PHENOMENON AND COMPLEX CALCULATION OF THE SLOTTED SPRING

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Abstract: *The areas of use of a slotted spring are given in the work by examples of applications from the point of view of modernity and the promising development of technological structures. With the advent of the design of a slotted spring in the middle of the last century, its calculations were usually limited to calculation for compression-stretching. Torsion and bending in complex loads were neglected, stipulating this in the calculation assumptions. In connection with modern high-precision applications of a slotted spring in aerospace engineering and nuclear power, more rigorous calculation approaches are needed. For this purpose, the analytical dependences of the calculation of a slotted spring on the torque are derived. The accepted design scheme uses the generally known provisions of the theory of resistance of materials and this is illustrated in detail by author's sketches. As in the traditional approaches to calculating a slotted compression-tension spring, the outer rings are considered as separate semi-rings with fixed ends. The contribution of the associated bending of the rings and the displacement of the bridges are taken into account in calculating the total torsion angle in the proposed solution of the stress-strain state of a slotted spring under the torque load.*
Keywords: slotted spring, torsion, calculation, high-precision application.

1. Introduction

Technical and technological relevance, modern workability and prospects of further design and calculated developments of slotted springs are shown in modern dilogy [1, 2] devoted to the history and prospects of their development in machine-building applications technologies including with respect to innovative improvement of cutting machinery and machining.

However, the greatest interest in the development and calculation of innovative designs of slotted springs in their entire history is observed in the aerospace industry and in the military-industrial segment of the real economy, as well as in the properly oriented intellectual support environment of the higher technical school.

This statement is confirmed by a list of institutions of work of the authors of most research on scientific and technical problems and applied applicability of a slotted spring. As follows from the review [1], the Moscow Higher Technical School (now the Moscow State Technical University or MSTU) named after N. E. Bauman, which is widely known as the intellectual and educational center of Soviet and Russian astronautics, takes a leading position among them. The authors and editors of the fundamental works on a slotted spring from the scientific school of applied mechanics led by S. D. Ponomarev [3, 4] received a higher technical education at this university and (or) subsequently worked there. The names of prominent developers and organizers of space research, adherents and popularizers of a slotted spring are included in the scrolls of professors of this university: I. A. Birger (Central Institute of Aviation Motors named after P. I. Baranov or CIAM, deputy chief) [5, 6] and V. N. Chelomei (Design Bureau 52 - Scientific and Production Association of Mechanical

Engineering, Director and General Designer of Missile, Artillery and Aerospace Engineering) [7].

The slotted springs belong to the class of rigid springs [8-10]. They are used in those cases where the radial dimensions should be small, and the bearing capacity should be large [10]. According to a source from the scientific school of applied mechanics of MSTU [11], a slotted spring was the brainchild of technical challenges of the period of the Second World War. Today, slotted springs work effectively in various critical devices of general and special instrument engineering and machine building, which are operated in oil and gas exploration drilling machines, production of construction materials, robotics and other applications [2].

Potential cognitive, creative and applied inexhaustibility of the technical ideology of the slotted spring is demonstrated by the wide-functional modern invention [12] based on more than a half-century prototype [13], in essence a new class of slotted springs as rigid elastic elements of increased compliance, born in the walls of the All-Russian Scientific Research Institute of Nuclear Physics and providing progressive transformation of the design scheme from two support pinching of the curved beam to consideration of the cantilever structure.

In modern finite-element studies of a slotted spring [14] numerical calculations draw attention to the inadequacy of the existing analytical apparatus to describing its behavior under load for high-precision applications. Analyzers of tensile-compression of a slotted spring indicate to involvement of torque in its causal relationship as the type of the load tested, and the torsional stresses themselves, as well as flexural and shear stresses [3, 11, 10]. The proposed study attempts to somewhat extend the analytical possibilities of investigating a slotted spring in this direction in order to contribute to an adequate prediction of its behavior in high-precision applications at the design development stage.

In the alphabetic identification of the proposed analytical analysis of the stress-strain state of a slotted spring under the action of torque uses both Latin and Cyrillic, it is unified with basic work on the tensile-compression of this spring performed by the researchers from the scientific school of applied mechanics of the MGTU and CIAM [3-6].

2. Analytical research

The general view of the sections and the scheme of the considered torsion of the slotted spring when the torque M_k is loaded are shown in Fig. 1.

The extreme rings (1 and n) are connected by rigid bridges to the rigid support part, therefore the outer rings are considered as separate semirings with clamped ends.

The middle rings (2, 3, ..., i, i+1, ..., n–1) are connected by rigid bridges to springing rings.

