THE MOTION LAWSSYNTHESIS IN THE POSITIONING MECHANISM MOTOR COUPLES 3R

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Abstract: *The manipulator mechanism with 3D working space is characterized by the configuration of three motor rotation couple. In this worksheet we consider R ⊥ R ∥ R configuration on which characteristic point with first rotation couple ax determine one plan which represent movement plan at the last two elements. With the cinematically modulations you can calculate command functions (position, velocity and acceleration in motor couple) from the condition that tracer point make a movement after a law named movement law. Numerical results are obtained with a program and they are put in diagrams.*

1. Introduction

Generally, a robot ensures the positioning and orientation of a rigid solid in a determined workspace. The mechanical structure can be unitary for the two functions, or decomposable in two independent mechanisms, called positioning mechanism and, respectively, orientation mechanism [1], [9].

The positioning mechanism, also called the robot mechanism, is an open mechanism with rotating R-motors and T-translation, the number of which is compatible with the degree of mobility and the working space.

The positioning mechanism, also called the robot mechanism, is an open mechanism with rotary drive motors - R and translation - T, their number being compatible with the degree of mobility and the working space.

There is a class of three-dimensional work space positioning mechanisms made by serially binding two R or T single-dimensional (MB1) of type R or T and two-dimensional (MB2) (R∥ R),), (R⊥T), (T⊥R), (T⊥T). In this way, a three-dimensional base module (MB3) is obtained which admits as a kinematic model one of the variants [11]: R⊥(R∥R), R⊥(R⊥T), R∥(T⊥R), R∥(T⊥T) , T∥(R∥R) , T∥(R⊥T), T⊥(T⊥R) , T⊥(T⊥T). The analysis of all possible variants of trajectory generating mechanisms in the three-dimensional space is done in the paper [10].

In the present paper, the three-dimensional three-position rotary drive mechanism has the configuration R ⊥ R ∥ R (figure 1). The position, speed and acceleration parameters in the motor couplers are determined so that the tracer point M moves on a given trajectory after a certain motion law.

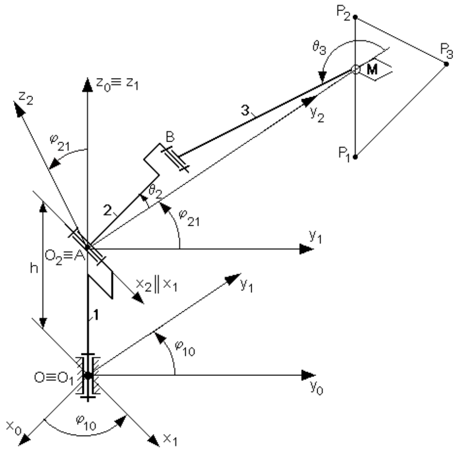


Fig. 1. Scheme of the 3R manipulator

2. Synthesis of the movement law on the trajectory of the tracer point M

The movement of the traversing point M of the positioning mechanism is determined if known: trajectory (Γ), motion law (LM), and travel time (t_m The trajectory is considered to be a straight segment determined by two points P_i and P_{i+1} given in the fixed system. The trajectory can also be a polygonal contour when crossing multiple points. Moving on each side (straight or curve) of the polygon represents an "i" work sequence. At the ends of the polygon it is necessary to stop (manipulate) th manipulator and therefore it is necessary to know the residence time (t_s).

$$\overline{H} = H\overline{\delta} \tag{2.1}$$

Where $H = \sqrt{(x_{P_{i+1}} - x_{P_i})^2 + (y_{P_{i+1}} - y_{P_i})^2 + (z_{P_{i+1}} - z_{P_i})^2}$ (2.2)

$$\overline{\delta} = \cos\alpha \cdot \overline{i} + \cos\beta \cdot \overline{j} + \cos\gamma \cdot \overline{k} \tag{2.3}$$

$$\cos\alpha = \frac{x_{P_{i+1}} - x_{P_i}}{H}; \cos\beta = \frac{y_{P_{i+1}} - y_{P_i}}{H}; \cos\gamma = \frac{z_{P_{i+1}} - z_{P_i}}{H} \tag{2.4}$$

