DETERMINATION OF PHYSICAL EQUATIONS USED IN PROCESS INDUSTRIES USING THE BUCKINGAM π THEOREM

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Abstract: *Technological process, however complex, can be broken down into a succession of distinct component processes, in which the input materials undergo changes in shape (mechanical processes), pressure, temperature, concentration, state of aggregation (physical processes) or structure molecular (chemical and biochemical processes). The unitary operations of most process phases in process industries are based on three fundamental processes: pulse transfer, heat transfer, and mass transfer.*

Key Words: principle of dimensional homogeneity, equation, mathematical model

1.Introduction

Fluid mechanics problems can be addressed through dimensional analysis, which is essentially a mathematical procedure that studies exclusively the dimensions of physical quantities. It starts from the understanding of flow phenomena in order to determine the parameters that influence it and it is possible to group these parameters into dimensional combinations, to better knowledge and explanation of the phenomena. Dimensional analysis is of real use in experimental studies because it can indicate the magnitudes or parameters that really influence the development of physical phenomena.

According to the principle of dimensional homogeneity, all mathematical relations, which express physical phenomena, must be dimensionally homogeneous (all equation terms must have the same dimensions).

If the terms of a dimensionally homogeneous equation are shared with a quantity that is expressed in the same dimensions, a dimensioning of the terms will result, the equation becoming a dimensionless relationship between groups of numbers and a simpler form. This is done in a dimensional analysis, grouping all the variables involved in an equation that contains groups of non-dimensional numbers, avoiding experimental research, with the non-dimensional groups being much smaller than the variables.

Dimensional analysis applications consist of:
- transformation from one system of units to another;
- establishing equations;
- reducing the number of variables required for an experimental program;
- establishing the principles of designing a model.

2. The mathematical model

Buckingham's π theorem is a generalization of the determination of physical equations used in process industries by dimensional analysis. A physical relationship involving m dimensions and dimensional constants can be expressed as a relation between $i = m - n$ dimensional groups, where n represents the number of fundamental units of the unit of measurement system used.

In this way, a physical equation such as:

$$f_1(x_1, x_2, \dots, x_m) = 0 \tag{1}$$

is reduced to an equation of the type:

$$f_2(\pi_1, \pi_2, \dots, \pi_i) = 0 \tag{2}$$

where each adimensional group π depends on the maximum *(n + 1)* constant sizes and dimensions.

The number of adimensional groups π is equal to *(m - n)*, the difference between the number *m* of physical quantities plus the dimensional constants involved in the phenomenon unfolding and the number *n* of fundamental sizes involved in the units of the physical and dimensional constants:
n = 3 for mechanical phenomena (length, weight, time);
n = 4 for thermal phenomena (length, mass, time, temperature);
n = 4 for electrical phenomena (length, mass, time, power intensity);
n = 5 for thermoelectric phenomena (length, mass, time, temperature, electric current intensity).

To find the desired function, Buckingam's π theorem applies in the following way:
1 – All physical quantities and dimensional constants, which from mechanical, thermodynamic conditions, etc. go through. it is appreciated that it influences the phenomenon studied;
2 – Write the dimensional formula for each of the physical dimensions and dimensional constants considered;
3 –We choose the *n* fundamental sizes between the physical quantities and the dimensional constants that occur in the problem. The choice is made in such a way that all the quantities and constants chosen contain at least once all the fundamental dimensions of the problem;
4 –The groups π_1, π_2, π_3, π_4, ..., π_i, are formed, consisting of each of the products of the n values chosen in item 3 plus one of the other sizes and constants. An arbitrary exponent is associated with each dimension and dimensional constants in each group π.
5 – Determine the value of these exponents, providing that each group π is adimensional.

3. CASE STUDY

With theorem π, we will determine the non-dimensional groups that interfere with the isothermal flow of fluids. It is appreciated that the phenomenon is influenced by the sizes presented in Table 1.

Table 1 – Sizes influencing the flow of fluids

Size	Symbol	Dimensional formula
Length	l	L
Flow speed	v	LT^{-1}
Density of the fluid	ρ	ML^{-3}
Fluid viscosity	μ	$ML^{-1}T^{-1}$
pressure difference	ΔP	$ML^{-1}T^{-2}$
Gravitational acceleration	g	LT-1

Thus, m = 6 physical and dimensional constants and n = 3 fundamental values (M, L, T). The result is $i = m - n$ dimensional groups π. The relationship sought, according to the π theorem, will be:

$$f(\pi_1, \pi_2, \pi_3) = const \tag{3}$$

For grouping, the first three sizes are chosen, l, v, ρ as common sizes, plus each other, all being influenced by the exponents of a_i, b_i, c_i, d_i.