Since the bridges are rigid to bend, the angle of rotation of the cross-section along the bridges for adjacent rings will be the same, i.e. the compatibility equation for deformation has the following form:

$$\theta_i = \theta_{i+1}\,, \qquad (1)$$

where θ_i and θ_{i+1} are the angles of rotation of the cross-section along the bridge, respectively, for the rings i and i +1.

The individual rings of slotted spring are subjected to bending when torsion.

As shown at the Fig. 1, the distance h between the longitudinal axes of adjacent rings is equal to:

$$h = h_1 + h_2, \qquad (2)$$

where h_1 is the height of bridges, m; h_2 is the height of the cross-section of the rings, m.

The bending moment M is determined by the formula:

$$M_u = (M_k \cdot h)/2R, \tag{3}$$

where R is the average radius of the ring, m; according to the Fig. 1:

$$R = (d - \delta)/2. \tag{4}$$

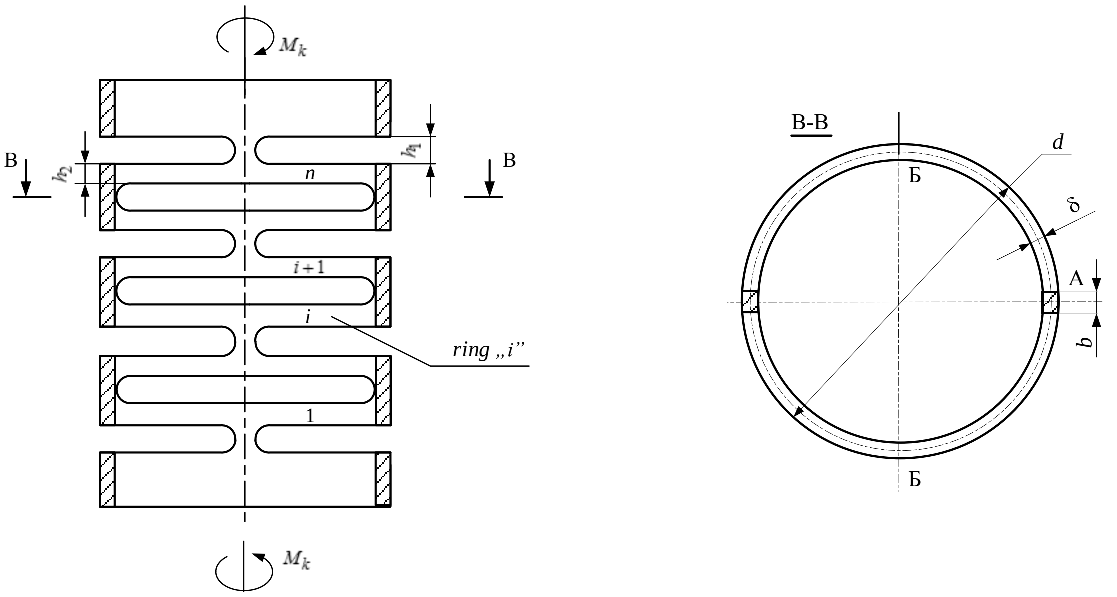

***Fig.* 1**. *General view of sections and the torsion scheme of a slotted spring*

The bending moment M_u is distributed between adjacent rings according to their flexural rigidity:

$$M_u = \sum_{i=1}^{i=n} M_i, \tag{5}$$

where i = 1, …, n; n is the number of rings.

A generalized schematization of the loading of an arbitrary ring of a slotted spring according to the representation of its cross-section in Fig. 1 is shown in Fig. 2.

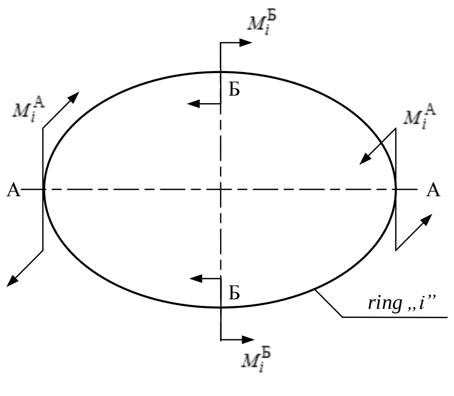

Fig. 2. *Generalized schematization of the loading of an arbitrary ring of a slotted spring*

A generalized schematization of the loading of an arbitrary ring of a slotted spring according to the representation of its cross-section in Fig. 1 is shown in Fig. 2.

Since the slotted springs belong to the class of rigid springs, the principle of the independence of the action of forces can be adopted [15]. The design schemes for the intermediate and outer rings are respectively shown at the Fig. 3 and Fig. 4.