For position, speed and acceleration determination, three subprograms were created in FORTRAN:

a). The LEGI subprogram (X, Y, DY, D2Y, LM) delivers the law of motion in the dimensionless way through the LM counter. Formal parameters are:

$$X = x = \frac{t}{t_m}; Y = y = \frac{s}{H}; DY = \frac{dy}{dx}; D2Y = \frac{d^2y}{dx^2} \tag{2.5}$$

b). The TRASOR subroutine (H, XP, YP, ZP, TM, ALFA, BETA, GAMA, Y, DY, D2Y, PM, VM, AM, N) determines the position coordinates, contained in the matrices:

$$[PM] = \begin{bmatrix} x_{0M} \\ y_{0M} \\ z_{0M} \end{bmatrix}; [VM] = \begin{bmatrix} v_{0M}^x \\ v_{0M}^y \\ v_{0M}^z \end{bmatrix}; [AM] = \begin{bmatrix} a_{0M}^x \\ a_{0M}^y \\ a_{0M}^z \end{bmatrix} \tag{2.6}$$

c). The STA subroutine (XP, YP, ZP, PM, VM, AM, J) specifies the point M parameters in the final position of a sequence, counted by the J = N + 1 counter. Position coordinates correspond to fixed coordinates XP (J); YP (J); ZP (J), and speeds and accelerations are null.

In the case where the movement of the stake is N work sequences [6], it is necessary indexing H (N), XP (N), YP (N) ZP (N), ALPHA (N) BETA (N) ,GAMA (N).

3. Synthesis of command functions

The positioning mechanism considered in Figure 1 is centered and in this form is encountered in the structure of several types of manipulators and welding robots. For reverse kinematic analysis, the mechanism is structured into two "modules", namely:

- RR module with two-dimensional workspace;
- the R module that provides the tracer point to the three-dimensional workspace.

Fixed reference systems $OX_0Y_0Z_0$ and mobile systems $O_1X_1Y_1Z_1$; are considered; $O_2X_2Y_2Z_2$ attached to the two modules. The tracer point is positioned in the three systems by the coordinates (x_{0M} y_{0M} z_{0M}); (x_{1M} y_{1M} z_{1M}); (x_{2M} y_{2M} z_{2M}). Among these are relations established:

$$\begin{bmatrix} x_{1M} \\ y_{1M} \\ z_{1M} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_{21} & -\sin\varphi_{21} \\ 0 & \sin\varphi_{21} & \cos\varphi_{21} \end{bmatrix} \begin{bmatrix} x_{2M} \\ y_{2M} \\ z_{2M} \end{bmatrix} \tag{3.1}$$

$$\begin{bmatrix} x_{0M} \\ y_{0M} \\ z_{0M} \end{bmatrix} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1M} \\ y_{1M} \\ z_{1M} \end{bmatrix} \tag{3.2}$$

Due to the centric solution of the mechanism it results:

$$\begin{aligned} & x_{1M} = 0; y_{1M} = \sqrt{x_{0M}^2 + y_{0M}^2}; z_{1M} = z_{0M} \\ & x_{2M} = 0; y_{2M} = \sqrt{y_{1M}^2 + (z_{1M} - h)^2}; z_{2M} = 0 \\ & \varphi_{21} = \operatorname{arctg}\left(\frac{z_{1M} - h}{y_{1M}}\right) \end{aligned} \tag{3.3}$$

