A COPULA APPLICATION FOR MECHANICAL PROPERTIES

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Abstract: *Based on copula applications, the work points out the use of cumulative distribution function for the characteristics bivariate cases and the connections among them. The analyzed variables, Brinell Hardness and Tensile Strength, are joined in a cumulative distribution function (CDF) to present and forsee the output variable, the copula function. A method using the bivariate density and cumulative distribution function with the Clayton copula, and Gamma distribution is herein analyzed. The dependence between two important mechanical properties is studied and assed.*

KEYWORDS: mechanical properties, dependence, copula

1. Introduction

"In high-dimensional statistical applications copulas allow to easily model and estimate the distribution of random vectors by estimating marginal of explanatory variables as well as their dependence" (Paris et al., 2015).Copula requires only marginal CDFs and correlation parameter τ in order to approximate the joint bivariate modeling. The proposed statistical model has many applications for technical and economical prognoses and schedules. Correlation coefficient used to measure dependence and copulas are an very important tool in modelling the non-linear dependence structures. In decision support, properly accounting and modeling these dependencies and correlations are essential in deriving reliable valuations. It has proven, the copulas technique is a superior tool for modeling dependence structures (Nelson, 2006), (Najjari et al., 2013).The copula function offers new opportunities for advanced engineering design and can model correlated structures; in other words can describe time varying and nonlinear features of statistical dependence of marginal distributions (Paris and Târcolea, 2016). This methodology was applied to study the dependence between two mechanical properties.

Chen analysed (Chen, 1994) a bivariate process example, based on data (Sultan, 1986), with the characteristics the Brinell Hardness (H) and Tensile Strength (S). New materials rendered possible an advance engineering design, but in view of enabling a rational selection, based only on a few characteristics (Paris and Târcolea, 2014). A sample of 25 paired data, with these two mechanical properties, is given in the table 1.

Table 1: Paired values of (H) and (S) of output of the bivariate process (Sultan, 1986)

H	143	200	160	181	148	178	162	215	161	141	175	187	187
S	34.2	57.0	47.5	53.4	47.8	51.5	45.9	59.1	48.4	47.3	57.3	58.5	58.2
H	186	172	182	177	204	178	196	160	183	179	194	181	
S	57.0	49.4	57.2	50.6	55.1	50.9	57.9	45.5	53.9	51.2	57.5	55.6	

Chen proposed "a multivariate process capability index over a general tolerance domain, a generalization of the rectangular and ellipsoidal area" (Chen, 1994).

The study applied the principal component analysis to obtain the process capability index. It is resulted "that the first principal component can be used to evaluate the capability of the two dimensional process because it is explained 97% from the total variability" (Wang and Chen, 1998-1999).

Pan and Lee (Pan and Lee, 2009) proposed the indices NMCp and NMCpm, using the dependence between quality characteristics.

CASE STUDY

Essentially, it make use of the distribution function Gamma for the components H, S (table 2) and the correlation coefficient τ-Kendall of paired data, to determine the values of the two-dimensional distribution function, using Clayton type copula, while the data cloud is adequate to the this one (fig.1).

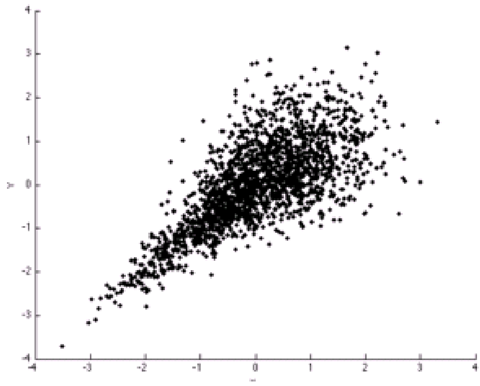

Figure 1. Clayton copula with its distinctive tail and expanding cloud (Web-1)

Table 2 The parameters of the Gamma distribution of components

	Parameter	Estimated Value	Standard Deviation
H	Shape	106.454660410788	30.6744977408683
	Rate	0.595968100695313	0.172129642142312
S	Shape	139.272604792443	40.1521537150371
	Rate	2.62428232133273	0.757937677194376

The *goodness-of-fit* test (Kolmogorov–Smirnov test) was applied to compare of the empirical values and the theoretical Gamma ones (table 3):

For H: because $D=0.13<0.27=$ critical value, the null hypothesis is accepted at the level $\alpha=0.05$;

For S: because $D=0.17<0.27=$ critical value, the null hypothesis is accepted at the level $\alpha=0.05$.

