DETERMINING THREE LOCI
BY USING THE CONTOURS' METHOD

Professor Iulian POPESCU, Faculty of Mechanical Engineering, University of Craiova, 107, Calea Bucuresti, 200478, Roumanie, iulianpopescucraiova1@gmail.com
Lecturer PhD Alina Elena ROMANESCU, Faculty of Mechanical Engineering, University of Craiova, 107, Calea Bucuresti, 200478, Roumanie, romanescualinaelena@yahoo.com
Associate Professor PhD Ludmila SASS, Faculty of Mechanical Engineering, University of Craiova, 107, Calea Bucuresti, 200478, Roumanie, ludmila_sass@yahoo.com

Abstract: *The contours method used in the Theory of Mechanisms can be applied easily for problems addressing loci, through the synthesis of equivalent mechanisms and their analysis. The start point consists in a loci problem in which loci of three points have to be determined. Firstly the equivalent mechanism is found and its analysis is performed for different positions, by using the method of projections across axis of the vector contours. Numerous curves obtained for different sizes of the mechanism are provided. They have different shapes and consist in pairs of branches.*

keywords: loci, contours' method at mechanisms.

1. Introduction

The method of vector contours projected along the system's axis, used in the Theory of Mechanisms, makes possible the solving of many problems dealing with loci, which are difficult to solve through the classical geometric methods.

Many problems approaching loci were solved along time by using the above mentioned problems. For example a step by step demonstration in this sense is presented in [1].

Hundreds of mechanisms are presented in [2]. They present interest due to their generation relying on geometrical aspects.

An impressive mathematic library [3] presents details on curves obtained as loci. In certain cases animations are presented as well.

Numerous examples of simple loci are exposed by [4]. They are accompanied by the corresponding geometric figures, values of parameters required for their generation and associated curves. In certain cases the corresponding graphical constructions are also indicted and various cases are analyzed whilst analytical relations are provided for other cases.

Many examples of loci problems are clearly exposed in [5], accompanied by many pictures. Details on problems addressing loci are also included in [6], along with drawings and relations.

Other examples of loci and corresponding animated pictures, are given by [7].

A detailed and well argued paper [8] contains theoretic aspects of loci along with examples, relations and curves.

A basic and very old work dedicated to spatial loci [9] presents the theory and equations for various cases. This paper was republished in [10].

In [11] and [12] loci are considered as starting points for finding equivalent mechanisms which describe them as trajectories (rod curves). A study of a loci problem is presented below. It is solved by using a mechanism with three dyads.

2. The loci problem

In a common triangle ABC (fig. 1), the point A is fix and the side AB rotates around it. The point C slides along a fix side AC of variable length. Constant angles BAD and ADE are built. The loci for F,D and E have to be determined considering that D slides along AD and E slides along BC.

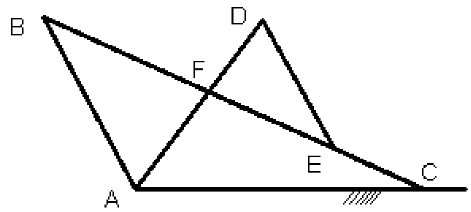


Fig. 1. The proposed problem.

3. The equivalent mechanism

The sides AB and BC have constant values whilst C is moving toward AD. As a consequence, a coulisse is placed in C (fig. 2). AD and BE are variable and therefore coulisses are placed in D and E. As long as the point E is placed in the intersection of AD and BC, two coulisses, able to rotate one around the other, are placed there.

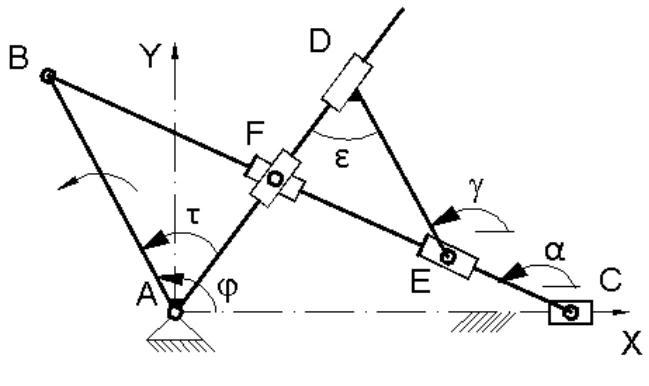


Fig. 2. The equivalent mechanism.