For kinematic element 1 of module R, position parameters, speeds and accelerations are determined with the relations:

$$\begin{aligned} & \varphi_{10} = \operatorname{arctg}\left(\frac{-x_{0M}}{y_{0M}}\right) \\ & \omega_{10} = \frac{v_{0M}^x \cos\varphi_{10} + v_{0M}^y \sin\varphi_{10}}{x_{0M}\sin\varphi_{10} - y_{0M}\cos\varphi_{10}} \\ & \varepsilon_{10} = \frac{a_{0M}^x \cos\varphi_{10} + a_{0M}^y \sin\varphi_{10} - 2\omega_{10}(v_{0M}^x \sin\varphi_{10} - v_{0M}^y \cos\varphi_{10}) - \omega_{10}^2(x_{0M}\cos\varphi_{10} + y_{0M}\sin\varphi_{10})}{x_{0M}\sin\varphi_{10} - y_{0M}\cos\varphi_{10}} \end{aligned} \tag{3.4}$$

These parameters are calculated with the subprogram R: R(PM,VM,AM,FI10,OM10,EPS10).

The positional parameters of elements 2 and 3 are determined in the reference system $O_2X_2Y_2Z_2$, with the relations:

$$\theta_2 = \arccos\left(\frac{l_2^2 + y_{2M}^2 - l_3^2}{2 \cdot l_2 \cdot y_{2M}}\right); \theta_3 = \arccos\left(\frac{l_2^2 - y_{2M}^2 - l_3^2}{2 \cdot l_3 \cdot y_{2M}}\right) \tag{3.5}$$

In the $O_1Y_1Z_1$ mobile plane, elements 2 and 3 [4] are positioned at the angles:

$$\varphi_2 = \varphi_{21} + \theta_2 \;;\; \varphi_3 = \varphi_{21} + \theta_3 \tag{3.6}$$

Elements 2 and 3 of the RR module with Y_1Z_1, plane motions, kinematic modeling of the RR module for M point moves, are reduced to the kinematic model of RRR [3].
Output parameters of the diade, in number two, are included in the one-sided matrices:
X - the positional parameter matrix;
Y - the matrix of speeds;
Z - the acceleration matrix.
The matrix X comprises the parameters φ_2 and φ_3 the matrix Y comprises the parameters ω_2 and ω_3, and the matrix Z comprises the parameters ε_2 and ε_3. Gear and acceleration equations are obtained from position equations by repeated derivation over time and are shown below.

Equations of positions:
$$\begin{aligned} Y_2 + Y_3 &= y_{1M} - y_{1A} \\ Z_2 + Z_3 &= z_{1M} - z_{1A} \end{aligned} \tag{3.7}$$

Speed equations:
$$\begin{aligned} -Z_2\omega_2 - Z_3\omega_3 &= v_{1M}^y - v_{1A}^y \\ Y_2\omega_2 + Y_3\omega_3 &= v_{1M}^z - v_{1A}^z \end{aligned} \tag{3.8}$$

Acceleration equations:
$$\begin{aligned} -Z_2\varepsilon_2 - Z_3\varepsilon_3 &= a_{1M}^y - a_{1A}^y + Y_2\omega_2^2 + Y_3\omega_3^2 \\ Y_2\varepsilon_2 + Y_3\varepsilon_3 &= a_{1M}^z - a_{1A}^z + Z_2\omega_2^2 + Z_3\omega_3^2 \end{aligned} \tag{3.9}$$

For the calculation of the parameters, a subprogram RRR [5]: RRR (PB, VB, AB, PM, VM, AM, H1, H2, X, Y, Z, ER), in which: $X(1) = \varphi_2$; $Y(1) = \omega_2$; $Z(1) = \varepsilon_2$ and $X(2) = \varphi_3$; $Y(2) = \omega_3$; $Z(2) = \varepsilon_3$
Of course, for another structure of the two-dimensional module: RT, TR or TT, the kinematic model is reduced to an appropriate diadem: RTR, TRR and TTR [6], [7] respectively.

