

# SOME GENERALIZATIONS AND REPRESENTATIONS OF THE POSSIBILITIES OF MULTI-PARAMETRIC MAPPING OF AFFIN SPACE IN AN APPENDIX TO THE CURVILINEAR FORMING AND TRANSMISSION MOVEMENT

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***Abstract:** The works of Prof. B. A. Perepelitsa from Kharkov Polytechnic Institute and his disciples to develop an applied methodology of multiparameter mappings in relation to the profiling and functioning of complex curvilinear objects and transmission mechanisms in mechanical engineering, mainly with examples of gears, are presented. The completed review is gratefully devoted to the 10th anniversary of *Fiability & Durability Journal*, which has repeatedly provided its platform to the authors to highlight this development of the wide Romanian, European and world scientific community.*

**Keywords:** multiparameter mappings, curved surfaces, variable curvature, complex shaping, gears and gearing.

## 1. INTRODUCTION TO REVIEW AND BEGINNINGS OF DEVELOPMENT

Historically, Ukraine has been traditionally noticeable in the global market for the development of the theory and practice of complex shaping, especially in the current challenges of the aerospace, nuclear power engineering, defense industry, including in terms of compact gears and CVTs for armored military and advanced civil equipment. So, in the world chronological list of works of scientists who made the most notable contribution to the theory of gears according to the modern version [1] (S. P. Radzevich, 2018), the first seven, followed by the representation of Italy (L. Da Vinci, 1493), Russia (L. Euler, 1754), England (R. Willis, 1841), France (T. Olivier, 1842) and the USA (F. Reuleaux, 1861, and J. W. Gibbs, 1863), closes H. I. Gofman's master degree work on mechanics [2]. It developed the analytical form of arbitrary gears and the basics of the modern analytical theory of gearing, created engineering methods for calculating and designing gearing, developed new types of gearing, the theory of kinematic pairs and chains, investigated the structure and derived the general equations of mechanisms, first classified them based on objective features which are determined by structure levels and movements.

The same prestigious list of works by 60 scientists includes Kharkov researcher A. I. Pavlov [3] and Hungarian researcher I. Dudas [4], who collaborates with the Kharkov scientific school in the applied development of the mathematical apparatus of multi-parameter mappings of space for gear wheels and gears [5]. The works [6-8] and some others in the list [1] also represent a number of ideologically close to development presented here on the mathematical apparatus and approaches used in it. In our experience, [9-14] also should be considered in this series of works.

At the same time, some well-known developments to enhance the adaptive capabilities of the mathematical apparatus of multiparameter mappings of affine space, for which the improvement of the theory and practice of gears is not an end in itself, but one of the directions of applied implementations, i. e. subordinate theme, are missing in the selected

series of particularly significant works on the theory of gears [1]. Most likely, this is what explains the absence. Development of the Kharkov scientific school of Prof. B. A. Perepelitsa and his disciples at the Kharkov Polytechnic Institute (NTU "KhPI") [15-21] refers precisely to such works of a wide range of applications, including with respect to the description, improvement and synthesis of new gears and gearings. The history of this development is especially rich in fruitful cooperation with the Kiev Polytechnic Institute (P. R. Rodin, academic fundamentalist of the theory of instrumental production for complex cutting, and others at the NTUU "Igor Sikorsky KPI") [22, 23] and the Institute for Superhard Materials (A.V. Krivosheya, world class coryphaeus in the theory and practice of tooth processing, now the leading specialist in Ukraine in this object-oriented area of applied development of the general theory of multiparameter mappings, and others at the V. Bakul ISM of the NAS of Ukraine) [24-27]. The significant results of this collaboration are jointly summarized in [28] by the key participants mentioned above.

## **2. BRIEF EXCURSE TO GENERAL THEORY AND METHODOLOGY**

The fundamentals of the new direction in the theory of shaping and designing of cutting tools and objects to be processed by them, based on the formalizing and generalizing capabilities of the mathematical apparatus of multiparameter mappings of affine space, were developed to the beginning of the 80-s of last century [15-17].

It was established that a general mathematical model of shaping during cutting can be a regulated multiparameter mapping. The general character of the model is that the workpieces with their edges are images, and the shaping movements of any complexity are one-parameter mappings. Consequently, it means to build one or another model of shaping by synthesize the corresponding mapping, that is, to completely determine its components. Initially, this requires completely, but in general terms, to determine the structure of the model, parameters and operators, the nature of the pre-image, operators, and functional connections between the parameters. Then it is necessary to assign or calculate the specific numerical content of all components of the model in accordance with the initial data. In the synthesis of the model, it is necessary to take into account the known regulatory conditions of contact and intersection, as well as the conditions for the completeness of shaping.

