NEW FLAT CURVES

Professor Emeritus Iulian POPESCU, PhD, University of Craiova

Abstract: *It gives a small history of flat curves and their correlation with the evolution of the technique is emphasized. Mathematical details are given on the study of flat curves and their properties. Then there are many new, original flat curves and their parametric equations. The directions for further investigation of these curves are given.*

Key words: flat curves; parametric equations

1. Introduction

Many plane curves were known in antiquity. Over time, many mathematicians have created different plane curves to which they studied their properties. The geometry textbooks treat a plurality of plane curves [3]. The evolution of the technique has greatly influenced the finding and studying of many plane curves, these being applied to different mechanisms of different machines. Many achievements in this field have been made by Reuleaux [5], Cebashev, Artobolevski [1] and many other researchers. There are many curved sites on the Internet, Ferreol is the most complete one [2]. In [4] there are many white curves, i.e. plane curves generated by some mechanisms.

2. Mathematical considerations

A flat curve can be explicitly mathematically expressed as:

$y=f(x)$,

so that the definition domain can be determined, the asymptotes equations are determined (if any), the derivatives of the orders one and two are calculated, there are the intersection points with the axes, the function variation table is drawn, the curve is plotted.

The parametric equations of the curves are also used:

$x=x(t)$, $y=y(t)$,

so that each function is studied with the procedures of the previous case.

Another expression is in polar coordinates:

$\rho = \rho(\theta)$,

studying as above.

Mathematics offers methods of studying curves by calculating the curvature, studying the contact of two curves, establishing the wrap, unfolded, or curve-leader.

3. New flat curves

Below are the parametric equations of original plane curves and the shapes of these curves. The values a = 30, b = 40 mm are used for all curves and the following notations:

$s = \sin(\varphi)$; $c = \cos(\varphi)$, $t = \tan(\varphi)$.

In my researches we have obtained many curves, some known but not given here, although they are not given in the books at all the parametric equations. There are no studies on these curves, but only their shapes and parametric equations, and they will be studied in other papers.

Some curves obtained are similar in shape to other known curves, but no checks have been made if they are the same or similar, and will be studied in detail in other papers. Under the trace curves are given the parametric equations with the notations above.

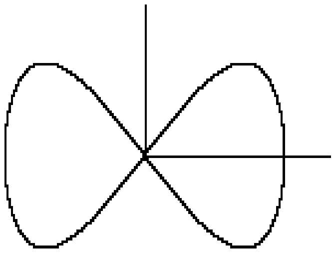
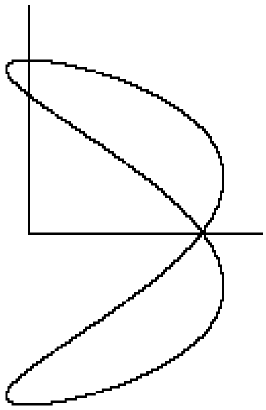
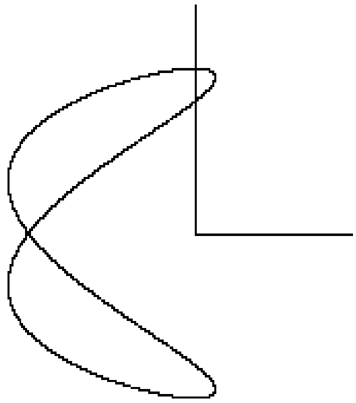




Fig.1: x=a.s;y=b.s.c Fig. 2: x=a.s.c+b.s.s; y=b.c Fig.3: x=a.s.c-b.s.s; y=b.c

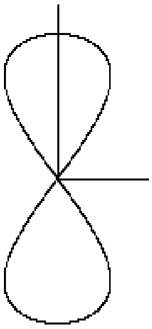
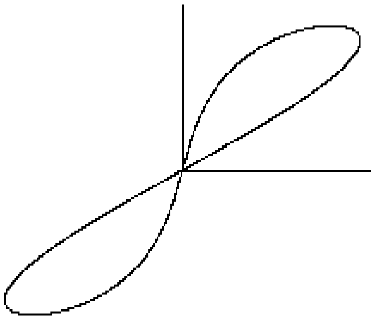
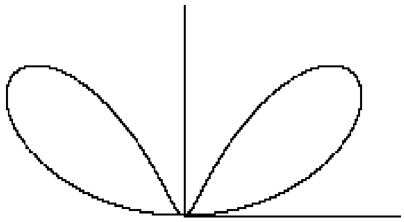