It results that the data follows respectively, a specified Gamma distribution with two parameters. The correlation coefficient τ-Kendall of paired data was calculated as 0.7057; therefore the initial value of the Clayton copula parameter Θ_0 is 4.7958.

Table 3 The values of the cumulative distribution functions

F(G,H)	F(G,S)	C(V, W, Θ =1.65)
0.896974	0.811729	0.750048
0.149742	0.10538	0.081255
0.584526	0.541856	0.420648
0.03572	0.119029	0.033114
0.516323	0.374373	0.30655
0.180407	0.050636	0.047387
0.979745	0.907218	0.891508
0.164644	0.14975	0.104464
0.011648	0.096903	0.011443
0.446866	0.828365	0.42261
0.70986	0.884795	0.660193
0.70986	0.872175	0.654342
0.690371	0.811729	0.611104
0.378242	0.211026	0.179582
0.606644	0.822933	0.552652
0.493209	0.299629	0.255281
0.930196	0.683414	0.656825
0.516323	0.323905	0.275308
0.852891	0.858577	0.756052
0.149742	0.041198	0.038594
0.628344	0.585263	0.462201
0.539296	0.348854	0.295996
0.826399	0.838887	0.72345
0.584526	0.720817	0.499682

Starting with this value Θ_0 it was approximates an optimal value for Θ with the maximum likelihood method (a plot of data in figure 2), for the Clayton density, given by the formula:

$$c(u,v;\Theta) = (\theta+1) * (u*v)^{-1-\theta} *(u^{-\theta}+v^{-\theta}-1)\hat{}(-\frac{2*\theta+1}{\theta}) \quad (1)$$

From the figure 3 it was determinate Θ ≈ 1.65 and with the copula Clayton,

$$C(u,v) = (u^{-\theta}+v^{-\theta}-1)^{-1/\theta}, \quad \theta > 0 \quad (2)$$

it obtained the values of the bivariate distribution function of the vector (H,S): F(H,S, Θ=1.65) (third column in the table 3) (fig.2)..

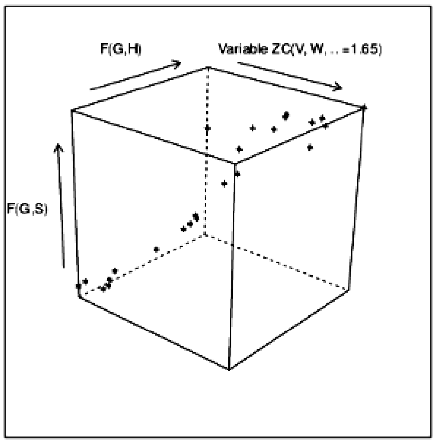

Figure 2 3D scatter plot of the data from table 3

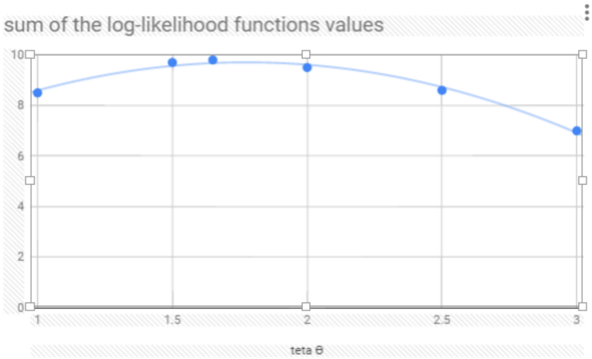

Figure 3. The plot of the sum of the log-likelihood function values with respect to ϴ

Figure3 presents the plot of the sum of the log-likelihood function values with respect to ϴ, used for the graphical estimate. A very accessible application for this is Google spreadsheet, a freeware software, available even in Romanian language as "foi de calcul" , especially adapted for smart phones. An easy fitting solution here is a second degree function, which gives a close solution for ϴ, with a difference less than 10% from the graphical estimate.

With the Kolmogorov–Smirnov test was compared the difference values between F(H,S, ϴ=1.65) and theoretical Gamma function: because the maximal difference D_{max} = 0.16<0.27 = critical value, the null hypothesis is accepted at the level α=0.05 and the data follow the specified Gamma distribution with two parameters too.

CONCLUSION

The bivariate cumulative Gamma distribution associates two controlled mechanical properties, hardness Brinell and tensile strength with the Clayton copula. The resulted Gamma joint distribution was calculated based on Gamma univariate marginal distribution functions of the components and the dependence structure between the initial variables (Paris et al., 2016). Copula requires only marginal CDFs and correlation parameters in order to approximate the joint outcome variable. A big variety of copula end many distributions can offer important advantages, coupled with simple statistical software, for the mechanical experimental field.

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