4. Numerical application

For the determination of position, speed, and acceleration functions in motor couples, the movement of the tracer point on a polygonal contour defined in the fixed reference sister, the points P_1(0; 1.2; 0.84); P_2(-1.2; 1.2; 0.84); P_3(-1.2; 1.5; 0.84). Movement times corresponding to sides P_1P_2; P_2P_3; P_3P_1 have values $ts_1 = 5$ s, $ts_2 = 5$ s, $ts_3 = 5$ s. The motion laws on the three work sequences are: sinusoidal, cosine and linear. The numerical values of the kinematic parameters of the motor couplers A_0, A and B (tab.1) are obtained with a calculation program and transposed into the graphs of Fig. 2, Fig. 3 and Fig. 4.
In conclusion, the kinematic analysis of the three-dimensional workspace manipulators, elements of which belong to the MB1 and MB2 base modules, can be computerized by calling STANDARD subprograms designed for static subsystems determined by the Diadue type.

Table 1

t[s]	Poz.Pct. M	φ_1 [grade]	ω_1 [s^{-1}]	ε_1 [s^{-2}]	φ_2 [grade]	ω_2 [s^{-1}]	ε_1 [s^{-2}]	φ_3 [grade]	ω_3 [s^{-1}]	ε_3 [s^{-2}]
0	P1	0.000	0.0000	0.0000	60.000	0.0000	0.0000	120.000	0.000	0.000
2		2.784	0.0689	0.0593	59.961	- 0.0019	0.0281	120.039	0.001	- 0.028

4		17.037	0.1654	0.0054	58.470	-0.0311	-0.0216	121.530	0.031	0.022
6		34.743	0.1221	-0.1043	52.520	-0.0650	-0.0180	127.480	0.065	0.018
8		43.572	0.0363	-0.0658	46.360	-0.0329	0.0246	133.640	0.032	-0.025
10	P2	45.000	0.0000	0.0000	45.000	0.0000	0.0000	135.000	0.000	0.000
15	P2	45.000	0.0000	-0.0062	45.000	0.0000	-0.0062	135.000	0.000	0.006
17		44.324	-0.0113	0.0064	44.308	-0.0118	-0.0053	135.692	0.011	0.005
19		42.629	-0.0171	0.0146	42.415	-0.0204	-0.0022	137.585	0.020	0.002
21		40.675	-0.0159	0.0157	39.902	-0.0221	0.0029	140.098	0.022	-0.003
23		39.199	-0.0092	0.0116	37.711	-0.0146	0.0076	142.289	0.014	-0.008
25	P3	38.659	0.0000	0.0048	36.833	0.0000	0.0088	143.167	0.000	-0.009
30	P3	38.659	-0.0390	0.0295	36.833	0.0684	-0.0035	143.167	-0.068	0.003
32		33.690	-0.0481	0.0282	43.854	0.0550	0.0012	136.146	-0.055	-0.001
34		27.552	-0.0594	0.0234	49.567	0.0449	0.0028	130.433	-0.044	-0.003
36		19.983	-0.0730	0.0141	54.180	0.0356	0.0010	125.820	-0.035	-0.001
38		10.784	-0.0875	0.0021	57.694	0.0256	-0.0042	122.306	-0.025	0.004
40	P1	0.000	-0.1000	-0.0050	60.000	0.0144	-0.0088	120.000	-0.014	0.009
45	P1	0.000	0.0000	0.0000	60.000	0.0000	0.0000	120.000	0.0000	0.000

