PROBLEM OF LOCUS RESOLVED BY THE THEORY OF MECHANISMS

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Abstract: *A problem of locus is transformed into a R-PRP-RPP type mechanism whose synthesis is made. With an initial set of data, is obtained the mechanism in successive positions and the geometric locus, ie two opposite parabola curves. Further on, constant angles and some lengths are changed, resulting in a wide range of curves as loci, but with different dimensions and positions relative to the axis system.*

Keywords: locus, synthesis of mechanisms, PRP dyad, RPP dyad

1. Introduction

In [1] we analyze in detail how many problems of loci can be resolved by the Theory of Mechanisms. The geometric and kinematic synthesis of the mechanisms fulfilling the geometric conditions required by the loci problems is made, then the movement of the mechanisms is modeled, resulting in different curves representing the loci sought. The loci for the intersection points of mediators, heights, medians, bisectors of the triangles are analyzed. The locus of the intersection point of the diagonals of a quadrilateral is also analyzed.

The problem is dealt with for other geometric loci in [2]; in the case of a quadrilateral there are loci of intersection points of heights, medians, mediators, bisectors. Similarly, the loci of the intersections of different lines for other mechanisms are studied.

Geometry textbooks deal, some more detailed, others more brief, with many loci problems. For example, in [3] there are many examples of loci problems, indicating how to solve them.

In [4] are presented many problems of loci, these being in the end circles or straight.

In [5] Robert Ferréol, Jacques Mandonnet and Alain Esculier present many cases of geometry with curves obtained as loci, for some being given animations too.

Numerous examples of loci are given in [6], resolved by analytical or graphical methods.

Below, it starts from a question of locus and comes to a mechanism, tracing loci that change when changing some lengths or angles of the mechanism.

2. The locus problem

A fixed line, AC (fig. 1) has a fixed point A, around it rotating the stright line AB with variable length. The line BC slides with point B on AB and with C on AC, so the angle ABC = λ to be constant. The line EF can rotate in E, and its F point slips to the line BC. It is asked the locus of point F.

3. Synthesis of the mechanism

The abscissa is adopted in the AC direction, and in points A and E rotation joints are set. The point B slips on AB, so in B we provide a slider that is welded to the BC element to ensure λ constant. Point C is bound to move on the AC, which means that a slider is set on AC in C, but BC rotates relative to C, which means that a rotational joint in C is also required. The length EF is variable so that it requires in F to mount a slider in the direction EF, but F moves on the BC direction as well, so a slider in this direction is necessary, both being welded in the point C. In this way it was obtained the mechanism in figure 2.

The structure of the mechanism consists of the driving element AB of type R, the dyad BC type PRP and the dyad EF of type RPP.

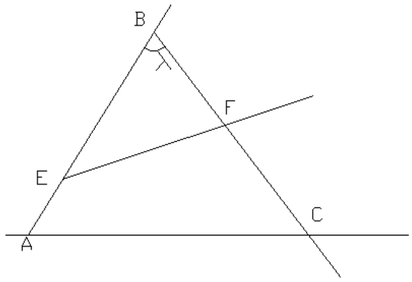


Fig. 1. The initial problem

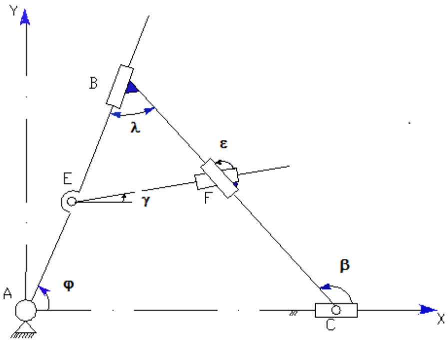


Fig. 2. The mechanism obtained

4. Relationships and Outcomes

Based on fig. 2 the following relationships are written:

$$\mathrm{AB}\ \cos\varphi = x_C + BC\cos\beta \tag{1}$$
$$\mathrm{AB}\ \sin\varphi = BC\sin\beta \tag{2}$$
$$\beta = \varphi + \lambda \tag{3}$$
$$\gamma = \beta - \varepsilon \tag{4}$$
$$x_E = AE\cos\varphi\,;\ y_E = AE\sin\varphi \tag{5}$$
$$x_F = x_E + EF\cos\gamma = x_C + CF\cos\beta \tag{6}$$
$$y_F = y_E + EF\sin\gamma = CF\sin\beta \tag{7}$$

From (2) it results AB, and from (1), x_C. Due to the welds on the kinematic scheme, the angles are correlated by equations (3) and (4). From (5) we get the position of E, and from (6) and (7) we obtain the position of F, the point that generates the locus sought.