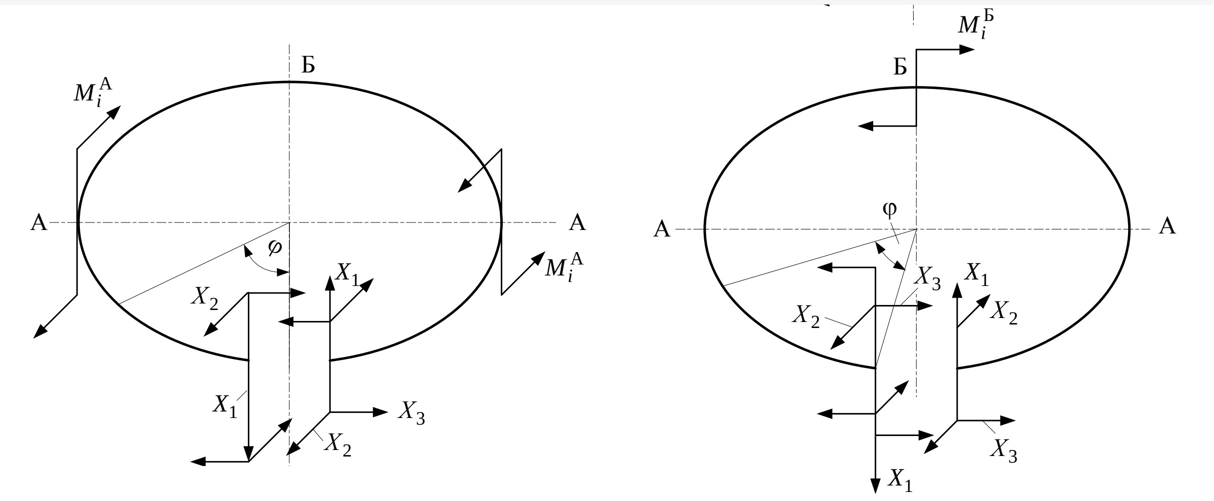

***Fig.* 3**. *Design schemes for the intermediate rings*

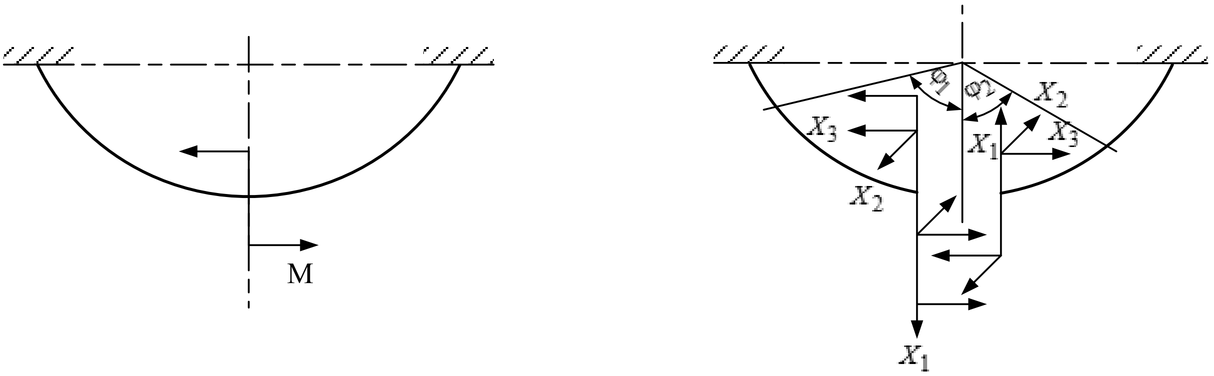

***Fig.* 4**. *Design schemes for the outer rings*

System of deformation compatibility equations:

$$\theta_1^А = \theta_2^А, \theta_2^Б = \theta_3^Б, \ldots, \theta_i^А = \theta_{i+1}^А, \theta_{i+1}^Б = \theta_{i+2}^Б, \ldots, \theta_{n-2}^А = \theta_{n-1}^А, \theta_{n-1}^Б = \theta_n^Б, \qquad (6)$$

where $\theta_i^А$ is the angle of rotation of the section *A* of the ring *i*, rad;

$$M_1^А + M_2^А = M,\ M_2^Б + M_3^Б = M, \ldots, M_i^А + M_{i+1}^А = M,\ M_{i+1}^Б + M_{i+2}^Б = M, \ldots, M_{n-2}^А + M_{n-1}^А = M,\ M_{n-1}^Б + M_n^Б = M. \qquad (7)$$