4. Relations

The projections' method yields the following relations:

$$\begin{cases} x_B = ABcos\varphi = x_C + BCcos\alpha \\ y_B = ABsin\varphi = BCsin\alpha \end{cases} \quad (1)$$

$$\begin{cases} x_D = ADcos(\varphi - \tau) = x_C + ECcos\alpha + DEcos\gamma \\ y_D = ADsin(\varphi - \tau) = ECsin\alpha + DEsin\gamma \end{cases} \quad (2)$$

$$\begin{cases} x_F = AFcos(\varphi - \tau) = FCcos\alpha + x_C \\ y_F = AFsin(\varphi - \tau) = FCsin\alpha \end{cases} \quad (3)$$

$$\gamma = \varphi - \tau + \varepsilon \qquad (4)$$

$$EC = \frac{DEsin\gamma - (x_C - DEcos\gamma)tg(\varphi - \tau)}{cos \propto tg(\varphi - \tau) - sin\alpha} \qquad (5)$$

$$AF = \frac{x_C tg\alpha}{\cos(\varphi - \tau)\, tg\alpha - \sin(\varphi - \tau)} \qquad (6)$$

$$\begin{cases} x_E = x_C + ECcos \propto \\ y_E = ECsin \propto \end{cases} \qquad (7)$$

Eqs. (1) allows for the obtaining of x_B and y_B , sin α and x_C being computed afterward. Eqs. (2) are used to obtain the values for AD and EC, x_D and y_D whilst γ can be determined from (4). AF and FC are obtained from (3), and x_F and y_F are evaluated after them. Eq. (5) yields EC and AF is determined by using eq. (6), whilst the coordinates of E are determined from the eqs. (7).

5. Graphical results

Based on the above mentioned draft, the following initial data were considered: AB=40; BC=105; DE=38; $\varphi = 125^0$; $\tau = 70^0$; $\varepsilon = 70^0$. The mechanism from fig. 3 was obtained. It came to certify the relations' correctness.

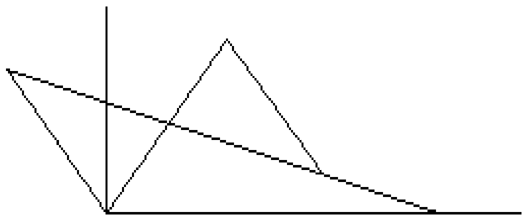


Fig. 3. The initial mechanism.

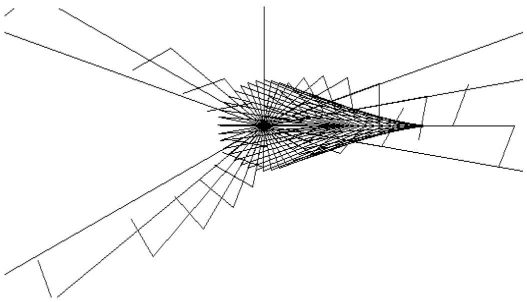


Fig. 4. Mechanism's subsequent positions.

Fig. 4 depicts the mechanism's subsequent positions. One can notice that the kinematic chain denoted by ABC has normal movements (AB describes full rotations). On the other hand, the chin ADE is operational only for few subintervals of the cycle.

Fig. 5 reveals jumps made by the point D during its movement, in two critical points. When AD overlaps with the system's axis, the trajectory of D tends to infinite. In the presented diagram, limits were settled for x_D and y_D in order to make room for the drawing in a limited frame.

For a real mechanism, the passing through these critical points involves inertia. In order to find loci, there is no need for building the real mechanism. In the above the mechanism is used only to facilitate the calculations.