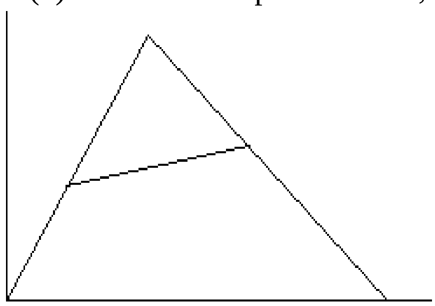


Fig. 3. The mechanism in one position

The following were taken as initial data: AE = 28; BC = 77; λ = 70; ε= 120 (lengths in millimeters and angles in degrees, as below). In fig. 3 it is presented the mechanism obtained for one position, and in fig. 4 are shown the successive positions of the mechanism; a detail in the central area of fig. 4 is shown in fig. 5.

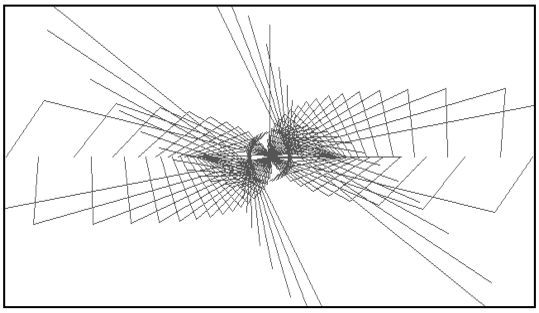


Fig. 4. Successive positions

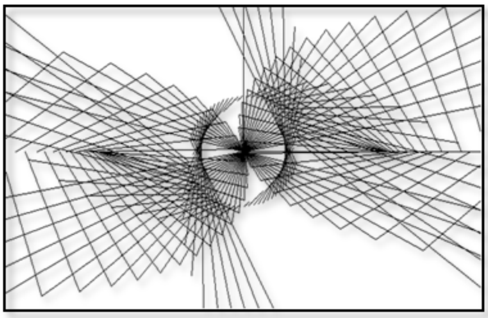


Fig. 5.The center detail of fig. 4

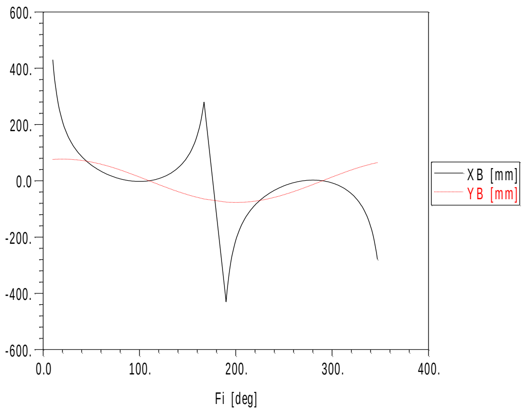


Fig. 6. Curves for coordinates of B

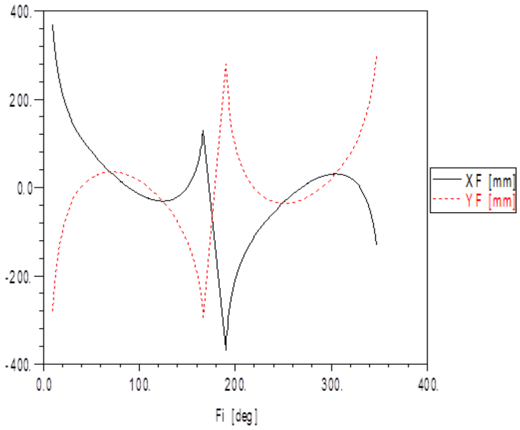


Fig. 7. Curves for point F

From fig.6 it is noted that in the area $\varphi = 180^0$ there is a jump on the curve of x_B caused by the slider in B, knowing that at PRP dyad occurs undeterminations or jumps to infinity. Similarly, in fig.7 is given the variation of the coordinates of F, with instability in the same position as above. For the diagram to be read, the values of y_F greater than 300 mm were avoided , otherwise some curves tended to infinity. The curves for AB and x_C were also obtained (fig. 8), where discontinuities occur for $\varphi = 180^0$. In fig. 9 it is shown the locus we are looking for (for the initial data). There are two parabola curves that intersect at the peaks.