Fig.4: x=a.s.c;y=b.c+c Fig. 5: x=a.c.s+b.s+s;y=b*s Fig. 6: x=a.c.s-b.s+s;y=b.s.s

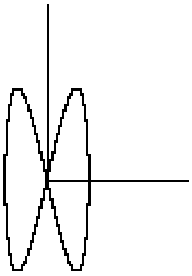
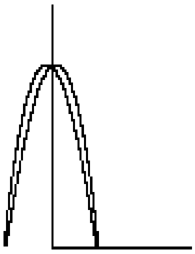
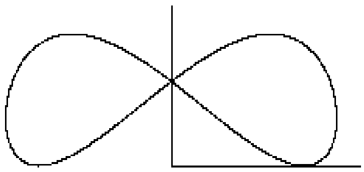




Fig. 7: x=a.s-b.s+s;y=b.c.s Fig. 8: x=a.c-b.c+s;y=b.s.s Fig. 9: x=a.s+b.c;y=b.s.s

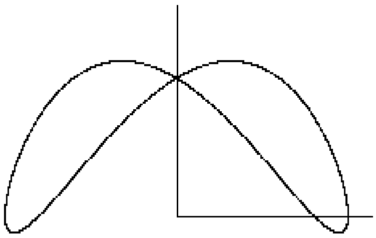
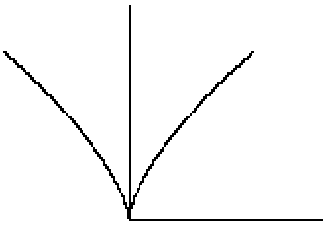
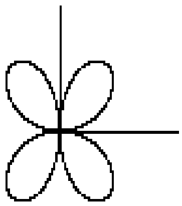
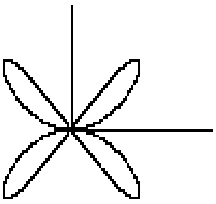
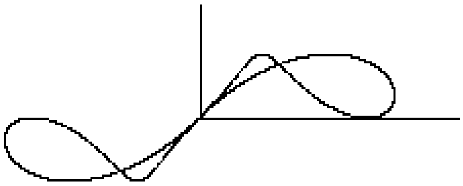




Fig.10: x=a.s-b.c;y=b.s.s+a.c.s Fig.11: x=a.s.s.s;y=b.s.s Fig.12: x=a.c.c.s;y=b.s.s.c





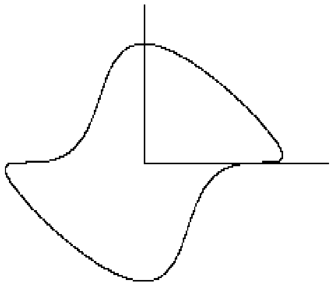


Fig.13:x=a.c.s;y=b.s.s.c Fig.14:x=a.c.c.s+b.c;y=b.s.s.c Fig.15:x=a.c.c.s+b.c;y=b.s.s.s