There are formulated the theoretical principles of modeling, which reflect different degrees of algorithmic generality: the principles of directional synthesis, coincidence, replacement, and inversion of mappings; principle of regulated intersection of images, etc. Based on these principles and the general model, systems of typical and specific models can be formed, each of which corresponds to a method or group of methods of real shaping, which potentially allows to predict and develop new processing methods.

Multiparameter mappings are used as a general methodological basis for geometric design and shaping. In both processes, the same theoretical principles of modeling and the general mathematical apparatus operate. This commonality allowed to develop an automated structural modeling system, which covers a wide range of seemingly different in nature modeling and control problems in the design and shaping.

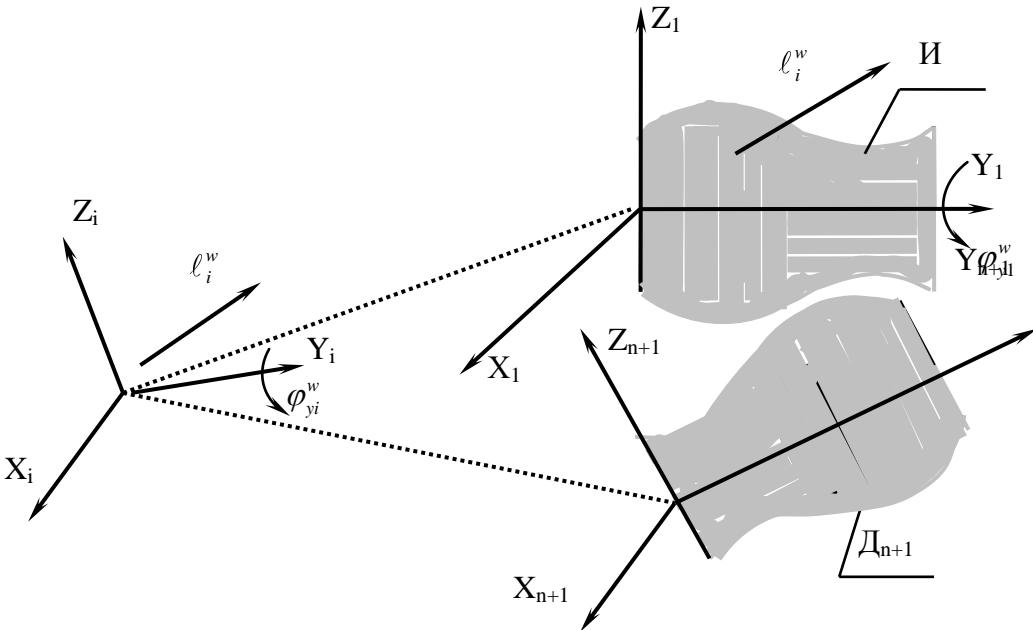
The basis of the universality of the system are a multiparameter mapping and a structural method based on the unification of parameters, operators and functional relationships. It is enough to specify a mapping in order to fully define the corresponding part or tool as a geometric shape.

By specifying the structural models of the workpiece, the machining tool and the shaping, as well as the corresponding numerical parameter arrays, this information can be

converted into a form convenient for technological realization, for example, for computer-controlled machines and systems equipped with microprocessors and mini-computers. At the same time, it is advisable to introduce unified operators into the structure of the forming surface at the design stage of the part, providing for the use of the same operators during processing. In other words, if necessary, it is possible to foresee and use an algorithmic generality between the design of the part and its shaping.

A technique has been developed for applying of mappings for the mathematical description and construction of parts and tools as limited multi-parameter images. For example, by successive one-parameter mappings with the help of two operators (rotation and parallel transfer), it is possible to synthesize a gear from a starting point as a multi-parameter area of space. Gear equations in vector, matrix and coordinate form can be obtained by substituting operators into the general equation of their mapping. The combination of these equations and parameter intervals fully describes the gear wheel as a set of its points (inside and on the surface). In the particular case when the operators of rotation and parallel translations are mapping, and the independent parameter is time, the one-parameter mapping represents motion. Consequently, the general mapping equation in this case is an equation of arbitrarily complex motion.

Schematically generalized kinematic scheme of shaping for gear links of various classes, types and types proposed by prof. B. A. Perepelitsa is represented in Fig. 1. The movements of the initial shape forming link in its own coordinate system are indicated in the first coordinate system, and movements of the first coordinate system with the moving initial shape forming link are indicated in the second coordinate system, etc.



