

# MATERIAL SELECTION WITH A STATISTICAL METHOD

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**Abstract.** *The paper presents an overview of the methods in the materials selection, offers some statistical heuristics in the decision process and a few ideas about a statistical approach for engineering applications, with some solved numerical examples. The complexity of the decisions in the present engineering problems imposes the use of new techniques, much more computer assisted, based on mathematics and psychology. A preliminary structuring of materials can be offered by cluster analysis: the agglomerative hierarchy gives a starting point in the classification practice. Further, a graphical selection method, based on eigenvalues and -vectors, was detailed and exemplified. Principal Component Analysis (PCA), a standard technique to reduce multivariate data sets to lower dimensions, was used in the selection of materials, with the possibility of simplification and implantation on a computer. The multiple-criteria decision-making (MCDM) or multiple-criteria decision analysis (MADM) , incorporating analytic hierarchy process (AHP), cover the high expansion of materials in the field of material classification.*

**Keywords:** multiple-criteria decision-making, materials

## 1. Introduction

The multi criteria decision making applications have developed modelling very different area like business, industry, healthcare, and education from the introduction by Thomas L. Saaty in the 1970s. „MCDM is the composition of set of multiple criteria, set of alternatives and their comparison in some manner“[10]. “Based on the generic elements of MCDM, one can distinguish two contrasts: individual versus group decision-making and decisions under certainty versus decision under random (e.g. probabilistic, fuzzy)”[1]. The design process starts considering for each material as many attributes as possible, while next it is taken only a few "virtual" attributes (usually two or three), given by principal component analysis [11]. The properties of materials can be metrical, ordinal and scalar. For the last two types it is necessary to have quantification. A scale with marks from 1 to 5, where 1 is the best, is a possible order. Zhu and Xu, proposed a 1-9 scale, or alternatively a 0.1-0.9 scale [14].

## 2. Materials Classification Methods

### 2.1. Cluster Analysis

A possible preliminary arrange of materials data is the grouping of values using cluster analysis: the objects in the same group (cluster) are more similar to one another. The result of a hierarchical clustering calculation is the dendrogram, a tree-structured graph (fig.1): here is plotted to visualize the likeness and difference between nine materials for sliding bearings [5].

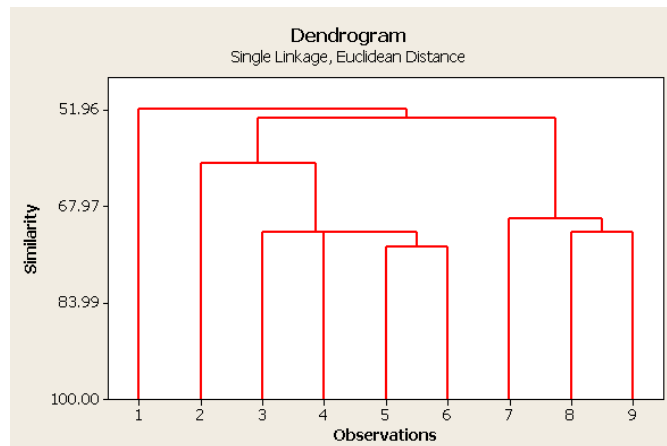


Figure 1. The dendrogram of objects [4]

The dendrogram points out that the nearest objects (materials) are the ones notated 5 and 6, and at the next level are 3 si 4, connected with class {5;6}. If the number of the objects is very high it is useful to apply the cluster methods to form classes of related objects. Since the class is fixed, its ranking will continue with improved methods.

## 2.2 Preference Index Value

The objective of this method is to find the best selection on the basis of statistical and mathematical knowledge of MDCM, namely the computation of preference index [9].

An important step of the algorithm is to determine objective weights of the attributes. The objective weight for an attribute is computed by dividing the variance value of the attribute with the total value of variance of all attributes. The most important step of the applied methodology is the determination of the preference indices of materials, which should be obtained by multiplying the normalized data matrix by the transposed weights of the attributes matrix [7].

The correlation coefficient between Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and primary ranking with only two properties is important for any simplification.

## 2.3 A PCA Application

PCA proposes new factors, but artificial ones, and the subspace contains the minimum deformation in comparison with the original cloud. Based on eigenvectors of the attribute variables, material properties are related on a few artificial factors. The variance of the original cloud of analysed materials is maximal in a new representation; the Jöreskog' technique is used for the dimensional reduction in a bivariate subspace for a family of materials [13]. To apply the Pearson method of PCA [5, 12] , for the same family of materials, the data were processed with XLSTAT 2009 software [3]; new relevant factors are obtained, and the results are comparable [2].

## 2.4 A Bivariate Statistical Selection of Materials

In the real conditions some properties may be important, and it is interesting to analyse the variation of the properties in time, at the changes of temperature, etc. [8].