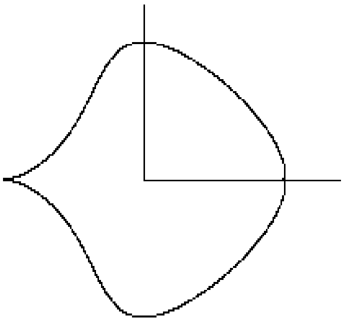


Fig. 16: x=a.c.c.s.s+b.c;y=b.s.s.s

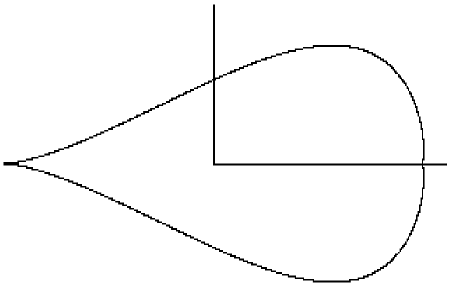


Fig. 17: x=a.c+b.s.s+b.c;y=b.s.s.s

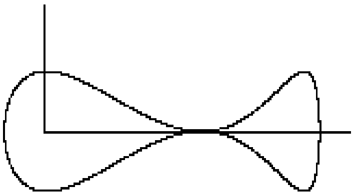


Fig. 18: x=a.c.c+b.s.s+b.c;y=b.s.c.c

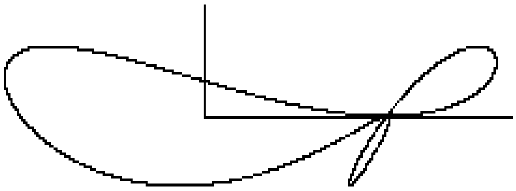


Fig. 19: x=a.c.s+b.s.s+b.c;y=b.s.c.c

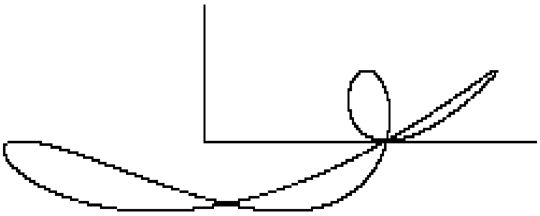


Fig. 20: x=a.c.s+b.s.s+b.c;y=b.s.s.c

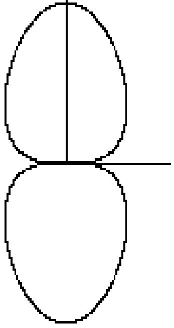


Fig. 21: x=a.c.s;y=b.c.c.c

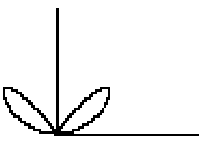


Fig. 22: x=a.s.c.c;y=b.c.s.c.s

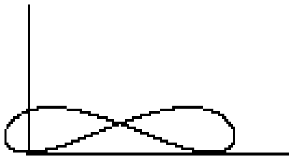


Fig. 23: x=a.s.c+b.c.c;y=b.c.s.c.s

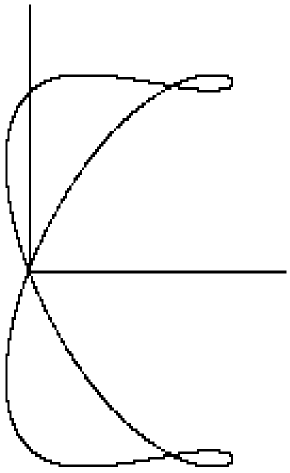


Fig. 24: x=a.s.c+b.c.c;y=b.c+b.s.c.s