The canonical equations have the following form:

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{1M} = 0; \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{2M} = 0; \\ \delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{3M} = 0. \end{cases} \quad (8)$$

For intermediate rings after determining the coefficients and solving the system:
a) section A angle of rotation:

$$\theta_i^A = \frac{\pi M_i^A R}{8EI_x} + \frac{M_i^A R}{2C} - \frac{M_i^Б R}{4EI_x} + \frac{M_i^Б R}{2C}\left(-\frac{2}{\pi} + \frac{1}{2}\right); \quad (9)$$

b) section $Б$ angle of rotation:

$$\theta_i^Б = \frac{\pi M_i^Б R}{8EI_x} + \frac{M_i^Б R}{2C}\left(-\frac{2}{\pi} + \frac{\pi}{4}\right) - \frac{M_i^A R}{4EI_x} + \frac{M_i^A R}{2C}\left(-\frac{2}{\pi} + \frac{1}{2}\right). \quad (10)$$

The bending (M_u) and torque (M_k) moments on the cross sections of the intermediate rings in the segments φ (Fig. 3) are summarized in the Table 1.

***Table 1**: M_u and M_k in segments φ for the intermediate rings*

Segment φ	M_u	M_k
[0, π/2]	$-\frac{M_i^A}{2}sin\varphi + \frac{M_i^Б}{2}cos\varphi$	$\frac{M_i^A}{\pi} - \frac{M_i^A}{2}cos\varphi + \frac{M_i^Б}{\pi} - \frac{M_i^Б}{2}sin\varphi$
[π/2, π]	$\frac{M_i^A}{2}sin\varphi + \frac{M_i^Б}{2}cos\varphi$	$\frac{M_i^A}{\pi} + \frac{M_i^A}{2}cos\varphi + \frac{M_i^Б}{\pi} - \frac{M_i^Б}{2}sin\varphi$
[π, 3π/2]	$\frac{M_i^A}{2}sin\varphi - \frac{M_i^Б}{2}cos\varphi$	$\frac{M_i^A}{\pi} + \frac{M_i^A}{2}cos\varphi + \frac{M_i^Б}{\pi} + \frac{M_i^Б}{2}sin\varphi$
[3π/2, 2π]	$-\frac{M_i^A}{2}sin\varphi - \frac{M_i^Б}{2}cos\varphi$	$\frac{M_i^A}{\pi} - \frac{M_i^A}{2}cos\varphi + \frac{M_i^Б}{\pi} + \frac{M_i^Б}{2}sin\varphi$

We introduce [16]:

$$K_1 = \frac{\pi\left(\frac{1}{EI_x} + \frac{1}{C}\right)^2 - \left(\frac{\pi}{EI_x} + \frac{\pi-4}{C}\right)\left(\frac{1}{EI_x} - \frac{1}{C}\right)}{\pi\left(\frac{\pi}{EI_x} + \frac{3\pi-8}{C}\right)\left(\frac{1}{EI_x} + \frac{1}{C}\right) - \left(\frac{\pi}{EI_x} + \frac{\pi-4}{C}\right)^2} \quad (11)$$

and

$$K_2 = \frac{\left(\frac{1}{EI_x} + \frac{1}{C}\right)^2\left(\frac{\pi}{EI_x} + \frac{\pi-4}{C}\right) - \left(\frac{1}{EI_x} - \frac{1}{C}\right)\left(\frac{\pi}{EI_x} + \frac{3\pi-8}{C}\right)}{\left(\frac{\pi}{EI_x} + \frac{\pi-4}{C}\right)^2 - \left(\frac{1}{EI_x} + \frac{1}{C}\right)\left(\frac{\pi}{EI_x} + \frac{3\pi-8}{C}\right)^2}, \quad (12)$$

where E is the modulus of elasticity, Pa; I_x is the moment of inertia of the section of the rings relative to the central transverse axis, kg·m^2, $I_x = \delta \cdot h_2^2/12$; C is torsional rigidity, N/m, is determined in the system of empirical dependencies:

$$\begin{aligned} C &= \alpha \cdot C_\tau \cdot \delta^4, \text{ when } \delta \le \alpha, \\ C &= \alpha \cdot C_\tau \cdot h_2^4, \text{ when } \delta > h_2; \end{aligned} \quad (13)$$

where C_τ is the shear modulus, Pa; α is a dimensionless scale factor, is determined by tables [5, 17, 18] depending on the ratio h_2/δ of the sides of the cross-section of the rings as a rectangle.