**Fig. 1.** The generalized kinematic scheme of shaping for gear links [17]:  
 $\mathbb{H}$  and  $\mathbb{D}_{n+1}$  – forming and formable gear links, respectively

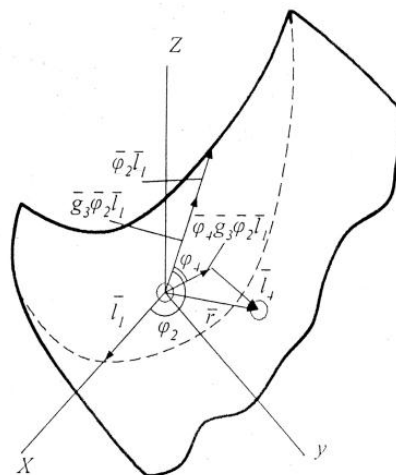
According to the generalized kinematic scheme of shaping (Fig. 1), its generalized unified mathematical model, using 4th order matrices, can be represented as (original identification [17] is preserved):

$$\left. \begin{aligned}
 & m_{r\partial(n+1)H/\Delta} = m_{V_n^w c_n^w} m_{\varphi_n^w \ell_n^w} \dots \\
 & m_{V_i^w c_i^w} m_{\varphi_i^w \ell_i^w} \dots m_{V_{i+1}^w c_{i+1}^w} m_{\varphi_{i+1}^w \ell_{i+1}^w} m_{r_{u1}} \\
 & \left. \begin{aligned}
 & \varphi_{1A}^u < \varphi_1^u < \varphi_{1A}^u \\
 & \varphi_{1A}^v < \varphi_1^v < \varphi_{1B}^v \\
 & \varphi_{1A}^w < \varphi_1^w < \varphi_{1B}^w \\
 & \varphi_i = f_{\varphi_i}(\varphi_1^w) \\
 & \ell_i = f_{\ell_i}(\varphi_1^w)
 \end{aligned} \right\} \quad (1)
 \end{aligned}$$

In system (1), the last two equations are the equations of communication;  $\varphi_1^u, \varphi_1^v$  – independent parameters of the surface of the gear rims of the forming gear link;  $\varphi_{1A}^u, \varphi_{1B}^u$  and  $\varphi_{1A}^v, \varphi_{1B}^v$  – respectively, the initial and final values of the independent parameters of the surface of the gear rims of the forming gear link;  $\varphi_1^w$  – independent parameter of the movement of the forming gear link;  $\varphi_{1A}^w, \varphi_{1B}^w$  – the initial and final value of the independent parameter of the movement of the forming gear link;  $m_{r_{u1}}$  – matrix equation of the forming surface or line of the forming gear link in the first coordinate system;  $m_{\varphi_i^w \ell_i^w}$  – motion matrix of the forming gear link in the coordinate system  $i$ ;  $m_{V_i^w c_i^w}$  – position matrix, i. e. matrix of coordinate transformations in the transition from the coordinate system  $i$  to the coordinate system  $i + 1$ ;  $m_{r\partial(n+1)H/\Delta}$  – the matrix equation of motion of the forming link ( $H_1$ ), specified in the 1st coordinate system relative to the fixed formable gear link ( $\Delta$ ), written in the coordinate system  $n + 1$ , i. e. in the coordinate system of the taken form gear link.