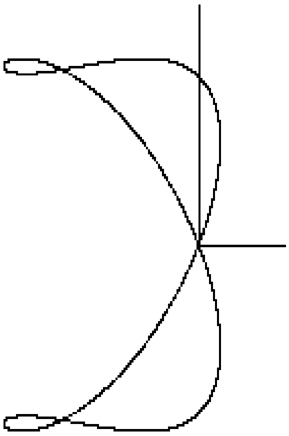


Fig. 25: x=a.s.c-b.c.c;y=b.c+b.s.c.s

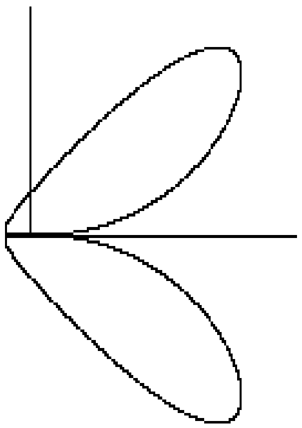


Fig. 26: x=a.s.c+b.c.c;y=b.c-b.s.c.s

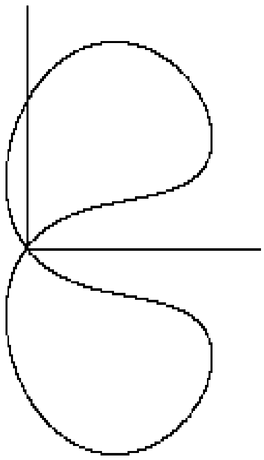


Fig. 27: x=a.s.c+b.c.c;y=b.c-b.c.c.s

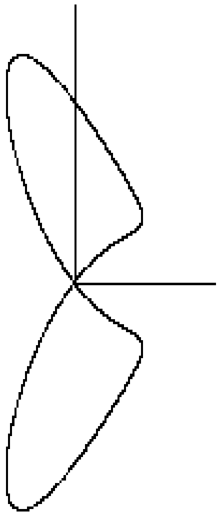


Fig. 28: x=a.s.c;y=b.c-b.c.c.s

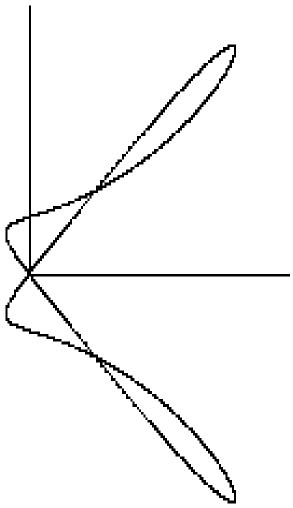


Fig. 29: x=a.s.c+b.c.c;y=b.c+b.c.c.s

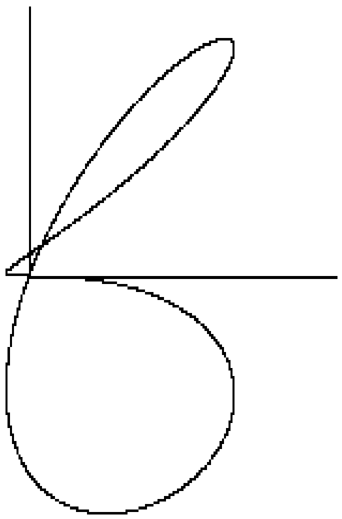


Fig. 30: x=a.s.c+b.c.c;y=b.c+b.c.s

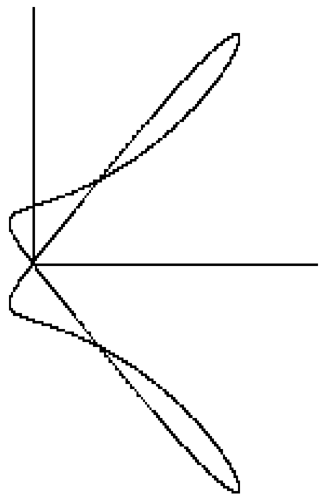


Fig. 31: x=a.s.c+b.c.c;y=b.c+b.c.s.c

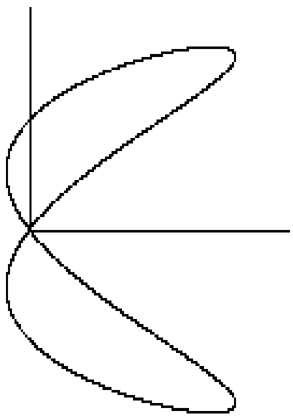


Fig.32: x=a.s.c+b.c.c;y=b.c

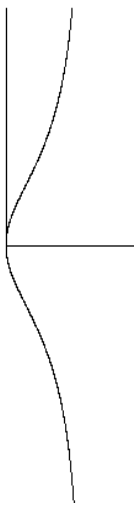


Fig.33: x=a.s.s;y=b.t