Then the angle of rotation of the section *A* or *B* for the outer rings is written as:

$$\theta_1^A = \theta_n^Б = \frac{M_i^A R}{EI_x}\left(-\frac{K_1+K_2}{2}+\frac{\pi}{8}\right)+\frac{M_i^A R}{C}\left(-\frac{K_2+K_1}{2}+\frac{\pi}{8}\right). \quad (14)$$

The bending (M_u) and torque (M_k) moments on the cross sections of the outer rings in the segments φ (Fig. 4) are summarized in the Table 2.

***Table 2**: M_u and M_k in segments φ for the outer rings*

Segment φ_1, φ_2	M_u, M_k
$0 \le \varphi_1 \le \frac{\pi}{2}$	$M_u = -(K_1+K_2)\,M_i^A sin\varphi_1 + \frac{M_i^A}{2} + cos\varphi_1;$ $M_k = -(K_1+K_2)\,M_i^A cos\varphi_1 - \frac{M_i^A}{2}\,sin\varphi_1 + K_1 M_i^A$
$0 \le \varphi_2 \le \frac{\pi}{2}$	$M_u = (K_1+K_2)\,M_i^A sin\varphi_2 - \frac{M_i^A}{2}\,cos\varphi_2;$ $M_k = -(K_1+K_2)\,M_i^A cos\varphi_2 - \frac{M_i^A}{2}\,sin\varphi_2 + K_1 M_i^A$

Substituting the obtained analytical expressions (9) and (10) for θ_i^A and $\theta_i^Б$, and also (14) for θ_1^A and $\theta_n^Б$ into system (6), and solving it together with (7), we obtain M_i^A, $M_i^Б$, M_1^A, $M_n^Б$..

Then substituting the resulting numerical values of the moments into formulas (9), (10), (14), we obtain the numerical values of the angles of rotation of the bridges *A* and *B*.

Then the angle of torsion of the slotted spring from the bending of the rings is:

$$\varphi_{из} = (\theta_1^A + \theta_2^Б + \theta_3^A + \theta_4^Б + \cdots + \theta_i^A + \theta_{i+1}^Б + \cdots + \theta_{n-1}^A + \theta_n^Б)h \cdot \frac{1}{R}. \quad (15)$$

The bridges of the slotted spring undergo shear stress, which can be deformationally significant at relatively high altitudes.

Neglecting the curvilinearity of the lateral boundaries of the bridge, the relative shift of γ is determined by the well-known formula:

$$\gamma = Q/(F \cdot C_\tau), \quad (16)$$

where Q is the shear force, N, $Q = M_k/2R$; F is the shear area, m^2, $F = \delta \cdot h_1$.

The absolute shift Δ with the same neglect of the curvilinearness of the lateral boundaries of the bridge is defined as

$$\Delta = \gamma \cdot \delta. \quad (17)$$

Then the calculation formula for determining the twist angle $\varphi_{сд}$ of the slotted spring from the shift $(n + 1)$ pairs of bridges,

$$\varphi_{сд} = \Delta \cdot (n+1)/R, \quad (18)$$

taking into account (17), (16) and $Q = M_k /2R$, becomes:

$$\varphi_{\text{сд}} = M_k \cdot (n + 1)/(2R^2 \cdot h_1 \cdot C_\tau). \tag{19}$$

The pure torsion of annular sections of a slotted spring by a certain angle $\varphi_{\text{к}}$ is estimated in total by the well-known approach of estimating the state of such structures without taking into account the sections of the slits:

$$\varphi_{\text{кр}} = M_k \cdot (l_{on} + n \cdot h_2)/(C_\tau \cdot J_\rho), \tag{20}$$

where $l_{\text{оп}}$ is the total length of the support (end) sections, m; J_ρ is the polar moment of inertia of the annular section, m^4, $J_\rho \approx 0{,}1d^4[1 - (d - 2\delta)^4/ d^4]$.

The total torsion angle of the slotted spring from the moment M_k is the result of the summation of the calculations for (15), (19), (20):

$$\varphi_{Mk} = \varphi_{\text{из}} + \varphi_{\text{сд}} + \varphi_{\text{кр}}. \tag{21}$$

3. Conclusion

The performed research and the obtained results are proposed for use in in-depth design predictive calculations and studying the behavior of slotted springs for high-precision applications.

The relevance of a slotted spring in innovative technology in the era of the emergence of the modern, sixth technological order [19] first of all follows from the interest in the development and calculation of innovative designs of slotted springs in institutions of the state level for solving problems of nuclear physics and atomic engineering [13], as well as the obvious [01] not exhausting attention to it in the aerospace industry, the last century flagship in the development of scientific and technological progress of modern civilization.

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