$$\pi_1 = l^{a_1} \cdot v^{b_1} \cdot \rho^{c_1} \cdot g^{d_1} \tag{4}$$

Dimensional,

$$[\pi_1] = L^{a_1} \cdot (LT^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot (LT^{-2})^{d_1} = M^{c_1} \cdot L^{(a_1+b_1-3c_1+d_1)} \cdot T^{(-b_1-2d_1)} \tag{5}$$

For π_1 to be dimensionless, it is necessary that the exponents of the fundamental sizes M, L, T be null, so:

$$\begin{cases} c_1 = 0 \\ a_1 + b_1 - 3c_1 + d_1 = 0 \\ -b_1 - 2d_1 = 0 \end{cases} \tag{6}$$

The system (6) of 3 equations with 4 unknowns resolves in relation to d1 considered equal to the unit. Is obtained; $a_1 = 1$; $b_1 = -2$; $c_1 = 0$; $d_1 = 1$.

Substituting in (4) results:

$$\pi_1 = l^1 \cdot v^{-2} \cdot \rho^0 \cdot g^1 = \frac{l \cdot g}{v^2} = Fr \tag{7}$$

The adimensional group $\frac{l \cdot g}{v^2}$ is called the number (criterion) of Froude, symbolized by Fr.

Similarly,

$$\pi_2 = l^{a_2} \cdot v^{b_2} \cdot \rho^{c_2} \cdot (\Delta P)^{d_2} \tag{8}$$

$$[\pi_2] = L^{a_2} \cdot (LT^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot (ML^{-1}T^{-2})^{d_2} = M^{c_2+d_2} \cdot L^{(a_2+b_2-3c_2-d_2)} \cdot T^{(-b_2-2d_2)} \tag{9}$$

Since π_2 is adimensional, the system is obtained:

$$\begin{cases} c_2 + d_2 = 0 \\ a_2 + b_2 - 3c_2 - d_2 = 0 \\ -b_2 - 2d_2 = 0 \end{cases} \tag{10}$$

By imposing $d_2 = 1$, we obtain: $a_2 = 0$; $b_2 = 2$; $c_2 = -1$ and the relation (9) becomes:

$$\pi_2 = l^0 \cdot v^{-2} \cdot \rho^{-1} \cdot (\Delta P)^1 = \frac{\Delta P}{\rho \cdot v^2} = \mathrm{Eu} \tag{11}$$

The adimensional group $\frac{\Delta P}{\rho \cdot v^2}$ is called the number (criterion) of Euler, symbolized by Eu.

Similarly,

$$\pi_3 = l^{a_3} \cdot v^{b_3} \cdot \rho^{c_3} \cdot \mu^{d_3} \tag{12}$$

$$[\pi_3] = L^{a_3} \cdot (LT^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot (ML^{-1}T^{-1})^{d_3} = M^{c_3+d_3} \cdot L^{(a_3+b_3-3c_3-d_3)} \cdot T^{(-b_3-d_3)} \tag{13}$$

By making the condition of the adimensional of π_3 obtain the system:

$$\begin{cases} a_3 + b_3 - 3c_3 - d_3 = 0 \\ -b_3 - d_3 = 0 \\ -c_3 + d_3 = 0 \end{cases} \tag{14}$$

By imposing $d_3 = 1$, we obtain: $a_3 = -1$; $b_3 = -1$; $c_3 = -1$ and the relation (12) becomes:

$$\pi_3 = l^{-1} \cdot v^{-1} \cdot \rho^{-1} \cdot (\mu)^1 = \left(\frac{\rho \cdot v \cdot l}{\mu}\right)^{-1} = \frac{1}{Re} \tag{15}$$

The adimensional group $\frac{\rho \cdot v \cdot l}{\mu}$ is called the number (criterion) of Reynolds, symbolized by Re.

Formula generală a funcţiei care descrie curgerea fluidelor (depinzând doar de variabilele l, v, ρ, g, μ şi ΔP) are forma:

The general formula of the function describing fluid flow (depending only on the variables l, v, ρ, g, μ şi ΔP) has the form:

$$f(Fr, Eu, Re) = const$$

4. Conclusions

The buckingam π theorem is the domain that deals with establishing relationships between dimensional formulas of different physical sizes. Based on these relationships, we can sometimes determine approximate forms of valid laws in certain experimental situations. Even though formulas determined using dimensional analysis are only approximate, they can be of great help in simplifying the experiments to determine the correct form of those laws. Also, The buckingam π theorem can highlight dimensional reports of physical quantities, called criteria, which are used to characterize the predominance of a particular physical effect with respect to another.

References

[1] Gavrilă, L – Operaţii unitare în industria alimentară şi biotehnologii, Universitatea Bacău, 2001

[2] Gavrilă, L – Fenomene de transfer, vol. I,II, Editura Alma Mater, Bacău, 2000

[3] Nioaţă Alin - Optimization of dimensional processing process through complex electric erosion with introducing of the electrolite through the transfer object - FIABILITY & DURABILITY, Nr.2/2017, ISSN 1844-640X, pp. 92-98.