CONSIDERATIONS ON COMPUTER ASSISTED WEIBULL FUNCTION FITTING TO EXPERIMENTAL RELIABILITY DATA

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Abstract *The mechanical reliability uses many statistical models to fit experimental data. Weibull distribution offer a flexible and practical instrument in this domain, but implies extended use of specialized applications, from simple freeware to complex, expensive software; the paper proposes an investigation of the capabilities of some useful software, applying the regression analysis of experimental data.*

Keywords: reliability data, Weibull software solution

1.Introduction

Reliability has emerged as an extension of quality and safety in operation. In the 1950s, with the accelerated development of electronics, a rigorous study of products behavior over time was required, complemented by related areas of maintainability and availability. The reliability theme exploded immediately in all engineering activities. Mechanical systems, especially subject to wear patterns, were initially covered by normal distribution, a well-known model through its wide spread. With the strong impact of electronics reliability, the exponential model was imposed, as model of the useful life of products. A possible overview of the methods developing in reliability engineering is presented in table 1 [1].

1945	1945-1950	1952	1953	1960s	1970s	1980s	1990s	2000s
V-1 missile is developed Quantitative measure for reliability is established "Weakest Link" reliability design concept is developed	U.S. Armed Forces Only 30% of the electronic devices are successful in missions Cost of maintenance and repair is 10 times the original cost Dec. 7, 1950 Ad hoc groups are established Reliability engineering for electronics begins	U.S. DOD AGREE (Advisory Group of the Reliability of Electronic Equipment) is formed System RMA is established to meet the government procurement needs RMA requirement specification is standardized	Exponential distribution is popular approach Infant mortalities are screened out to get almost constant failure rate Degradation was not an issue for electronic systems Analyses are simple	Exponential distribution declines in popularity because: It is not practical in many applications It is sensitive to departure from the initial assumptions It leads to high chance of accepting systems with poor mean time to failure Other statistical distributions are used to get more realistic hazard rate	Birth of fault tree analysis is motivated by nuclear safety considerations Monte Carlo methods for finding minimum cut sets are avoided Combinatorial algorithms are developed for analyzing very large fault trees Bayesian approach allows development of new state of knowledge form prior experience	Common cause failure analysis and modeling dependencies are implemented System-level reliability assessment is needed Bayesian statistics become more advanced Accelerated life testing is implemented and facts from real cause of failure is used in statistical models	Physics of failure approach is widespread Facts from root-cause failure processes are used to prevent the failure of the products Robust design approaches are implemented Better manufacturing practices are implemented	The age of hybrid physics-statistics approaches begins Simulations are performed instead or in addition to testing

Tab. 1:History of the methods in reliability engineering [1]

2. The Weibull Era

However, to accommodate a wider range of product performance, the need for a more complex function for reliability with 2-3 parameters has been introduced, compared to the unique lambda parameter of the exponential function, which remains a particular case, as well as the Rayleigh distribution.

In January 1951 Wollodi Weibull proposed a new distribution function (1) with the parameters x_u, x_0 and m [8]: with this he modelated yield strength of a Bofors steel and many others real processes including manufacturing [5, 6].

$$F(x) = 1 - e^{-\frac{(x-x_u)^m}{x_0}} \tag{1}$$

The Weibull distribution is the most frequently used model for time (or strength via pressure) to failure [4] and has a very important place between the reliability models [7] and many specialised books [2].

For the moment the Weibull function has two basic notations, due to the lack of standardization, and even her parameters can have different symbols upon many authors, with the general acceptant significations for β as shape and η as scale factor. Even if apparently this is a small difference, actually the data processing is modified and is important to fix the formula from the beginning.

$$F(t) = 1 - e^{-\frac{(t-\gamma)^{\beta}}{\eta}} \tag{2}$$

$$F(t) = 1 - e^{-(\frac{t-\gamma}{\eta})^{\beta}} \tag{3}$$

Usually it should consider γ =0, as it will be considerate further, and it eliminates the case of the earliest failures, before t=0.

The MTBF expression will vary, depending on the used expression (2 or 3): for a better understanding it will be use (3), with the statistical defining measures:

$$MTBF = \eta\Gamma(1 + \frac{1}{\beta}) \tag{4}$$

$$variance = \eta^2[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2] \tag{5}$$

In the industrial practice for the study of the reliability it useful to analyze the field data, respective times between failures [3].

Having this data, and possibly after a elimination of outliers, the question is to fitting the data to the Weibull model. Here the problem becomes complicated because, unlike the normal and exponential distributions, the Weibull parameters of the distribution function, β and η, cannot be calculated directly from the data. The original solution, in the '70s, '80s, and

'90s, was the graphical solution, namely the use of double logarithmic scales that arranges experimental data on a line, if the Weibull model is adequate (fig 1) [11].

To plot the observations on probability paper it is necessary to use an estimate of $F(x_i)$, where the variable x_i is usually time between failures (TBF_i), being the i-th order statistic in the sample. Several methods were proposed as estimates for $F(x_i)$, described and compared in [14], and frequently it should be used median or mean rank.