A frequent situation is when are given limits of some independent properties: it may be considered a random vector, which takes values within a “rectangle”, with the sides parallel to the axes. For example, in the bivariate case, the probability density function is:

$$f(x, y) = \frac{\rho^2}{\pi E_1 E_2} \exp[-\rho^2 (\frac{x^2}{E_1^2} + \frac{y^2}{E_2^2})],$$

$$\text{where } E_1 = \rho \sigma_1 \sqrt{2}, E_2 = \rho \sigma_2 \sqrt{2}.$$

Then the probability of the random vector (X,Y) to take the values in the area:

$$D = \{(x, y), \alpha \leq x \leq \beta, \gamma \leq y \leq \delta\}$$

is given by the equation:

$$P[(x, y) \in D] = \frac{1}{4} [\Phi(\frac{\beta}{E_1}) - \Phi(\frac{\alpha}{E_1})] [\Phi(\frac{\delta}{E_2}) - \Phi(\frac{\gamma}{E_2})],$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution.

In the particular case, when the components are independent, and

$$\alpha = -l_1, \beta = l_1, \gamma = -l_2, \delta = l_2, \text{ the formula becomes:}$$

$$P[(x, y) \in D] = \Phi(\frac{l_1}{E_1}) \Phi(\frac{l_2}{E_2}).$$

In the general case the joint probability density function is:

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \exp\left\{-\frac{1}{2(1-r^2)} \left[ \frac{(x-m_1)^2}{\sigma_1^2} - 2r \frac{(x-m_1)(x-m_2)}{\sigma_1\sigma_2} + \frac{(y-m_2)^2}{\sigma_2^2} \right]\right\}$$

This function keeps a constant value on the ellipses, i. e.:

$$\frac{(x-m_1)^2}{\sigma_1^2} - 2r \frac{(x-m_1)(x-m_2)}{\sigma_1\sigma_2} + \frac{(y-m_2)^2}{\sigma_2^2} = k^2,$$

where  $k$ =constant, and those ellipses are of equal probability.

The probability  $P(k)$ , so that the random vector (X,Y) takes values in the ellipses area, is given by:

$$P(k) = 1 - \exp\left(-\frac{k^2}{2(1-r^2)}\right).$$

Numerical values for  $k$  should be determined given values for  $P(k)$ . In this way it results statistical domains of ellipses, so that contains materials having the values of characteristics in imposed area, for a given confidence level. As example in the fig.2 are plotted the probability ellipses and materials [6], in the case of mechanical characteristics, breaking strength and resilience, for a few types of steels, taking values in the area bounded by the ellipses.

In this case the equation of ellipses, reported to the eigenvectors:  $v=(-4; 7.23)$ ;  $w=(4; 2.3)$ , is:

$$\frac{v^2}{7.33 * 10^3} + \frac{w^2}{2.1 * 10^{-4}} = k^2$$

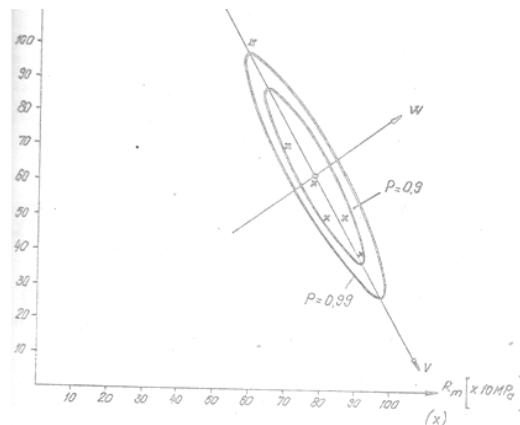


Figure 2 Plot of ellipses[6]

The statistical softwares (XLSTAT, SPSS, MINITAB, etc) are a useful tool to facilitate the selection of materials [2, 3].

### 3.CONCLUSION

Materials selection methods have been in progress for more than 20 years. The presented methods, applied by the authors, simplify the materials design, with other possible extensions in the design process. Revising the up-to-date problems of the materials selection, it was emphasized on the statistical approach.

MCDM analysis offers a lot of solutions for materials selection using different attributes such as the presence of multiple non-commensurable criteria, and the presence of quite different alternatives. The paper reports related elements of TOPSIS and Preference Index Value for classification of a variety of engineering materials and, as an alternative possibility, the use of PCA and the Jöreskog' technique.

Also the paper describes a bivariate analysis using probability distribution function for mechanical characteristics and the relationship between them, in association with some factors and their interactions, ensemble which can be modelled by the joint distribution of multiple random variables and graphics, with the plotted ellipses for different probabilities. The problem can be extended in the multidimensional space.

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