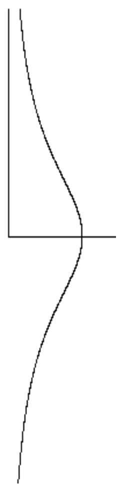


Fig.34:x=a.s.s;y=b/t

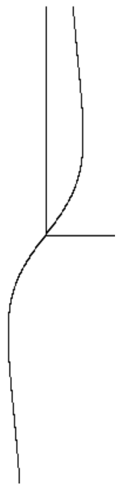


Fig.35: x=a.s.c;y=b.t

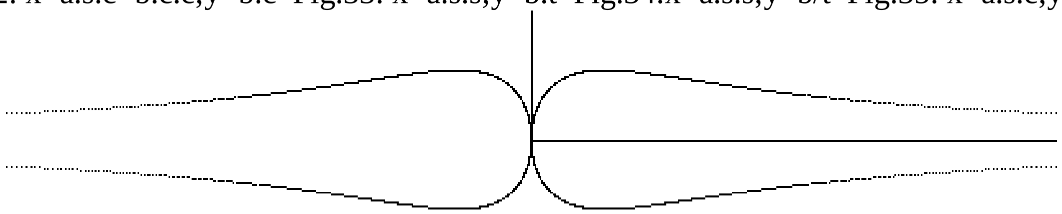


Fig. 36: x=a.s.t;y=b.s.c

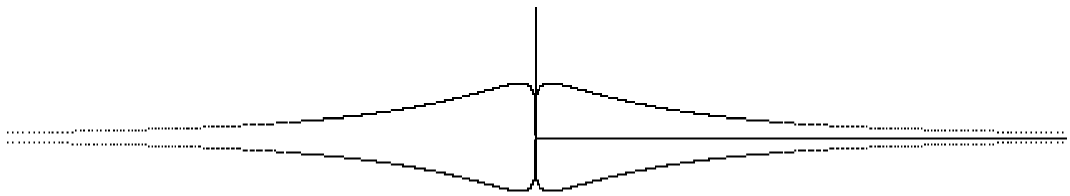


Fig. 37: x=a.c.c.c/t;y=b.s.s.c

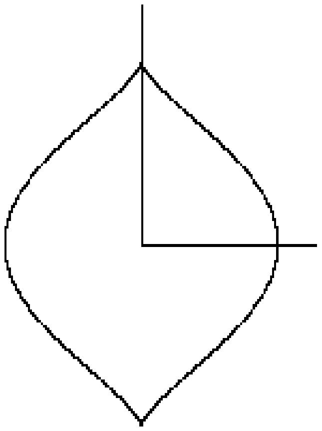


Fig. 38: x=a.s.s.s;y=b.c

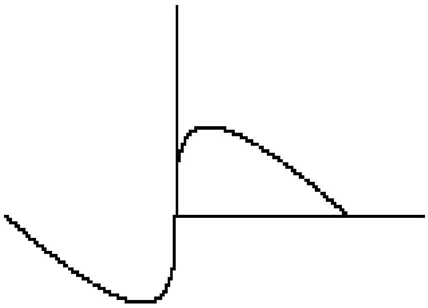


Fig. 39: x=a.s.s.s;y=b.c.s.c

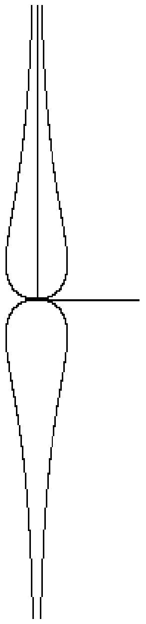


Fig. 40: x=a.c.s.s.s;y=b/t.c

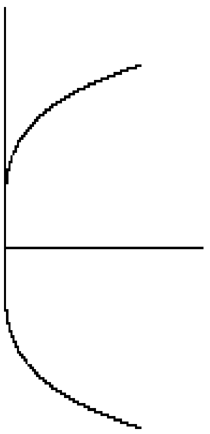


Fig. 41: x=a.c.c.c.c;y=b.c

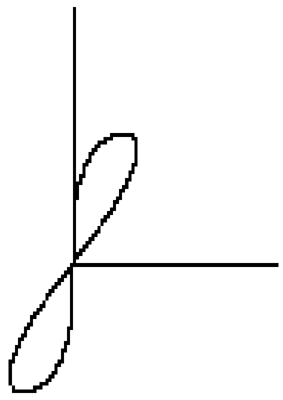


Fig. 42: x=a.c.s.c.c;y=b.c.s

The following values were then chosen a = b = 40 and the following curves were obtained.