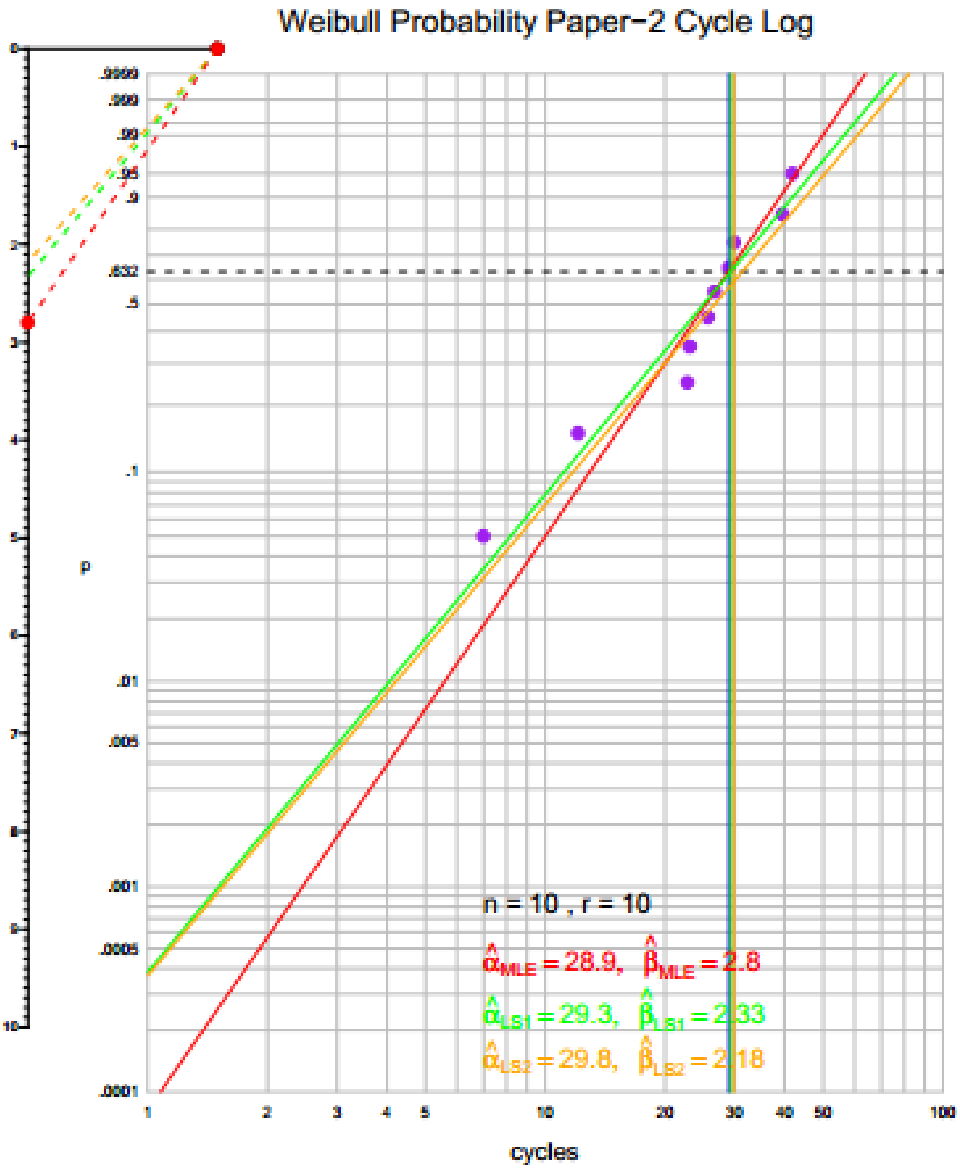

Fig. Example of Weibull probability paper [11]

Some professional statistical software solutions of the data processing companies such as Weibull, Reliasoft, Unistat, Minitab, OriginLab [12], etc. were developed, but assuming affordable costs only to businesses with intense activity in the field. The accelerated use of reliability studies imposed the development of new, cheaper software, like CurveExpert [10] or LAB Fit [13], started even as freeware [3].

The growing pressure of Linux applications has led to an increasingly fierce battle, forcing even the usual spreadsheet software (eg MSExcel) to explain computational models.

The impact of smart phones implied the transfer of Weibull application too: an example for iOS is [15].

A new trend is introduced by Open Source software, like the statistical environment and language R: with a few programming knowledge is possible to make the most complicated data processing for free [4, 9], applying maximum likelihood, moment or quantile matching estimations, even for censored data.

3.Practical approach

For a more complete explanation of the Weibull data fitting below will be explain a practical utilization of the calculus methods.

The maximum likelihood approach is a useful way to estimate the parameters of the Weibull distribution, starting with the probability density function:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^{\beta}} \tag{6}$$

$$L(\beta,\eta) = \prod_{i=1}^{n} \frac{\beta}{\eta}\left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{x_i}{\eta}\right)^{\beta}} \tag{7}$$

For the processing of data is more practical to maximize the logarithm of the above expression:

$$LL(\eta, \beta) = \ln L(\eta, \beta) \tag{8}$$

It results an implicit equation, with two parameters to determine, β and η. It exist many applications for this kind of problem, inclusive for smart phones, and an accessible example is the function SOLVER of MSExcel. Similar i

t is possible to apply the method of moments, based on estimates of the mean μ and standard deviation σ of the population from the data with the function GOAL SEEK of MSExcel. Any way it is match easier to use an specialized software, but for the few reliability evaluations remains to expensive.

4. Conclusions

The extended use of quality and reliability imposed the development of more and more new software applications for specific practical problems and mathematical models. Weibull distribution is an important one. It is obvious the necessity to pass, in reliability statistical calculus, from the use of mathematical tables or probability papers to the computer assisted reliability data processing for Weibull models fitting.. The extend pallet of such software, with more and more freeware, gives a good chance for students or single researchers to fructify any set of reliability data, to model a diversity of processes and phenomena. The present paper offers only a small overview of the explosion in reliability software and models.

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