### 3. ANNEX TO SURFACES OF COMPLEX FORM

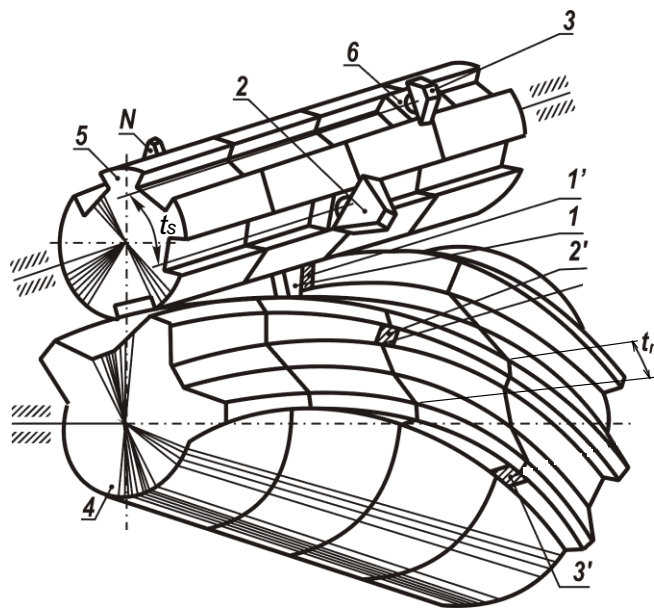
The development of structural mathematical modeling in relation to the processes of shaping the cutting of parts of complex shape leads to the development of matrix formalization of geometric objects of processing, that allow, firstly, on a uniform algorithmic basis, to consider both the geometric design of surfaces, and their shaping, and with a significant reduction in the amount of processed information; secondly, not only to model, but also to investigate the geometric properties of structures, to predict and develop on the basis of this new surfaces (Fig. 2), tools and kinematic schemes of shaping; thirdly, to provide the necessary prerequisites for processing complex-shaped parts without a drawing, using structural mathematical models as an intermediate storage medium. Based on the previous national experience of the calculation and methodological support for the processing of complex non-linear surfaces on machine tools with numerical



**Fig. 2.** 3D model of the turbine blade surface in the structure with mapping operators [29].

control [30], a consistent development of a common method for calculating the trajectory and coordinates of instantaneous points of contact, applied to any shaped instrumental surface and based on special improvements and applied system development of the mathematical apparatus of multiparameter mappings of affine space, is carried out [18-20]. The addition of the existing methods of analytical description of discretely defined surfaces by the developed method of virtual modeling made it possible to expand the range of types and profiles of tool surfaces used on CNC machines.

#### 4. ANNEX TO GEAR AND GEARING



**Fig. 2.** General schematics of double-link variator on the base of bevel gear with equidistant tooth lengthwise curves: 1, 2, 3, ...  $N$  – movable tooth of spur gear; 1', 2', 3' – applicable contact zones; 4 – bevel gear with constant normal pitch; 5 – spur compound gear; 6 – sliding elements jointed with tooth [31&21]

The relevance of improving the mathematical apparatus of analysis and synthesis of gearing, especially from the last quarter of the last century, respectively, and the development response, is fueled by the globalization of competition in the automotive industry, the demands of the aerospace industry and the military-industrial complex to create gears and variators of increased compactness and (or) supporting capacity, in particular two-parameter [31-35] (Fig. 3), with the spherical design of the teeth carrying elements [36, 37], etc.

#### 5. DIRECTIONS OF INTEGRATION IN MECHANICAL ENGINEERING

Founded by Prof. B. A. Perepelitsa, with a brief overview of presented here related publications, working systematically develops applied principles and virtual concretization tools for a scientific theory of multiparameter mappings of affine space in the application to shaping of complex curvilinear surfaces, including gearing, that makes it possible to synthesize novel gears with the desired performance.

Ways of further integration of development in modern engineering, in accordance with its achieved theoretical level and applied assimilation, should be primarily associated with the triad of continuously relevant organizational, technical and technological tasks of increasing the competitiveness of enterprises of the engineering complex with mechanical and laser shaping of constructional parts and forming tools for aircraft, transport, energetics and others branches of the economy.

First, it is a virtual modeling of unitary objects and technical systems with new design

solutions of increased functional efficiency, including using the developed structural approach to the mathematical description and visualization of kinematic curves [21, 38].

Secondly, it is the integration into the design of individual machining operations with varying degrees of certainty at the micro level kinematic-geometric schematics, including the prediction of the output microcutting parameters for optimizing the processing with abrasive tools at the macro level, by analogy with the developed adaptation of the general methodology that takes into account the functional specifics interference objects set forth in the works of the scientific school of prof. B. A. Perepelitsa [19, 23, 39].

Thirdly, is to use development in the creation of applied mathematical software for renovation algorithms and the original integrated design of highly efficient multi-operational production processes and complexes of such processes of an industrial enterprise, including in the framework of the implementation of the concept of simulation modeling in accordance with the experience of development of the NTU "KhPI" in this direction [26, 40].

## 6. CONCLUSION

The history of publication presentations of the development and adaptation of the mathematical apparatus of multiparameter mapping of affine space in application to curvilinear shaping and transfer motion in the journal *Fiability & Durability* [29, 34, 35, 38] goes back to its first issue [29]. The authors heartily thank the "Constantin Brancusi" University of Targu Jiu, which is the publisher, and its *Fiability & Durability* Editorial Board for the opportunities to cover this and other developments of the wide Romanian, European and world scientific community in the first decade of the journal and along with it.

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