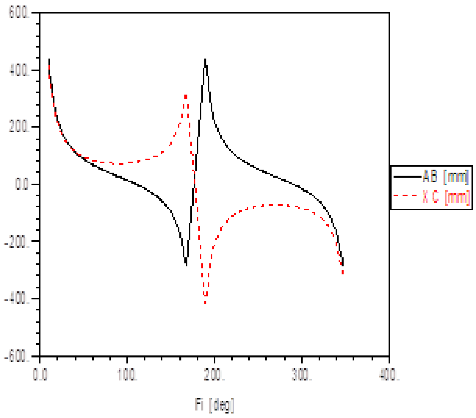


Fig. 8. Curves for AB and x_C

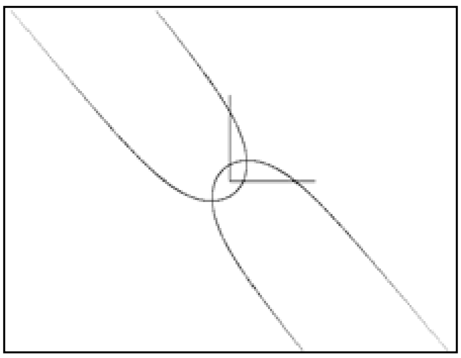


Fig. 9. The locus for initial data

5. Some other geometric locus for different data

Next, the original data was maintained, changing only the angle λ .
For λ= 0 a circle is obtained as a locus (fig. 10). Next, the resulting loci for different values of λ are given . So, for $\lambda = 10^0$ a circle with two tangents is obtained (figure 11).

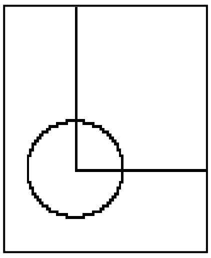


Fig. 10. The locus for λ = 0

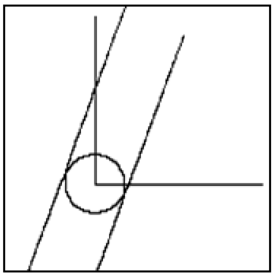


Fig. 11. The locus for λ = 10

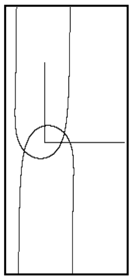


Fig. 12. λ = 30

Further on, similar curves to those in fig. 9, but positioned differently and with other sizes, are obtained.

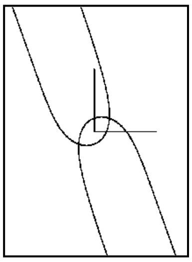


Fig. 13. λ = 50

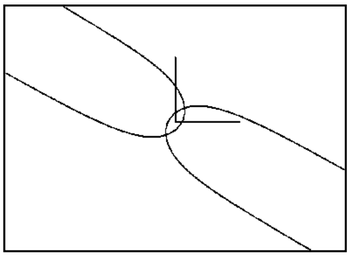


Fig. 14. λ = 90

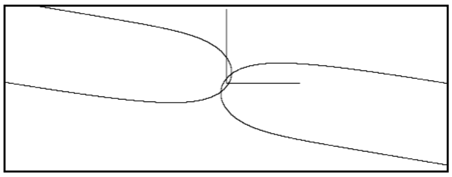


Fig. 15. λ = 110

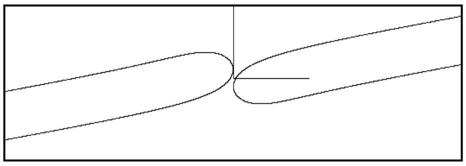


Fig. 16. λ = 130
Here, the curves no longer intersect.

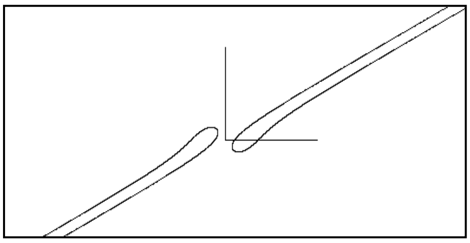


Fig. 17. λ = 150
Here, the curves are elongated, with deformations at the ends.

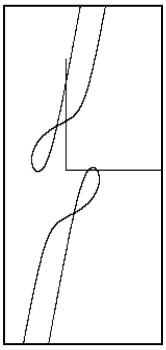


Fig. 18. λ = 200

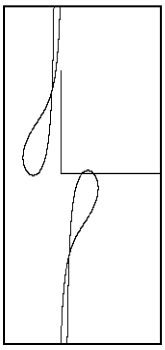


Fig. 19. λ = 210
These curves do not touch, and have loops.

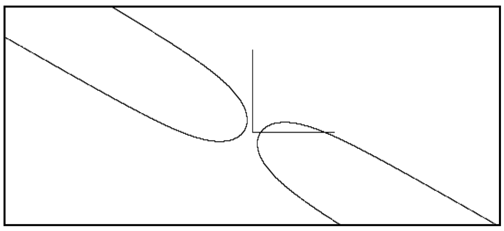
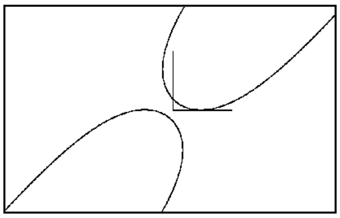
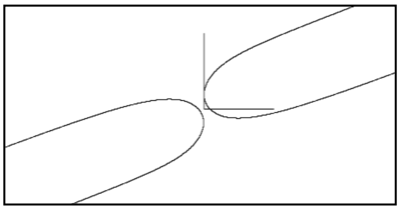


Fig. 20. λ = 270
In this case the curves are similar to the case λ = 90, but here they are narrower.