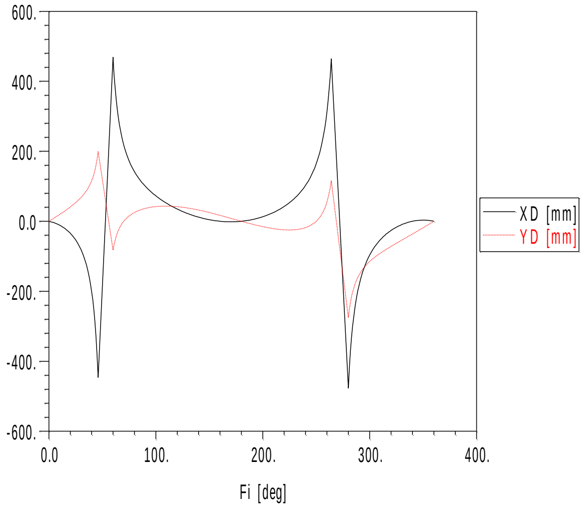


Fig. 5. Coordinates of the point D.

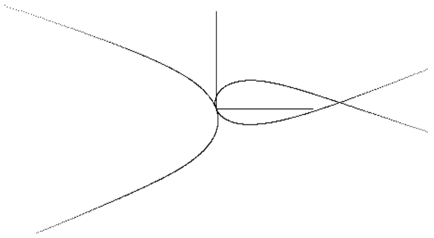


Fig. 6. Loci of the point D.

Fig. 6 depicts the loci of D. It is represented by a curve with two different branches, tangent in the system's origin.

Fig. 7 depicts the loci corresponding to the point E, which is a branch of the curve similar to that presented above for the point D. The other one is different. The loci of point F (fig. 8) has two branches, which are similar but more narrow.

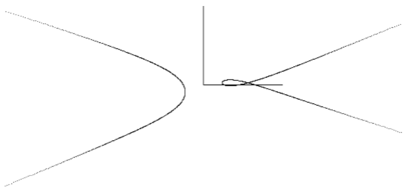


Fig. 7. Loci of the point E.

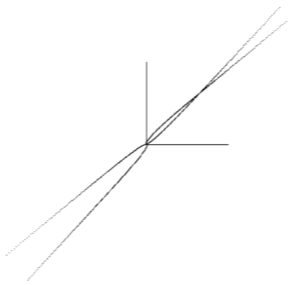
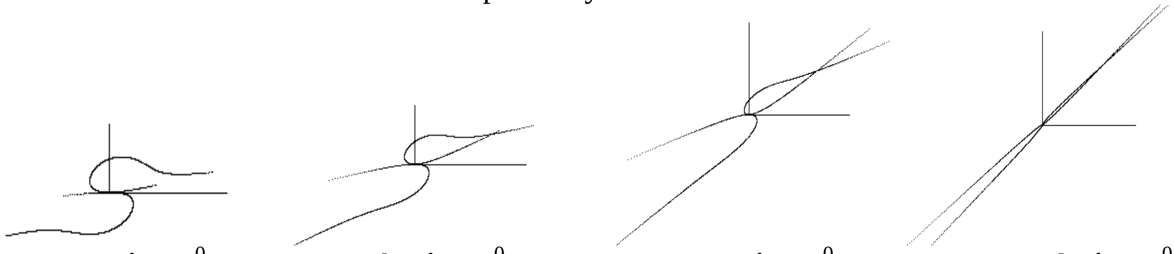


Fig. 8. Loci of the point F.

6. Loci obtained after resizing

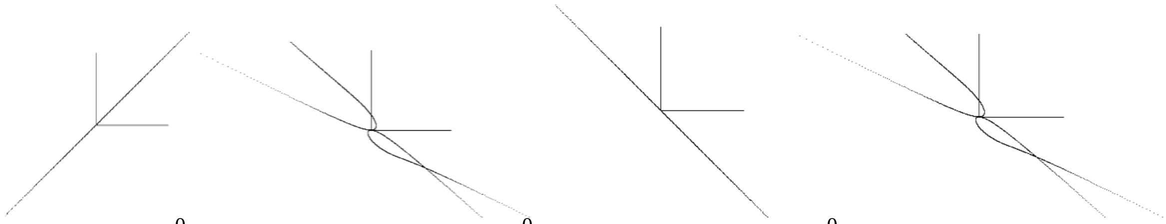
Different loci are obtained when certain sizes are modified.