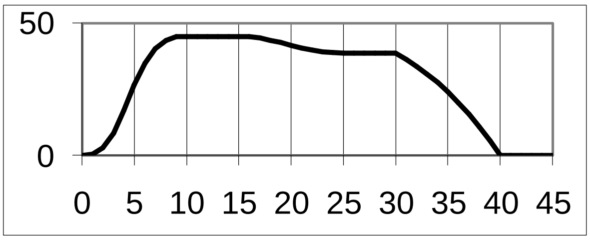


Fig.2a Angular displacement $\varphi_1(t)$

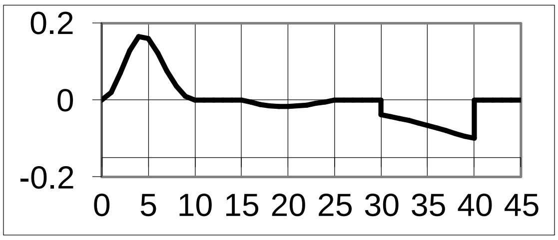
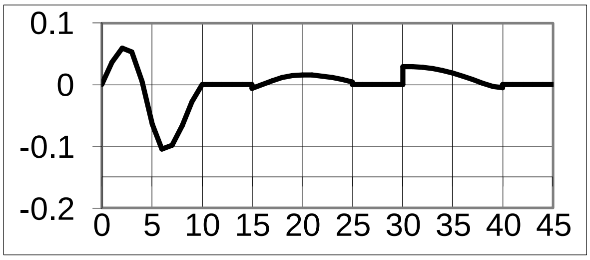


Fig.2b Angle speed $\omega_1(t)$



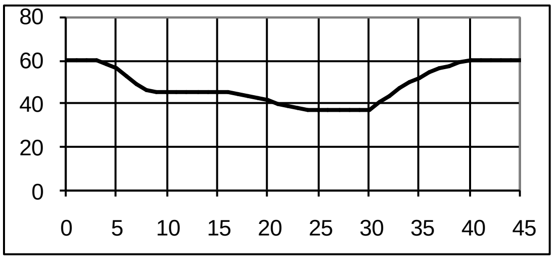
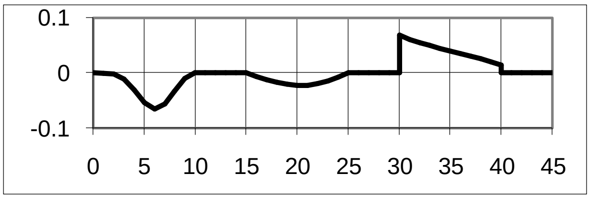


Fig.2c Angular acceleration $\varepsilon_1(t)$ Fig.3a Angular displacement $\varphi_2(t)$



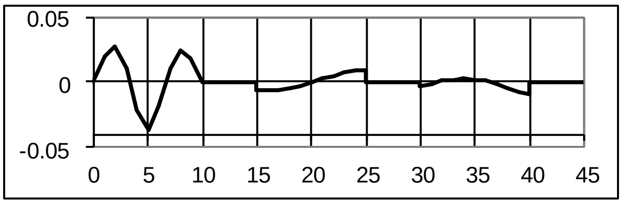
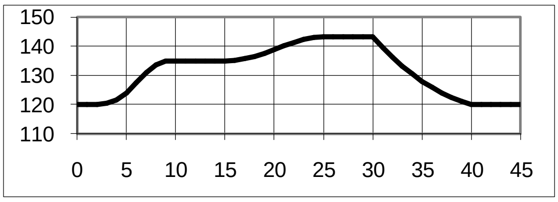


Fig.3b Angle speed $\omega_2(t)$ Fig.3c Angular acceleration $\varepsilon_2(t)$



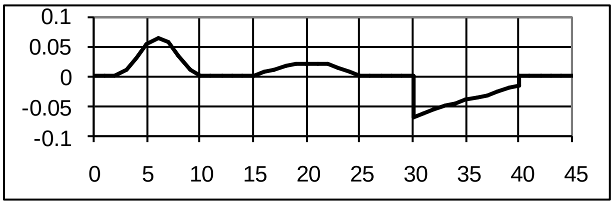


Fig.4a Angular displacement $\varphi_3(t)$ Fig.4b Angular speed $\omega_3(t)$

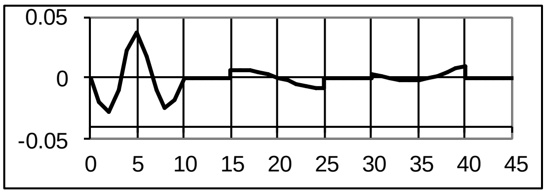


Fig.4c Acceleraţia unghiulară $\varepsilon_3(t)$

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