Next, the angle ε between the sliders in F, was looped.





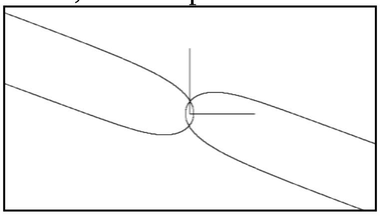
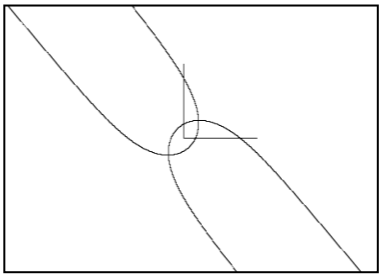


Fig. 21. ε = 20 Fig. 22. ε = 50 Fig. 23. ε= 90 and 270



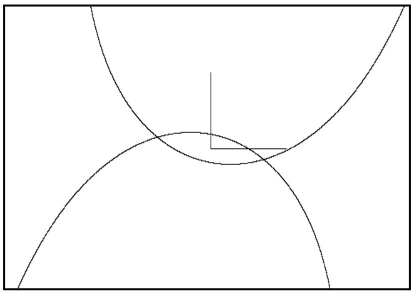
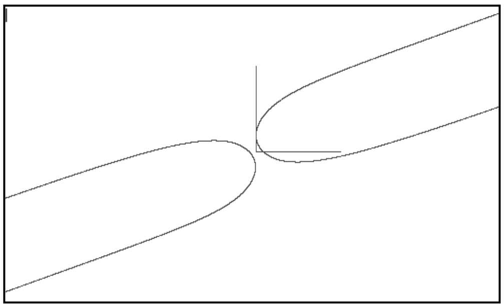


Fig. 24. ε = 120 Fig. 25. ε = 170



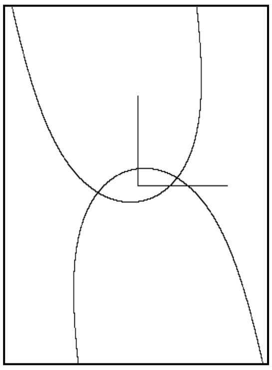
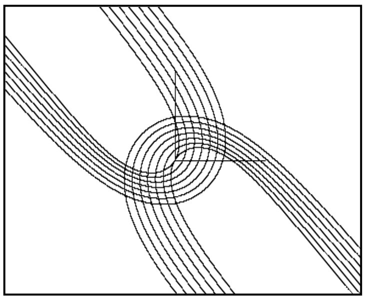
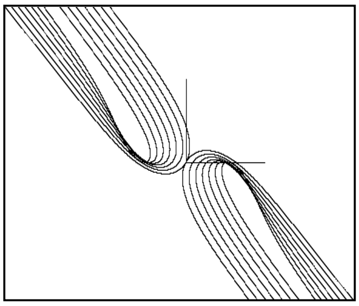


Fig. 26. ε = 230 Fig. 27. ε = 330

The curves are similar to those of λ variation, but with other dimensions and other positions.
Next, geometric loci were plotted on the same figure, for AE variation from 0 to 100 with step 15, resulting in fig. 28 the curves evolving from the ones inside to the outside.





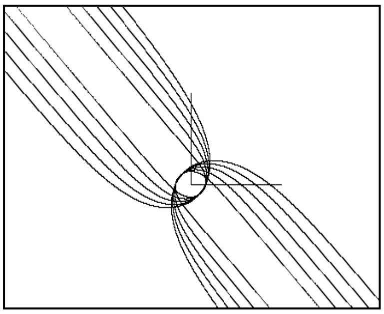


Fig. 28. AE = 0 ... 10 Fig. 29. AE = - 10 ... - 100 step - 1 Fig. 30. BC = 10 ... 100 step 15

It was taken in consideration the variation of AE for negative values, from -10 to -100 with step -15, resulting the curves in fig. 29 evolving also from the ones inside to the outside

The curves are similar to the ones before , but are positioned parallel and with variable shape at the ends.

Further on the length BC was also looped, resulting the curves in fig. 30. In this case, the first curve is the one inside, as well .

5. Conclusions

It started from a locus problem and it was made the synthesis of the equivalent mechanism, type R-PRP-RPP, which allows tracing the locus. For a set of initial data, there were plotted the mechanism obtained in a position and its successive positions. The curve representing the searched locus was obtained. Next, two angles and two lengths considered fixed were successively looped, resulting in a large number of curves, similar to parables with the close ends. Curves are similar in all cases, differing in size, position, and loop at ends.

References

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