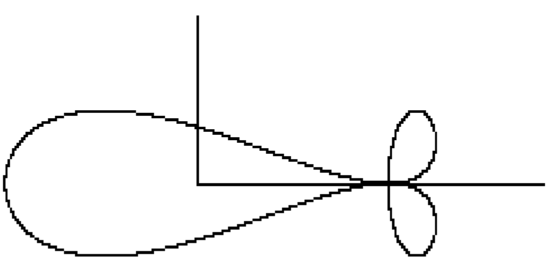


Fig. 43: x=40.s+40.c.c;y=40.c.s.s

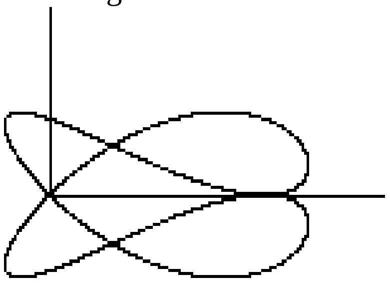


Fig. 44: x=40.s.c+40.c.c;y=40.c.s.s

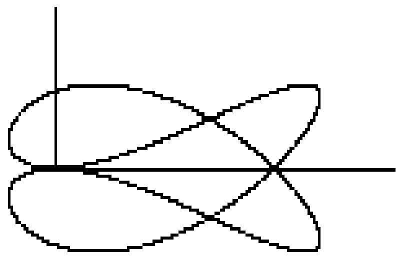


Fig. x=40.s.c+40.c.c;y=40.c.c.s

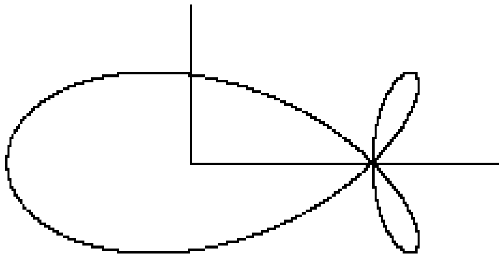


Fig. 45: x=40.s+40.c.c;y=40.c.s

3. Research directions

The research of these curves must solve the following problems:

- studying the curves with the methods of mathematics, i.e. finding the other forms of the equations and the properties of the curves;
- building geometric figures that meet the conditions imposed by the equations of these curves;
- finding equivalent mechanisms based on these geometric constructions and generating curves with mechanisms.

4. Conclusions

The parametric equations of some mathematical curves were obtained and these were plotted using computer programs. Some curves are similar to other known curves, but have not been shown to be curves of the same type, or are different. Many curves are absolutely new. They indicate which research directions are required for these curves, so that they also come within the range of mathematical curves.

References

[1] Artobolevskii, I. I. – Teoria mehanismov dlia vosproizvedenia ploskih krivâh. Izd. Akademii Nauk SSSR, Moskva, 1959.

[2 Ferreol, Robert – Mathcurve .[https://www.mathcurve.com/courbes2d/courbes2d.shtml]

[3] Ionescu, Gh. D. – Teoria diferenţială a curbelor şi suprafeţelor cu aplicaţii tehnice. Editura Dacia, Cluj-Napoca, 1984.

[4] Popescu, Iulian – Curbe de bielă în diferite plane şi locuri geometrice generate de mecanisme. Craiova, Editura Sitech, Craiova, 2017.

[5] Taimina, Daiana - Historical mechanisms for drawing curves. Cornell University, 2017

[6][https://www.researchgate.net/publication/267239483_Historical_Mechanisms_for_Drawing_Curves].