};\ \frac{1}{i_{02}} = 1 - \frac{\omega_4}{\omega_3} = 1 - \frac{\frac{\omega_4}{\omega_9}}{\frac{\omega_3}{\omega_9}};\ \frac{1}{i_{03}} = 1 - \frac{\omega_{4'}}{\omega_9} = 1 - \frac{\omega_4}{\omega_9} \qquad (18)$$

From formulas (18) deduced function (10) in the implicit and explicit form

$$i_{II} = \frac{\omega_1}{\omega_9} = 1 - i_{03} - i_{01} \cdot \frac{1 + i_{03}}{1 - i_{02}}; \qquad i_{II} = 1 + \frac{z_8}{z_{4'}} + \frac{z_3}{z_1} \cdot \frac{1 - \frac{z_8}{z_{4'}}}{1 + \frac{z_4}{z_6}} \qquad (19, 20)$$

In gear III ($\omega_1 = \omega_6; \omega_8 = 0$) transmission ratio is: $i_{III} = 1 - \frac{1}{i_{03}} = 1 + \frac{z_8}{z_{4'}}$ (21)

Gear IV ($\omega_3 = \omega_6 = \omega_8$) corresponded of direct transmission, without reduce: $i_{IV} = 1$ (22)

Gear V ($\omega_4 = 0; \omega_6 = \omega_8$) represented the returning (R), what corresponded the negative sign of transmission function:

$$i_V = i_{01}(1 - i_{03}) = -\frac{z_3}{z_1} \cdot \left(1 + \frac{z_{4'}}{z_8}\right). \qquad (23)$$

In the case of the *kinematic synthesis* of complex planetary mechanism (fig. 1) imposed the numerical values of the transmission ratios in gears I, II and V, while from the implicit formulas (15, 19, 23) calculated the transmission ratios i_{01}, i_{02}, i_{03} specific each the cylindrical planetary unit CPU1, CPU2 and CPU3.

From equations (19), (15) and (23) deduced formulas for the calculation of specific ratios for CPU1, CPU2 and CPU3:

$$i_{01} = \frac{i_I - 1}{i_I - i_V} . i_V ; \qquad i_{02} = 1 - \frac{i_{01}(1 + i_{03})}{1 - i_{03} - i_{II}} ; \; i_{03} = 1 - \frac{i_V}{i_{01}} \qquad (24, 25, 26)$$

Imposing the numerical values of the transmission ratios [2] $i_I = 4{,}75; i_{II} = 2{,}5; i_V = -6{,}85$ is obtained the transmission ratios, which are specific of each cylindrical planetary unit and transmission ratio in gear III:

$$i_{01} = -2{,}21; \; i_{02} = -3{,}05; \; i_{03} = -2{,}1; \; i_{III} = 1{,}48.$$

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