Figs. 9 and 10 depict loci for the point F, obtained after the modification of angle λ. Circles are obtained for $\lambda=0^0$ and respectively $\lambda=180^0$.



a. $\lambda=10^0$ b. $\lambda=20^0$ c. $\lambda=35^0$ d. $\lambda=80^0$

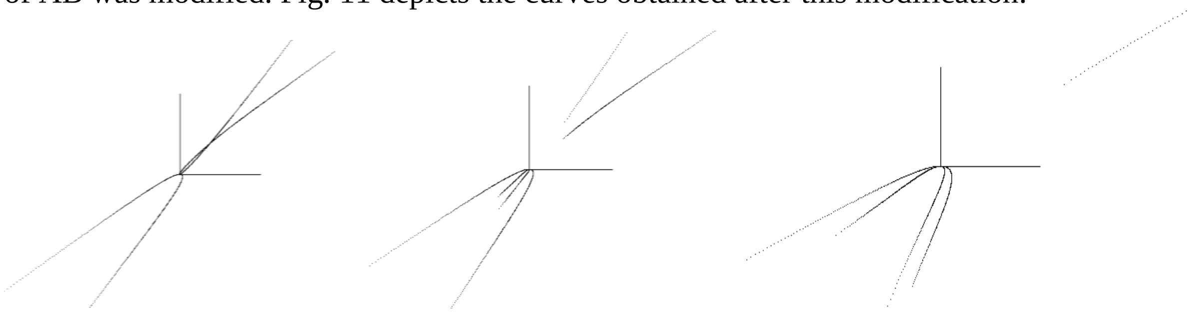
Fig. 9. Loci of the point F.



a. $\lambda = 90^0$ b. $\lambda = 220^0$ c. $\lambda = 270^0$ d. $\lambda = 320^0$

Fig. 10. Loci of the point F.

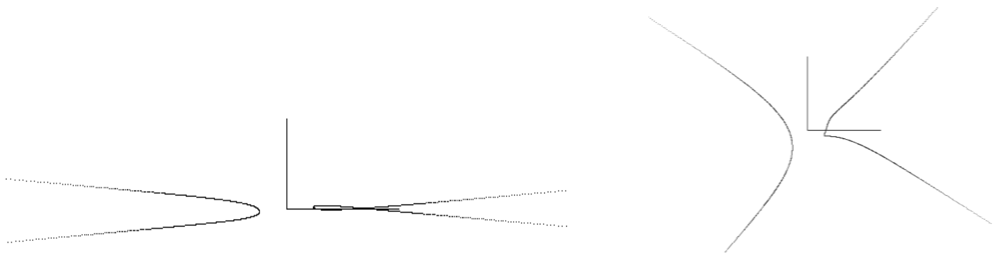
Interesting curves are obtained for small values of λ. When this parameter is increased, the branches become longer and shapes similar to those presented below are obtained. Straight lines with different slopes are obtained for $\lambda = 90^0$ and $\lambda = 270^0$. Further on, the size of AB was modified. Fig. 11 depicts the curves obtained after this modification.



a. AB=90 b. AB=120 c. AB=200

Fig. 11. Curves obtained as loci of F when AB was modified

The curves obtained as loci of the point F are different from those mentioned above with respect to their positions. Fig. 12 depicts the loci associated to the point E, when AB was modified



a. AB=10 b. AB=90

c. AB=130 d. AB=200

Fig. 12. Curves obtained as loci of E, when AB was modified.

Totally different curves were obtained this time.

One can notice that the yielded loci are different from one point to another, also being influenced by the modification of certain sizes of the equivalent mechanism. Many other curves can be obtained as loci by modifying other sized.

7. Conclusions

It was proved that loci can be obtained starting from complex problems and using the contours' method from the Theory of Mechanisms.

The synthesis of the equivalent mechanism leads to a problem of mechanism's synthesis.

The relations yielded by the contours' method made possible the plot of curves corresponding to loci for three points.

It must be mentioned that the solving of a geometric problem eliminates the necessity to build mechanisms, unless a real case requires this (e.g. the building of a toy). Otherwise the mechanism is used only as an artifice in order to find loci.

The loci presented in this paper are complicated, open curves, including two branches whose shapes are affected by the sizes of the mechanism's elements.

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